# Collective Transport in Charge and Spin Density Waves

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## ABSTRACT

We review the recent theoretical study of the Fröhlich conduction on charge density wave (CDW) and spin density wave (SDW). We limit ourselves to the threshold electric field and the electromechanical effect in these systems.

## 1. INTRODUCTION

It is well-established that CDW and SDW are described by quasi-two dimensional Fröhlich (Hubbard) model  $[1 \sim 4]$ . The mean field ground state in these models is the CDW (SDW). An essential ingredient is the imperfect nesting, which is controlled by pressure or by magnetic field. For example, such a model describes quite well the observed electron tunneling densities of two CDWs in  $NbSe_3$  [5].

Further, the same model describes the cascade of field induced spin density waves observed in  $(TMTSF)_2ClO_4$  and  $(TMTSF)_2PF_6$  under pressure  $[6 \sim 8]$ . Here again the imperfect nesting is crucial.

## 2. Phason Dynamics

The effective Hamiltonian density which describes the phason dynamics (e.g. the sliding motion of CDW and SDW) is obtained from the microsconic theory and written as [9]

$$H = \frac{1}{4} N_o f \left[ \frac{m^*}{m} \left( \frac{\partial \phi}{\partial t} \right)^2 + v_1^2 \left( \frac{\partial \phi}{\partial x} \right)^2 + v_2^2 \left( \frac{\partial \phi}{\partial y} \right)^2 + v_3^2 \left( \frac{\partial \phi}{\partial z} \right)^2 \right]$$
$$-enfQ^{-1} \phi E + V_{pin} (\phi) \tag{1}$$

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where

$$V_{pin}(\phi) = -2N_o\lambda^{-1}\Delta(T)V\sum_i \cos[\vec{Q}\cdot\vec{x}+\phi(\vec{x})]\delta(x-x_i).$$
<sup>(2)</sup>

and

$$V_{pin}(\phi) = -\left[\pi/2 \ N_o V\right]^2 \ \Delta(T) \quad tanh\left[\Delta(T)/2T\right]$$
$$\times \sum_i cos\left[2\left(\vec{Q} \cdot \vec{x} + \phi(\vec{x})\right)\right] \delta(x - x_i). \tag{3}$$

for a CDW and a SDW respectively. Further for a SDW, we have m \* /m = 1.

The electric charge and the current associated with the spatio- temporal variation of  $\phi$  is given by

$$\rho = -enf \ Q^{-1} \partial \phi / \partial x \tag{4}$$

and

$$J = enf \ Q^{-1} \ \partial \phi / \partial t \tag{5}$$

which satisfy the CDW (or SDW) charge conservation.

$$\frac{\partial \rho}{\partial t} + \frac{\partial J}{\partial x} = 0 \tag{6}$$

Indeed, the electric charge carried by CDW or SDW can be converted to that carried by quasi-particles only through topological defects like phase vortices (or dislocation) [10]. The function f here depends in general on  $\omega$  and  $\varsigma = vq$ , where  $\omega$  and q are the frequency and the wave vector associated with  $\phi$ . However, in the adiabatic limit f takes simple forms

$$f = \begin{cases} f_0 & \text{for } \omega > vq \\ f_1 & \text{for } \omega < vq \end{cases}$$
(7)

where  $f_o$  and  $f_1$  are called the dynamic and the static limit of the condensate density. In particular, in the limit of the perfect nesting the temperature dependence of  $f_1$  is the same as that of the superfluid density in a BCS superconductor. For the dc conductivity including the nonohmic regime  $f = f_1$  should be used unless E is extremely large, since  $\phi$  involved is dominated by spatial distortion. On the other hand, in the microwave conductivity  $f = f_o$  has to be used.

# 3. Threshold Electric Field

Perhaps, the most direct application of the phase Hamiltonian Eq (1) is the calculation of the threshold electric field  $E_T$  [11]. We follow the analysis by Fukuyama and Lee [12] and Lee and Rice [13] on a phenomenological model, since the structure of Eq(1) is the same as in the phenomenological model.

First, let us consider a CDW. It is important to distinguish two limiting cases; the strong pinning limit and the weak pinning limit. In the strong pinning limit a single impurity can pin down the local phase of the CDW. Then the threshold field is given as sum of individual contributions [11].

$$E_{T}^{s}(o) = 2Q/e\lambda \ (n_{i}/n)N_{o}V\Delta(o)$$
(8)

and

$$E_T^s(T)/E_T^s(o) = \left[\Delta(T)/\Delta(o)\right] \left[\rho/\rho_s(T)\right]$$
(9)

where  $n_i$  is the impurity concentration and we made use of the relation

$$f_1(T) = \rho_{\bullet}(T)/\rho \tag{10}$$

The present expression describes the sharp divergence of  $E_T$  at  $T = T_c$  observed in a number of CDWs [14] but cannot describe the increase of  $E_T$  at low temperatures. The latter effect is certainly due to the thermal fluctuation of  $\phi$  or the Debye-Waller factor of  $\phi$  and the inclusion of this effect gives [15]

$$E_T^S(T)/E_T^S(o) = e^{-T/T_o} \left[ \Delta(T)/\Delta(o) \right] \left[ \rho/\rho_s(T) \right]$$
(11)

where the anisotropic Fröhlich model gives

$$T_o = 2t_b t_c / \Delta(o) \tag{12}$$

where  $t_b, t_c$  are the electron hopping integrals in the transverse directions.

In the weak pinning limit, only a collection of impurities can pin the phase and we have [11]

$$E_T^{W}(o) \propto n_i^{2/4-D} \left[\Delta(o)\right]^{4/4-D}$$
(13)

and

$$E_T^W(T)/E_T^W(o) = \left[E_T^S(T)/E_T^S(o)\right]^{4/4-D}$$
(14)

where D is the spatial dimension of the CDW. We have shown already [11] that Eq(14) with D = 2 gives an excellent description of  $E_T(T)$  of the first CDW while Eq(12) does

that of the second CDW of  $NbSe_3$  determined by Fleming [16]. More generally, all of data collected by Monceau [14] can be fitted either by Eq(11) or Eq(14) except those for orthorhombic  $TaS_3$  and blue bronze. Though the origin of these discrepancies are not clear at the present moment, the change in Q towards the commensurable value at low temperatures may have a role in this.

Also, we don't understand why D = 2 should be appropriate. But we have other instances where D = 2 appears to be more appropriate. For example, for D = 2, the asymptotic values of the sliding velocity at  $E \to \infty$  and  $E \to E_T$  should be constant and  $(E - E_T)$  respectively in contrast to the result by Sneddon et al [17] and Fisher [18]. Indeed, the D = 2 result of the deformable CDW is consistent with Bardeen's tunneling expression [19] and therefore describes much better non-ohmic conductivity observed in CDWs. Finally, the electron density of states observed by tunneling technique [20] in CDWs of  $NbSe_3$  is more consistent with the 2 D model [5].

Now let us consider a SDW. A similar analysis gives in the strong pinning limit [11]

$$E_T^{S}(o) = Q/e (n_i/n) (\pi N_o V)^2 \Delta(o)$$
(15)

and

$$E_T^{s}(T)/E_T^{s}(o) = \left(\Delta(T)/\Delta(o)\right) tanh\left(\Delta(T)/2T\right)\left(\rho/\rho_{\bullet}(T)\right)$$
(16)

Unlike the case for a CDW,  $E_T$  increases monotonically to the value at  $T = T_c$ , since for Bechgaard salts the effect of the thermal fluctuation is completely negligible. Similarly, in the weak pinning limit, we obtain

$$E_T^W(o) \propto n_i^{2/4-D} \left[\Delta(o)\right]^{4/4-D}$$
(17)

and

$$E_T^W(T)/E_T^W(o) = \left(E_T^S(T)/E_T^S(o)\right)^{4/4-D}$$
(18)

The predicted temperature dependences of  $E_T$  are shown in Fig. 1 for the strong pinning limit and the weak pinning limit with D=2 and 3. Recently, the non-ohmic conductivity in SDWs of  $(TMTSF)_2NO_3$ ,  $(TMTSF)_2PF_6$  and in quenched  $-(TMTSF)_2ClO_4$ are observed by Tomić et al [21, 22] and Sambongi et al [23]. Both the observed value of  $E_T$  and the temperature dependence of  $E_T$  are qualitatively consistent with Eqs(15)  $\sim$  (18). However, the  $(TMTSF)_2PF_6$  with a clumped contact [21] exhibits a rather unexpected temperature dependence of  $E_T$ . Perhaps, the commensurability potential may have some role here though the SDW in  $(TMTSF)_2PF_6$  appears to be incommensurate in the transverse direction. If  $E_T(T)$  is due to the commensurability potential we would have



Figure 1. The temperature dependence of the threshold field is shown for the strong coupling limit (s), for the weak coupling limit with  $D = 2(W_2)$  and D = $3(W_3)$  and when the pinning is entirely due to the commensurability potential (C).

$$E_T^C(T)/E_T^C(o) = \left(\Delta(T)/\Delta(o)\right)^4 \left(\rho/\rho_*(T)\right)$$
(19)

Further  $E_T$  at T =0K, should be about an order of magnitude smaller than that due to impurities when the impurity concentration of a few ppm.

The observed  $E_T(T_c)/E_T(o)$  in  $(TMTSF)_2 PF_6$  with the normal contact [22] is ~2.5, which lies in between our weak pinning D=3 (3,13) and D=2 (1.77) values. However, the discrepancy is most likely due to the commensurability effect or due to  $\varepsilon_o$  (unnesting parameter, which we have so far neglected). The pressure dependence of  $E_T(T)$  (which controls  $\varepsilon_o$  of  $E_T(T)$  should be crucial to decide the origin of this discrepancy.

### 4. Electromechanical Effects

The softening of the elastic constants when the CDW (SDW) is depinned is another signature of the Fröhlich conduction first established experimentally by Brill et al [24] and by Mozurkewich et al [25]. We can interpret the electromechanical effect as follows [26, 27]. The ionic potential is screened by the conduction electrons in a normal metal. In a CDW (SDW), on the other hand, the potential is screened by the quasi-paticles and the phasons. However, when the CDW (SDW) is pinned, the phason cannot participate in the screening, while the quasi particle density decreases as the temperature decreases. Therefore, in the limit  $\omega << Dq^2$  where  $\omega$  and q are frequency and wave vector of the sound wave and D is the diffusion constant, the elastic constant (and the sound velocity) increases in a CDW (SDW) like

$$C = C_0 \left( 1 - \lambda (1 - f_1) \right)$$
<sup>(20)</sup>

when the CDW (SDW) is pinned where  $C_0$  is the bare sound velocity. For example, Eq(20) describes quite well the temperature dependence of the sound velocity in a SDW of  $(TMTSF)_2 PF_6$  observed by Chaikin et al [28]. Now, when the CDW (SDW) is depinned by application of the electric field, the phason contributes screening. Then in the high field limit C recovers  $C_N$  the sound velocity in the normal state, since the sum of two contributions is the same as the screening in the normal state. Indeed, this limiting behavior has been predicted earlier by Nakane and Takada [29], who neglected the pinning effect completely.

So far, we considered the limit  $\omega \ll Dq^2$ . On the other limit  $\omega \gg Dq^2$ , the situation is somewhat different. In this limit, the quasi particle contribution is of little importance to the screening of the ionic potential. Therefore, nothing happens to the elastic constant at the transition temperature if the CDW (SDW) is pinned. However, when the CDW (SDW) is depinned, the phasons still contribute in the screening at low temperatures. Therefore, the elastic constant decreases with electric field at low temperatures, we have

$$C_{pin} - C_{depin} = \lambda C_o \tag{21}$$

independent of  $\omega/Dq^2$ . This will provide a useful means to determine the electron-phonon coupling constant  $\lambda$ .

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### References

- 1. K. Yamaji, J. Phys. Soc. Jpn 51 2787 (1982); 52 1361 (1983)
- 2. Y. Hasegawa and H. Fukuyama, J. Phys. Soc. Jpn <u>55</u> 3979 (1986)
- 3. G. Montambaux, Phys. Rev. B 39 885 (1989)
- 4. K. Maki and A. Virosztek, Phys. Rev. B (in press)
- 5. X.-Z. Huang and K. Maki, Phys. Rev. B <u>40</u> 2575 (1989)
- 6. K. Yamaji, J. Phys. Soc. Jpn 54 1034 (1985) Synth Met 13 29 (1986)
- D. Poilblanc, M. Héritier, G. Montambaux, and P. Lederer, J. Phys. C 19 L 321 (1986)

- 8. A. Virosztek, L. Chen, and K. Maki, Phys. Rev. B 34 3371 (1986)
- 9. A. Virosztek and K. Maki, Phys. Rev. B 37 2028 (1988)
- 10. N. P. Ong and K. Maki, Phys. Rev. B 32 6582 (1985)
- 11. K. Maki and A. Virosztek, Phys. Rev. B 39 9640 (1989)
- 12. H. Fukuyama and P. A. Lee, Phys. Rev B 17 535 (1978)
- 13. P. A. Lee and T. M. Rice, Phys. Rev. B 19 3970 (1979)
- P. Monceau in Electronic Properties of Inorganic Quasi-One-Dimensional Materials, edited by P. Monceau (Reidel, Dordrecht 1985) p.139
- 15. K. Maki, Phys. Rev. B 33 2852 (1986)
- 16. R. M. Fleming, Phys. Rev. B 22 5606 (1980)
- 17. L. Sneddon, M. C. Cross and D. S. Fisher, Phys. Rev. Lett 49 299 (1982)
- 18. D. S. Fisher, Phys. Rev. Lett 50 1486 (1982)
- 19. J. Bardeen, Phys. Rev. Lett <u>42</u> 1498 (1979)
- 20. T. Ekino and J. Akimitsu, Jpn. J. Appl. Phys. 26 Suppl 625 (1987)
- S. Tomić, J. R. Cooper, D. Jérome, and K. Bechgaard, Phys. Rev. Lett 62 2466 (1989)
- 22. S. Tomić, W. Kang, J. R. Cooper and D. Jérome (to be published)
- T. Sambongi, K. Nomura, T. Shimizu, K. Ichimura, N. Kinoshita, M. Tokumoto and H. Anzai, Sol. State Commun (to be published); K. Nomura, T. Shimizu, K. Ichimura, T. Sambongi, M. Tokumoto, H. Anzai and N. Kinoshita, ibid (to be published)
- J. W. Brill and W. Roark, Phys. Rev. Lett <u>53</u> 846 (1984); J. W. Brill, W. Roark and G. Minton, Phys. Rev. B <u>33</u> 6831 (1986)
- G. Mozurkewich, P. M. Chaikin, W. G. Clark, and G. Grüner, Sol. State Commun 56 421 (1985)
- 26. K. Maki and A. Virosztek, Phys. Rev. B <u>36</u> 2910 (1987)
- 27. A. Virosztek and K. Maki, to be published
- 28. P. M. Chaikin, T. Tiedje and A. N. Block, Sol. State Commun. 41 739 (1982)
- 29. Y. Nakane and S. Takada, J. Phys. Soc. Jpn <u>54</u> 977 (1985)