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DIRAC EQUATION WITH TWO MASS PARAMETERS AND TWO-FLAVOUR NEUTRINO OSCILLATIONS

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Linear transformations of the Dirac equation with two mass parameters result in "standard" forms of the massive, massless and tachyonic equations. These equations are used to describe neutrino mass eigenstates, which, in turn, are linearly combined to obtain flavours. This paper examines the issue of neutrino oscillations in a particular case of the model.

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1. *Introduction*

In previous papers [1–6], the Dirac equation with two mass parameters and related topics were discussed. The approach was used to derive standard equations for massive, massless and tachyonic fermions. In particular, a massless equation was obtained, which differs from the usual one and does not produce a superfluous conserved current. In Ref. [7], the aforementioned results were reformulated and justified on the grounds of desirable features relating to the active symmetry operations (time reversal, spatial parity, etc.). Possible applications and a flavoured neutrino model were introduced in Refs. [1,3,4,7]. This paper examines the issue of flavour oscillations in the context of the neutrino model first proposed in Ref. [3]: the three flavours result as unitary superpositions of three neutrino mass eigenstates (massive, massless and tachyonic). The model is here specialized to a particular case of two-flavour oscillations.

The treatment is done before the second quantization, and notation is rather conventional. Specifically, and unless otherwise noted, Greek (Latin) indices run through the values $0, 1, 2, 3$ $(1, 2, 3)$ and the summation convention is applied to repeated up and down labels. An attempt is made at distinguishing powers from superscripts: for instance, $(m)^2$ and $| b |^2$ are powers, while x^2 indicates a specific variable with superscript 2. Units are such that $\hbar = c = 1$ (unless otherwise indicated).

2. *Neutrino flavours*

In a frame of reference $\mathcal{X}$ of real spacetime coordinates $x = \{x^\lambda\}$ and pseudoeuclidean metric $g^{\mu\nu} = \mathrm{diag}\{+1, -1, -1, -1\}$, the model described in Ref. [7] is based on the "standard" equations:

$$\not{P}\Theta_\omega(x) = \mathrm{M}(\omega)\Theta_\omega(x)\,, \qquad \omega = -1, 0, 1, \tag{1}$$

with

$$\not{P} = \mathrm{i}\gamma^\alpha \partial_\alpha \tag{2}$$

and

$$\mathrm{M}(\omega) = \frac{m}{2}\left[(I - \varepsilon\gamma^5) - \omega(I + \varepsilon\gamma^5)\right], \qquad m > 0\,, \tag{3}$$

where $\Theta_\omega(x)$ are complex four-spinors. The Dirac matrices γ^λ (in a fixed chosen representation) obey the usual rules

$$\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}I\,, \qquad (\gamma^\mu)^\dagger = \gamma^0\gamma^\mu\gamma^0\,, \tag{4}$$

with I being the 4×4 identity matrix. The matrix $\gamma^5 = \mathrm{i}\gamma^0\gamma^1\gamma^2\gamma^3$ is hermitian and unitary, and anticommutes with all γ^λ. For general reference on the Dirac equation and related topics, see, for instance Refs. [8–19]. The value of the sign $\varepsilon = (-1)^{T+S}$ depends on the frame of reference [1,20]. Namely, the time-index T and the space-index S of $\mathcal{X}$ are so defined: $T = 0$ if $t = x^0$ runs forward ($T = 1$ otherwise) and $S = 0$ if $s = \{x^\ell\}$ is a right-handed triplet ($S = 1$ otherwise).

The solutions $\Theta_\omega(x)$ of Eq. (1) are eigenstates of the squared four-momentum (SFM) operator

$$-\,\partial_\alpha g^{\alpha\beta}\partial_\beta \tag{5}$$

for the eigenvalues $-(m)^2\,\omega$. Thus, in this model, there are three neutrino mass eigenstates [7]: massive ($\omega = -1$), massless ($\omega = 0$) and tachyonic ($\omega = 1$). The flavour spinors $\Phi_f(x)$ are introduced by means of the linear superpositions:

$$\Phi_f(x) = b_f{}^\omega\Theta_\omega(x)\,, \qquad f = -1, 0, 1, \tag{6}$$

where $(b_f{}^\omega)$ is some 3×3 unitary matrix of mixing coefficients. The squared magnitude

$$|\, b_f{}^\omega \,|^2 \tag{7}$$

denotes the conditional probability that a flavour f will be measured to have a value $-(m)^2\,\omega$ of the SFM operator [7].

In this paper, the linear combinations (6) are specialized in the following manner:

$$b_f^{\omega} = \begin{cases} 1/\sqrt{2} & (f = \mp 1, \omega = -1) \\ 0 & (f = \mp 1, \omega = 0) \\ -1/\sqrt{2} & (f = -1, \omega = 1) \\ 0 & (f = 0, \omega = \mp 1) \\ 1 & (f = 0, \omega = 0) \\ 1/\sqrt{2} & (f = 1, \omega = 1) \end{cases}$$

which makes the flavour $f = 0$ strictly massless, as it coincides with the $\omega = 0$ mass eigenstate. The mass eigenstates $\omega = \mp 1$ are maximally superimposed in the flavours $f = \mp 1$: on the average, each of these two flavours is massless [7]. It is possible that these features might allow a massless treatment of all three flavours, whenever certain detailed behaviours (e.g., oscillations) need not be taken into consideration.

3. Oscillations

The $f = 0$ flavour cannot oscillate, since it is a mass eigenstate. The $f = \mp 1$ flavours were designed so that they should oscillate maximally: the calculation of these two-flavour oscillations is outlined in the following, as a variation of the method described in Ref. [21] and in numerous other papers. For the remainder of this section, the labels f (and, later on, f') and ω will be restricted to the values ∓ 1.

In the given frame of reference, define the energy [7] operator $\mathrm{i}(-1)^T \partial/\partial t$, and the (contravariant) momentum operator in the $z = x^3$ direction: $-\mathrm{i}\partial/\partial z$. Consider the corresponding eigenstates

$$\Theta_\omega(x) = \exp\{-\mathrm{i}(-1)^T E\, t\}\, \exp\{\mathrm{i} p_\omega z\}\, \Theta_\omega(0)\,, \tag{8}$$

with eigenvalues $E > 0$ and $p_\omega > 0$. These are $\omega = \mp 1$ plane waves of positive energy E, collimated in the positive z direction with momenta p_ω. The spinors $\Theta_\omega(0)$ satisfy the equations

$$\left[(-1)^T \gamma^0 E - \gamma^3 p_\omega\right] \Theta_\omega(0) = \mathrm{M}(\omega)\Theta_\omega(0)\,, \tag{9}$$

which entail

$$p_\omega = \left[(E)^2 + (m)^2\,\omega\right]^{1/2}\,, \qquad E > m\,. \tag{10}$$

For high energy ($E \gg m$), the approximation

$$p_\omega \approx E + \frac{(m)^2\,\omega}{2E} \tag{11}$$

may be used.

From Eq. (8), one obtains:

$$\begin{aligned}\Phi_f(x) &= \exp\{-\mathrm{i}(-1)^T E\,t\}\left[\frac{\exp\{\mathrm{i}p_{-1}z\} - f\exp\{\mathrm{i}p_1 z\}}{2}\,\Phi_{-1}(0)\right.\\ &+ \left.\frac{\exp\{\mathrm{i}p_{-1}z\} + f\exp\{\mathrm{i}p_1 z\}}{2}\,\Phi_1(0)\right], \qquad (12)\end{aligned}$$

having applied the definition of flavours (Sect. 2) both at point x and at the point $x = 0$. It is noted that the spinors (12) are eigenstates of energy, but not of momentum. At $z = D$ and for $f = 1$, the squared magnitude of the coefficient of $\Phi_{-1}(0)$ is indicated as $P(-1 \to 1)$, and is calculated as follows:

$$P(-1 \to 1) = \left[\sin\left(\frac{p_{-1} - p_1}{2}\,D\right)\right]^2. \qquad (13)$$

Similarly

$$P(1 \to -1) = P(-1 \to 1)\,, \qquad (14)$$

and

$$P(-1 \to -1) = P(1 \to 1) = 1 - P(-1 \to 1)\,. \qquad (15)$$

In the high energy approximation:

$$P(-1 \to 1) \approx \left\{\sin\left[\frac{(m)^2 D}{2E}\right]\right\}^2. \qquad (16)$$

The quantities $P(f \to f')$ are here intended to represent the probabilities of flavour oscillation ($f \neq f'$) or non-oscillation ($f = f'$) over a distance D between source and detector. A few comments follow.

The use of plane waves may be questioned, because both the production and the detection of neutrinos must involve some degree of spacetime localization; a related concern stems from the application of flavour definitions and probability interpretations at specific points of spacetime, rather than within neighborhoods of points [7]. However, this naive approach (to be viewed as an approximation) appears in most calculations of flavour oscillations. In the present model, further doubts are due to the unclear meaning of the tachyonic current [22,23], and to the lack of a well-established second quantization scheme for tachyons. It is hoped that the probability concepts introduced in Ref. [7] are consistent enough as to extrapolate to the intended interpretation of the quantities $P(f \to f')$. At any rate, see Ref. [24] for a similar treatment. See also Refs. [25–30] for a variety of opinions on tachyons and related issues.

4. Conclusions

The described model, compared with the data on atmospheric neutrinos [31], provides a good fit if $f = 0$ is identified with the electron neutrino, and if the value of m is chosen as

$$m \approx 3.3 \times 10^{-2}\,\mathrm{eV}, \tag{17}$$

in atomic units of energy. A distinctive feature of this model is that all flavours are massless (either strictly or on the average); the case $f = 0$ stands out as a "special" type of neutrino with respect to the other two flavours.

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DIRACOVA JEDNADŽBA S DVIJE MASE I NEUTRINSKIM OSCILACIJAMA DVAJU OKUSA

Linearnim transformacijama Diracove jednadžbe s dvije mase izvode se "standardni" oblici jednadžbi za masena, bezmasena i tahionska stanja. Te se jednadžbe primjenjuju za opis neutrinskih svojstvenih masenih stanja i linearno se slažu radi dobivanja okusa. Ovaj rad opisuje neutrinske oscilacije za poseban slučaj ovog modela.