ISSN 1330–0016
CODEN FIZBE7

FRAMEWORK FOR A THEORY THAT UNDERLIES THE STANDARD MODEL

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Received 23 July 1998; revised manuscript received 8 March 1999
Accepted 10 May 1999

We put forward the following, physically motivated premiss for constructing a theory that underlies the standard model in four-dimensional space-time: The Euler-Lagrange equations of such a theory formally resemble some equations of motion underlying fluid-dynamics equations in the kinetic theory of gases. In support of this premiss, we point out local and covariant Lagrangians whose Euler-Lagrange equations contain a subsystem equivalent to the Euler-Lagrange equations of the standard model with covariantly regularized propagators, and a subsystem describing faster-than-light effects.

PACS numbers: 12.60.-i, 03.30.+p, 03.40.Kf, 05.60.+w UDC 530.145, 530.17
Keywords: beyond standard model, regularization, faster-than-light effects

1. *Introduction*

The standard model, which provides an adequate description of all quantum-mechanical experiments so far performed, is generally considered to be only an effective field theory: a low-energy approximation to an underlying theory (UT). To obtain a UT, within the last fifteen years or so considerable effort has been put into various string theories. But it is still an open question whether this "top-down" approach leads to the observed low-energy physics. In this paper we put forward a new framework for an opposite, "bottom-up" approach to the construction of a UT in four-dimensional space-time. We base it on the analogy of the kinetic theory of gases and give basic assumptions in Sect. 2, the premiss in Sect. 3, and a transport-theoretic example in Sect. 4. As a starting point, we note that:

(A) Propagators of the standard model must be regularized to obtain physically meaningful results. Following Pauli [1], we presume there is a UT whose propagators

(i) do not need to be regularized, and (ii) can be regarded as such regularizations of standard-model propagators that reflect ultra-high-energy physics; for comments on this belief see Ref. [2]. In Sect. 4.3 we point out a possible Lagrangian of such a UT.

(B) Ever since Einstein, Podolski and Rosen published their gedanken-experiment some sixty years ago, physicists have been aware that, if we go beyond a strictly operational description of quantum phenomena, interpretations of certain results suggest the existence of *faster-than-light effects* (FTLEs) [3].[1] So we expect the mathematical formalism of a UT to exhibit some FTLEs. However, special relativity poses two serious conceptual problems in connection with FTLEs:

(i) When the relation between two events suggests FTLEs, different observers may not agree on what is the cause and what is the consequence! Suppose we observe two spacelike-separated measurements, say A and B, and we believe that the result of the earlier one determines the result of the latter one by a FTLE, e.g., by an instantaneous change of the quantum-mechanical state. In our frame of reference, let A precede B so that we believe B is determined by A. However, there are inertial frames where B precedes A, and there observers believe the opposite, that B determines A.

(ii) It is not clear how to model states that exhibit FTLEs without predicting that the present can influence the past! Suppose the relation between responses and their sources is covariant in the sense that to Lorentz-transformed source corresponds Lorentz-transformed response. So, if a part of the response due to some source is faster than light, then the corresponding parts of responses to sources that equal certain Lorentz transformations of this source precede their causes. Which is a very strong objection to FTLEs, since no physical phenomenon ever suggested the existence of "effects" that precede their causes.[1]

Resolution of such problems is often seen as the key to a better understanding of quantum phenomena [3]. We see no way around the first problem.[2] Regarding the second problem, we point out in Sect. 4 such Euler-Lagrange equations where one can avoid this problem without coming in conflict with special relativity.

2. Basic assumptions

The shape of a UT is quite unknown. However, we assume that *its Euler-Lagrange equations are local*[3] *and covariant, and the free-field equations* (i.e., Euler-

[1] A change in the state of a physical system that occurs at the space-time point $(ct_2, \mathbf{r}_2)$ and is attributed to a source at $(ct_1, \mathbf{r}_1)$ is refered to as: an effect (response) if $t_2 \geq t_1$; a faster-than-light effect (FTLE) if $|\mathbf{r}_2 - \mathbf{r}_1| > c(t_2 - t_1)$ and $t_2 \geq t_1$; an "effect" that precedes its cause if $t_2 < t_1$; an effect that satisfies Einstein's causality condition if $|\mathbf{r}_2 - \mathbf{r}_1| \leq c(t_2 - t_1)$.

[2] Aharonov and Albert [4] argued that one cannot use relativistic functions to describe FTLEs suggested by quantum mechanics.

[3] We label an equation of motion as local if it relates only the values of the state function and of finitely many of its time and space derivatives at the same space-time point.

Lagrange equations with all non-linear terms taken as external sources) *admit classical solutions that have: (i) properties that are propagated not faster than light according to covariant, regularized Green functions of basic field equations, (ii) unbounded front velocity, and (iii) no "effects" that precede their causes.* Therefore, we believe that an understanding of classical systems with such equations of motion would be invaluable in searching for and in constructing a UT.

To this end let us be more specific about the mathematical properties we expect from the above classical, free-field solutions in a particular, scalar system:

(A) An external source is described by a real function $j(x)$ of the space-time variable $x = (ct, \mathbf{r}) \in \mathrm{R}^{1,3}$. The set of possible sources is invariant under the inhomogeneous Lorentz transformations $x \to \Lambda x + a$, i.e., if a particular source $j(x)$ belongs to this set, then so does any "moving" source $j(\Lambda x + a)$.

(B) The total response of the system to the external source $j(x)$ is described by the state function determined by some local, linear, covariant, Euler-Lagrange equations of motion and subsidiary conditions. A certain part of this total response, described by the real function $\varphi(x)$ of $x \in \mathrm{R}^{1,3}$, is such that: (i) There is a Green function $G(x)$ such that $\varphi(x) = G * j$, where $*$ denotes the convolution with respect to x. (ii) The relation between $\varphi(x)$ and $j(x)$ is covariant [if $j(x)$ is the source of $\varphi(x)$, then a "moving" source $j(\Lambda x + a)$ causes $\varphi(\Lambda x + a)$]. (iii) Responses $\varphi(x)$ do not precede their sources $j(x)$. Thus, $G(x)$ is covariant in the sense that $G(\Lambda x) = G(x)$, and $G(x) = 0$ if $t < 0$, and also if $c^2t^2 < |\mathbf{r}|^2$ [5]. Consequently, (i) all responses $\varphi(x)$ exhibit Einstein's causality,[1] and (ii) the propagator

$$\widetilde{G}(k) \equiv \int \mathrm{d}^4x \, e^{-\mathrm{i}k\cdot x} G(x) \,, \tag{1}$$

where $k \in \mathrm{R}^{1,3}$ and $k \cdot x = \mathbf{k} \cdot \mathbf{r} - k^0 ct$, is covariant, i.e., $\widetilde{G}(\Lambda k) = \widetilde{G}(k)$.

(C) This propagator $\widetilde{G}(k)$ (i) can be adequately approximated by the propagator

$$\widetilde{G}_w(k) \equiv (k^2)^{-1} \tag{2a}$$

up to some extremely large value of $|k^2|$, and (ii) is regular in the sense that

$$\widetilde{G}(k) = O\big((k^2)^{-n}\big) \tag{2b}$$

as $k^2 \to \infty$, with constant $n > 2$ [1].

(D) The total response to any source $j(x)$ exhibits some FTLEs, but no "effects" that precede their causes. So state functions of the system in question depend causally on their sources; but, as pointed out in Sect. 1, this dependence cannot be covariant! Thus the subsidiary conditions that together with covariant equations of motion determine the state functions cannot be covariant.

(E) All inertial frames are equivalent: The properties (A) to (D), and the relations between state functions, responses $\varphi(x)$, and their sources $j(x)$ do not depend on the inertial frame of the observer. Sources $j(x)$ and responses $\varphi(x)$ are relativistic: functions $j(x)$ that represent the same source in different inertial frames are related by Lorentz transformations for scalar fields; and the same goes for the corresponding functions $\varphi(x)$, in agreement with the assumption (B). As certain Lorentz transformations of any state function exhibit effects that precede their causes, total responses are not relativistic—the relation between state functions in different inertial frames is open.[2]

By (C) above, the Fourier transform $\widetilde{G}_w(k)$ of the wave-equation Green function $G_w(x)$ adequately approximates $\widetilde{G}(k)$ up to extremely large values of $|k^2|$. So, the wave response $\varphi_w(x) \equiv G_w * j$ is a very good approximation to the response $\varphi(x)$ when the source $j(x)$ is varying slowly enough, both spatially and temporaly. In such a case, the wave equation $(c^{-2}\partial^2/\partial t^2 - \vec{\nabla} \cdot \vec{\nabla})\varphi_w(x) = j(x)$ is a good approximation to the unknown, covariant equation of motion for the response $\varphi(x)$. So we can say that the unknown equations of motion for the state function of the system in question underlie the wave equation in the sense that their causal solutions, though exhibiting FTLEs, propagate certain effects by a covariant, regular propagator $\widetilde{G}(k)$ that can be approximated by the wave propagator $\widetilde{G}_w(k)$ up to extremely high values of $|k^2|$.

3. *Premiss*

The above properties (A) to (E), however, do not even suggest whether such a classical system has a state function of only four continuous variables (ct and $\mathbf{r}$), and certainly give no indication about the nature of its equations of motion. So we went looking for classical physical systems that behave similarly in order to construct by analogy such equations of motion that underlie the basic free-field equations in the above sense. We found such systems in the kinetic theory of gases [5].

For example, take a non-relativistic gas. Its macroscopic state is described by macroscopic variables such as kinetic energy and density, slow changes of which propagate with a finite speed of sound, approximately according to some fluid-dynamics partial-differential equations. But its microscopic state is affected almost immediately everywhere by a localized source since the velocity of gas particles is not bounded in the non-relativistic theory. Only a finite number of local averages of the microscopic state are regarded as macroscopically observable, i.e., the macroscopic variables, which can also be defined independently with no reference to the microscopic state. The remaining properties of the microscopic state (i.e., infinitely many, macroscopically directly unobservable degrees of freedom) describe processes that manifest themselves (i) in fluctuations of macroscopic variables, and (ii) in the fact that fluid-dynamics equations are only asymptotically valid approximations for smoothly and slowly changing macroscopic variables.

For a rare gas of identical pointlike particles, the fluid-dynamics equations can be extended to model somewhat faster changes of macroscopic variables by introducing additional fields of spacetime variable, which have no direct significance within the framework of fluid dynamics, though they can be interpreted as local averages of the microscopic state, see, e.g., the Grad method of moments [6]. But eventually, these equations of motion cannot be improved this way any more, and one must resort to a more detailed description by the one-particle distribution, a function of time, position, and velocity, evolving according to the integro-differential Boltzmann equation [6]. So in the case of a rare gas, there is a characteristic length and time interval where a completely new physics appears with three additional independent variables: physics essentialy different from the macroscopic physics described by fluid-dynamics equations.

For various theoretical reasons, many theorists believe that the framework of present quantum field theories may not be appropriate for a theory of quantum phenomena valid at all energies. It was Feynman [7] who first suggested that the basic partial-differential equations of theoretical physics might be actually describing macroscopic motion of some infinitesimal entities he called X-ons. In addition, already Heisenberg [8] and Bjorken and Drell [9] expected that there is a characteristic energy (and length) beyond which quantum dynamics will be essentially different from the one described by the canonical formalism; so we expect the Euler-Lagrange equations of a UT to be very different from those of the standard model. All of which, together with the behaviour of a nonrelativistic gas, leads us to put forward the following premiss for a “bottom-up” approach to fundamental interactions: *The Euler-Lagrange equations of a UT formally resemble some of equations of motion underlying fluid-dynamics equations in the kinetic theory of gases.* We believe that this physically motivated premiss will help us (i) to construct a UT whose propagators do not need to be regularized, and (ii) to model quantum-mechanical FTLEs.

4. *Transport-theoretic example*

4.1. *Equation of motion*

Following the above premiss, we now consider a class of systems with properties (A) to (E), defined by covariant, linear, integro-differential equations of motion with a non-covariant causality condition. On the analogy with the linearized Boltzmann equation let us provisionally regard these equations of motion as modeling transport of some infinitesimal entities, X-ons, with arbitrary four-momenta, whose macroscopic motion evolves almost according to the wave equation.

We describe the state of X-ons in a given inertial frame by a real scalar state function $\Psi(x,p)$ of the space-time variable $x \in \mathrm{R}^{1,3}$ and of the four-momentum variable $p = (p^0, \mathbf{p}) \in \mathrm{R}^{1,3}$. As the equation of motion for $\Psi(x,p)$ we take the local, linear, transport equation

$$p{\cdot}\partial\Psi = \mathbf{S}\Psi + \mathbf{Q}\,, \tag{3}$$

where: (i) $p{\cdot}\partial = p^0 c^{-1}\partial/\partial t + \mathbf{p}\cdot\vec{\nabla}$ is the covariant, substantial time derivative. Thus Eq. (3) with $\mathbf{S} = 0$ and $\mathbf{Q} = 0$ is an analog of Newton's first law and describes free streaming of X-ons.

(ii) The scattering operator $\mathbf{S}$ describes the scattering of X-ons by the host medium—the vacuum. In the considered case,

$$\begin{aligned}\mathbf{S}\Psi \;\equiv\; & f_0(p^2)\int \mathrm{d}^4p'\, f_0(p'^2)\Psi(x,p') \\ & +f_1(p^2)p\cdot\int \mathrm{d}^4p'\, f_1(p'^2)p'\Psi(x,p') - t(p^2)\Psi(x,p)\,, \end{aligned}\tag{4}$$

where $f_0(p^2)$, $f_1(p^2)$ and $t(p^2)$ are real functions of $p^2 \in \mathrm{R}$; and the integral

$$\int \mathrm{d}^4p\, F(p) \equiv -\mathrm{i}\lim_{r\to\infty}\int_{-ir}^{ir}\mathrm{d}p^0 \int_{\mathbf{p}^2\le r^2} F(p)\,\mathrm{d}^3\mathbf{p} \tag{5}$$

for functions $F(p)$, $p \in \mathrm{R}^{1,3}$.

(iii) The source of all X-ons described by $\Psi(x,p)$ is given by

$$\mathbf{Q}(x,p) \equiv q_0 f_0(p^2) j(x)\,, \tag{6}$$

with q_0 being a real parameter. As we do not permit effects that precede their causes, we assume the causality condition: if $\mathbf{Q}(x,p) = 0$ for all $t \le t_0$, the corresponding state function

$$\Psi(x,p) = 0 \qquad \text{for all} \qquad t \le t_0\,. \tag{7}$$

The equation of motion (3) is covariant with respect to the inhomogeneous Lorentz transformations

$$x \to \Lambda x{+}a\,,\quad p \to \Lambda p\,,\quad \Psi(x,p) \to \Psi(\Lambda x{+}a,\Lambda p)\,,\quad \mathbf{Q}(x,p) \to \mathbf{Q}(\Lambda x{+}a,\Lambda p)\,. \tag{8}$$

However, like the Boltzmann equation, Eq. (3) is not invariant under the time reversal: the state function $\Psi(x,p)$ displays an arrow of time in the sense that the time-reversed $\Psi(x,p)$ is not a solution to (3) with time-reversed source $\mathbf{Q}(x,p)$. In contrast to the Einstein causality condition, condition (7) is not covariant. As a consequence, the relation between solutions $\Psi(x,p)$ to equation (3) and their sources $\mathbf{Q}(x,p)$ need not be covariant.

The equations of motion (3)–(6) are equal to the Euler-Lagrange equations of the local, covariant Lagrangian

$$(2q_0)^{-1}\int \mathrm{d}^4p\,\Psi(x,-p)[p\cdot\nabla\Psi - \mathbf{S}\Psi] - \int \mathrm{d}^4p\,\Psi(x,-p)f_0(p^2)j(x)\,. \tag{9}$$

4.2. Properties of the state function $\Psi(x,p)$

The total responses $\Psi(x,p)$ of the system in question to sources $j(x)$ are such that certain local averages, the macroscopic variables

$$\begin{aligned} \varphi[x;\Psi] &\equiv \int \mathrm{d}^4p\, f_0(p^2)\Psi(x,p)\,, \\ a[x;\Psi] &\equiv \int \mathrm{d}^4p\, f_1(p^2)\Psi(x,p)p\,, \end{aligned} \tag{10}$$

covariantly depend on sources $j(x)$ and exhibit Einstein's causality despite the non-covariant causality condition (7). To infer this, we proceed as in Ref. [5] to compute the Fourier transforms $\widetilde{\varphi}[k;\Psi]$ and $\widetilde{a}[k;\Psi]$ of $\varphi[x;\Psi]$ and $a[x;\Psi]$, and conclude that

$$\widetilde{\varphi}[k;\Psi] = \widetilde{G}_0(k)\widetilde{j}(k)\,, \tag{11a}$$

$$\widetilde{a}[k;\Psi] = \mathrm{i}k\widetilde{G}_1(k)\widetilde{j}(k)\,, \tag{11b}$$

where

$$\widetilde{G}_0(k) = q_0 D^{-1}(1 - I_3 - D)\,, \tag{11c}$$

$$\widetilde{G}_1(k) = q_0 D^{-1} I_2\,, \tag{11d}$$

with

$$D \equiv (1 - I_1)(1 - I_3) + k^2 I_2^2\,, \tag{11e}$$

$$I_1(k^2) \equiv (2\pi^2/k^2) \int_0^\infty f_0^2(y)t(y)[\sqrt{1 + k^2 y/t^2(y)} - 1]\mathrm{d}y\,, \tag{11f}$$

$$I_2(k^2) \equiv (\pi/k^2)^2 \int_0^\infty f_0(y)f_1(y)t^2(y)[\sqrt{1 + k^2 y/t^2(y)} - 1]^2\mathrm{d}y\,, \tag{11g}$$

$$I_3(k^2) \equiv (\pi/k^2)^2 \int_0^\infty f_1^2(y)t^3(y)[\sqrt{1 + k^2 y/t^2(y)} - 1]^2\mathrm{d}y\,. \tag{11h}$$

As $\widetilde{G}_0(k)$ and $\widetilde{G}_1(k)$ are covariant, the corresponding retarded Green functions $G_0(x)$ and $G_1(x)$ are covariant, and $\varphi[x;\Psi] = G_0 * j$ and $a[x;\Psi] = \partial G_1 * j$ satisfy Einstein's causality condition [5].

When $t^2(p^2)/p^2$ and its inverse are bounded for all $p^2 \geq 0$, the propagator $\widetilde{G}_0(k)$ has the following properties:

(A) If $a_4^{11} \neq 4$, $a_2^{00} = 1$ and $(4 - a_4^{11})(4q_0 - a_4^{00}) = (a_4^{01})^2$, where

$$a_r^{mn} \equiv \pi^2 \int_0^\infty f_m(y) f_n(y) [t(y)]^{m+n+1} |\sqrt{y}/t(y)|^r \mathrm{d}y \,,$$

then

$$\widetilde{G}_0(k) = 1/k^2 + O((k^2)^0) \text{ as } k^2 \to 0 \,.$$

(B) $\widetilde{G}_0(k) = O((k^2)^{-n})$ as $k^2 \to \infty$, where

$$n = 1 \quad \text{if} \quad a_1^{00} = 0, \qquad n = 3/2 \quad \text{if also} \quad 2a_0^{00} = -(a_2^{01})^2 \,,$$

$$n = 2 \text{ if also } a_{-1}^{00} = -4a_2^{01} a_1^{01} \,, \quad n = 5/2 \text{ if also } a_2^{11} a_0^{00} = 2(a_1^{01})^2 + 2a_2^{01} a_0^{01} \,,$$

$$\text{and } n = 3 \quad \text{if also} \quad a_{-3}^{00} = 8a_2^{01} a_{-1}^{01} + 32 a_1^{01} a_0^{01} - 16 a_0^{00} a_1^{11} - 4 a_{-1}^{00} a_2^{11} \,.$$

These results enable us to explicitly show that within the presented transport-theoretical framework, there are covariant propagators $\widetilde{G}_0(k)$ regularizing the wave propagator $\widetilde{G}_w(k)$. Namely, when $\sqrt{p^2}/t(p^2)$ has only two values for $p^2 \geq 0$, say τ_1 and $-\tau_2$, we can explicitly calculate the corresponding propagator $\widetilde{G}_0(k)$ as a rational function of $\sqrt{1 + \tau_j^2 k^2}$, $j = 1$, 2, whose six parameters are determined by integrals of $f_0(p^2)$ and $f_1(p^2)$. For $\tau_2 > \tau_1 > 0$, there are infinitely many f_0 and f_1 such that for some real q_0 the corresponding $\widetilde{G}_0(k)$ has the required properties: (i) it satisfies the conditions (A) and (B) with $n = 3$, (ii) $\widetilde{G}_0(k)$ is a decreasing function of $k^2 > 0$, (iii) the difference $|\widetilde{G}_0(k) - \widetilde{G}_w(k)|$ is a bounded function of complex k^2, e.g., for $\tau_2/\tau_1 = 2$, and (iv) for any μ_0 this difference can be made arbitrarily small for all $|k^2| < \mu_0$ by taking τ_1 and τ_2 sufficiently small.

By (3), (4), (5), (7) and (10), when $j(x) = 0$ if $t \leq t_0$, we can express the state function $\Psi(x, p)$ for $p^0 \neq 0$ in terms of the source $j(x)$ and fields $\varphi[x; \Psi]$ and $a[x; \Psi]$ [5]:

$$\Psi(x, p) = \Theta(t - t_0) \int_0^{c(t-t_0)/p^0} e^{-t(p^2)y} q(x - yp, p) \, \mathrm{d}y \,, \tag{12}$$

with

$$q(x, p) \equiv \{\varphi[x; \Psi] + q_0 j(x)\} f_0(p^2) + p \cdot a[x; \Psi] f_1(p^2) \,, \tag{13}$$

$\Theta(t < 0) \equiv 0$ and $\Theta(t \geq 0) \equiv 1$. By (10)–(13), the source $j(x)$ at $x = (ct_1, \mathbf{r}_1)$: (i) does not affect $\Psi(x, p)$ at $x = (ct_2, \mathbf{r}_2)$, $t_2 < t_1$, i.e., the system considered is causal; and (ii) affects $\Psi(x, p)$ at $x = (ct_2, \mathbf{r}_2)$, $t_2 > t_1$, for some values of four-momentum p no matter how small is the time interval $t_2 - t_1$ and/or how large is the distance $|\mathbf{r}_2 - \mathbf{r}_1|$, i.e., the physical system considered displays *everywhere arbitrary fast effects*: the front velocity of its state function $\Psi(x,p)$ is not bounded! Thus the dependence of $\Psi(x,p)$ on $j(x)$ is not covariant in contrast with the dependence of its properties $\varphi[x;\Psi]$ and $a[x;\Psi]$: the *covariance* (8) of the equation of motion (3) *is partly broken* by the non-covariant causality condition (7).

Regarding the relation between descriptions of X-ons in different inertial frames, we assume that (i) there is no preferred inertial frame, (ii) the source $j(x)$ is a scalar relativistic field, and, (iii) the independent variable p transforms as a four-momentum. In particular, when considering X-ons from an inertial frame whose space-time coordinates $x' = \Lambda x + a$, their four-momenta $p' = \Lambda p$, and their state function $\Psi'(x',p')$ is uniquely determined by (i) the equations of motion (3)–(6) with $\partial \to \partial'$, $p \to p'$ and $j(x) \to j'(x') = j(\Lambda^{-1}x' - \Lambda^{-1}a)$, and (ii) the non-covariant causality condition (7). The preceding results imply that $G_0'(x) = G_0(x)$ and $G_1'(x) = G_1(x)$ so that $\varphi'[x';\Psi'] = \varphi[x;\Psi]$ and $a'[x';\Psi'] = \Lambda a[x;\Psi]$; so these two local averages of the state function are relativistic scalar and vector fields. The state function itself is not relativistic; $\Psi'(x',p')$ is related to $\varphi'[x';\Psi']$, $a'[x';\Psi']$ and $j'(x')$ through the non-covariant relation (12).

The above results show that one can construct integro-differential equations of motion that can be regarded as the Euler-Lagrange equations underlying the wave equation in the sense specified at the end of Sect. 2. In the same manner, one can construct also integro-differential equations that can be regarded as Euler-Lagrange equations underlying other basic, differential free-field equations (see Appendix A).

4.3. Lagrangian for UT in accordance with the premiss

To construct a possible Lagrangian for a UT, we may proceed as follows:

(i) We take the Euler-Lagrange equations of the standard model and express them in terms of spin-0, spin-$\frac{1}{2}$ and spin-1 propagators.

(ii) We replace these propagators with propagators analogous to $\widetilde{G}_0(k)$ with properties (B) to obtain relations such as (11) with spin-0, spin-$\frac{1}{2}$ and spin-1 sources.

(iii) Combining Lagrangians that are related to the obtained relations as (9) is related to (11), we can then construct a possible transport-theoretic Lagrangian for a UT as specified in Sects. 2 and 3. Its local and covariant Euler-Lagrange equations comprise transport equations such as (3) with scalar, spinor, and vector sources.

For φ^4 theory, an example of such a construction is given in Appendix B. The question remains, however, which of the infinity of such transport-theoretic Lagrangians are physically relevant for constructing a UT. We considered quantum field theories defined by Feynman path integrals of such transport-theoretic Lagrangians in Ref. [10].

The Euler-Lagrange equations of a Lagrangian constructed as specified above contain a subsystem of equations equivalent to the Euler-Lagrange equations of the standard model with some covariantly regularized propagators. This subsystem determines the dynamics of all fields of the standard model in the classical approximation so that they exhibit no FTLEs, though solutions to the whole set of covariant transport-theoretic Euler-Lagrange equations do exhibit FTLEs. As in the classical approximation, the temporal dependence of the fields of the standard model describes the temporal dependence of its quantum states, FTLEs are absent there. How to use transport-theoretic FTLEs to explain FTLEs implied by certain quantum phenomena is open. Such an explanation would not require, as sometime suggested [11] that we abandon the traditional belief that the basic equations of motion are covariant, and all inertial frames are equivalent.

5. *Concluding remarks*

In this paper we have put forward a new framework for constructing a theory that may underlie the standard model in four-dimensional space-time. It requires that the Euler-Lagrange equations of this theory: (i) are local and covariant, (ii) have propagators that need not be regularized, (iii) describe some faster-than-light effects, and (iv) formally resemble equations of motion of some theory that underlies fluid dynamics in the kinetic theory of gases. Physical motivations for and details of this framework are given in Sects. 1 – 3.

To show that the proposed framework for modeling fundamental interactions is feasible, we have pointed out in Sect. 4.3 how one can construct Euler-Lagrange equations such that: (i) they are integro-differential equations defined in eight-dimensional $\mathrm{R}^{1,3} \times \mathrm{R}^{1,3}$ on the analogy with the Boltzmann transport equation, (ii) their causal solutions display FTLEs, and (iii) certain local averages of these solutions are propagated not faster than light by the Euler-Lagrange equations of the standard model whose propagators are covariantly regularized. These transport-theoretic Euler-Lagrange equations define for the first time *such a physically motivated class of classical models in four-dimensional space-time that (i) are not invariant under time reversal, (ii) have covariant, regular propagators, (iii) model certain FTLEs without predicting that present can influence the past, and (iv) are not in conflict with special relativity.*

Acknowledgements

We are grateful to Matjaž Poljšak and Igor Sega for many useful suggestions.

Appendix A: Spin-0 propagator

Propagator $\widetilde{G}_0(k)$ defined by Eq. (11a) can be made regular and approximate spin-0 propagator $\widetilde{G}_{KG}(k) \equiv (k^2 + m^2)^{-1}$, $m \geq 0$, as accurately as desired for all

k up to some extremely large value of $|k^2|$ by choosing q_0, $f_0(y)$, $f_1(y)$ and $t(y)$ so that: (i) conditions (B) in Sect. 4.2 are satisfied; (ii) at $y = -m^2$,

$$[1 - I_1(y)][1 - I_3(y)] = yI_2^2(y)\,, \tag{A1a}$$

$$q_0[1 - I_3(y)] = d\{[1 - I_1(y)][1 - I_3(y)] + yI_2^2(y)\}/dy\,; \tag{A1b}$$

and (iii) $|t(y)|$ is sufficiently large.

Appendix B: φ^4 theory

The Lagrangian of φ^4 theory reads

$$-\frac{1}{2}(\partial\varphi)^2 - \frac{1}{2}m^2\varphi^2 - \tfrac{\lambda}{4!}\varphi^4\,. \tag{B1}$$

Its Euler-Lagrange equation is

$$\partial^2\varphi - m^2\varphi = \tfrac{\lambda}{6}\varphi^3\,. \tag{B2}$$

According to Sect. 4.2, the Euler-Lagrange equations of the Lagrangian (9) with

$$j(x) = \tfrac{\lambda}{4!}\varphi^4[x;\Psi] \tag{B3}$$

contain in k-space the subsystem

$$\varphi^4[k;\Psi] = \tfrac{\lambda}{6}\widetilde{G}_0(k)\varphi^3\widetilde{[x;\Psi]}\,, \tag{B4}$$

which is equivalent to the Euler-Lagrange equation (B2) of (B1) if we replace $\widetilde{G}_0(k)$ with $\widetilde{G}_{KG}(k)$. So under conditions given in Appendix A, the Euler-Lagrange equations (B2) of the transport-theoretic Lagrangian (9) with (B3) contain a subsystem corresponding to the Euler-Lagrange equation (B2) of φ^4 theory with a regularized propagator.

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OKVIR TEORIJE KOJA JE OSNOVA STANDARDNOG MODELA

Predlažemo novu, fizički motiviranu postavku za razvijanje teorije koja je osnova standardnog modela u četiri-dimenzijskom prostoru–vremenu: Euler-Lagrangeove jednadžbe te teorije formalno sliče osnovnim jednadžbama dinamike tekućina u kinetičkoj teoriji plinova. Kao podršku toj postavci ističemo lokalnu i kovarijantnu Lagrangeovu funkciju čije Euler-Lagrangeove jednadžbe sadrže podsustav koji je jednakovrijedan Euler-Lagrangeovim jednadžbama standardnog modela s kovarijantno sređenim propagatorima i podsustav koji opisuje učinke iznad brzine svjetlosti.