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STRING ENERGY IN THE MONOPOLE NAMBU-JONA-LASINIO MODEL

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The string energy is calculated to the one–monopole loop approximation in the dual monopole Nambu-Jona-Lasinio (MNJL) model with dual Dirac strings by taking into account quantum fluctuations of the dual vector field and string shape fluctuations around the Abrikosov flux line. The quantum fluctuations are shown to increase the value of the string tension. The string shape fluctuations give contributions in the form of a Coulomb–like universal potential. Due to the London limit regime the scalar field exchange is taken into account in the tree approximation.

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1. Introduction

A dual superconductor is a very interesting model for the nonperturbative properties of QCD vacuum leading to quark confinement. In a dual $U(1)$ superconductor picture, magnetic "colour"-monopole $\chi(x)$ - antimonopole $\bar{\chi}(x)$ pairs condense to a more advantageous ground state forming a nonperturbative (superconducting) vacuum. The condensed phase is characterized by a non-zero value of the magnetic monopole condensate which is regarded as an order parameter of the system. The monopole condensation together with the included dual Dirac string between a quark and an antiquark pair leads to the formation of an Abrikosov flux line. Due to the Abrikosov flux line, we get a linearly rising confinement potential [1, 2].

2. String energy

The energy of a dual Dirac string in the superconducting phase can be divided into two terms [2]

$$W_{\text{string}} = W_{\text{tree}} + W_{\text{loop}}, \tag{1}$$

where the contributions $W_{\rm tree}$ and $W_{\rm loop}$ are defined by

$$\begin{aligned} W_{\rm tree} &= -\int \mathrm{d}^3x\,\mathcal{L}_{\rm string}\{C^\nu[\mathcal{E}(x)]\} \\ &= -\frac{1}{4}\,M_C^2\int\int \mathrm{d}^3x\mathrm{d}^4y\mathcal{E}_{\mu\alpha}(x)\Delta^\alpha_\nu(x-y,M_C)\mathcal{E}^{\mu\nu}(y), \end{aligned} \tag{2}$$

and

$$W_{\rm loop} = \int \mathrm{d}^3x^{(M)} < 0|\mathrm{T}\Big(\Big(-\frac{g^2}{2M_C^2}\,[\bar\chi_M(x)\gamma_\mu\chi_M(x)]$$
$$\times[\bar\chi_M(x)\gamma^\mu\chi_M(x)]\Big)\exp(-\mathrm{i}g)\int \mathrm{d}^4y\,\bar\chi_M(y)\,\gamma^\mu\,C_\mu[\mathcal{E}(y)]\,\chi_M(y)\Big)|0>^{(M)}. \tag{3}$$

Here C^ν is a dual vector field with mass M_C, forming an Abrikosov flux line and $\chi(x)$ the fermionic monopole field. Further,

$$\Delta^\alpha_\nu(x-y,M_C) = (g^\alpha_\nu + 2\partial^\alpha\partial_\nu/M_C^2)\Delta(x-y;M_C)\,,$$

where $\Delta(x-y;M_C)$ coincides with the Green function of a scalar field with mass M_C. The Abrikosov flux line $C_\mu[\mathcal{E}(x)]$ is related to the electric field $\mathcal{E}_{\mu\nu}(x)$ induced by a dual Dirac string via the relation:

$$C^\nu[\mathcal{E}(x)] = -\int \mathrm{d}^4x'\,\Delta(x-x'\,)\,\partial_\mu{}^*\mathcal{E}^{\mu\nu}(x'\,), \tag{4}$$

The computation of the string energy is first carried out for the static straight string of length L:

$$\vec{\mathcal{E}}(\vec{x}\,) = \vec{e}_z\,Q\,\delta(x)\,\delta(y)\left[\theta\left(z-\frac{1}{2}\,L\right)-\theta\left(z+\frac{1}{2}\,L\right)\right], \tag{5}$$

where a quark and an antiquark are placed at $\vec{X}_Q = (0,0,\frac{1}{2}\,L)$ and $\vec{X}_{-Q} = (0,0,-\frac{1}{2}\,L)$. The unit vector $\vec{e}_z$ is directed along the z–axis and $\theta(z)$ is the step–function. The field strength, Eq.(5), induces the following dual–vector potential:

$$<\vec{C}(\vec{x}\,)> = -\,\mathrm{i}\,Q\int\frac{\mathrm{d}^3k}{4\,\pi^3}\,\frac{\vec{k}\times\vec{e}_z}{k_z}\,\frac{1}{M_C^2+\vec{k}^{\,2}}\,\sin\left(\frac{k_zL}{2}\right)e^{\mathrm{i}\,\vec{k}\,\cdot\,\vec{x}}. \tag{6}$$

- The potential in the tree approximation for a sufficiently long string can be calculated as

$$W_{\rm tree} = L\frac{Q^2 M_C^2}{8\pi}\left[\ln\left(1+\frac{M_\sigma^2}{M_C^2}\right) + 2\,E_1(M_C L) - \frac{2}{M_C L}\left(1-e^{-M_C L}\right)\right] - \frac{Q^2}{4\pi}\,\frac{e^{-M_C L}}{L}\,. \tag{7}$$

The last term in Eq.(7) is the Yukawa potential appearing due to the contribution of the term proportional to the derivatives of the Green function $\Delta(x-y;M_C)$, i.e., $\partial^\alpha\partial_\nu\Delta(x-y;M_C)$. $E_1(M_C L)$ is an exponential integral function.

- The loop contributions to the string energy $W_{\rm loop}$ can be brought to the form:

$$W_{\rm loop} = L\,\frac{Q^2 M_C^2}{8\pi}\left(1+\frac{8g^2}{M_C^2}\frac{<\bar\chi\chi>}{M_\sigma}\right)^2 \times\left[\ln\left(1+\frac{M_\sigma^2}{M_C^2}\right) + 2\,E_1(M_C L) - \frac{2}{M_C L}\left(1-e^{-M_C L}\right)\right], \tag{8}$$

where $<\bar\chi\chi>$ is the monopole condensate. It is regarded as the order parameter of the superconducting phase in the MNJL model [3].

Adding up Eqs. (7) and (8), we arrive at the total string energy

$$W = L\,\frac{Q^2 M_C^2}{4\pi}\left(1+\frac{8g^2}{M_C^2}\frac{<\bar\chi\chi>}{M_\sigma}+\frac{32g^4}{M_C^4}\frac{<\bar\chi\chi>^2}{M_\sigma^2}\right)\times \left[\ln\left(1+\frac{M_\sigma^2}{M_C^2}\right) + 2\,E_1(M_C L) - \frac{2}{M_C L}\left(1-e^{-M_C L}\right)\right] - \frac{Q^2}{4\pi}\,\frac{e^{-M_C L}}{L}\,. \tag{9}$$

From Eq.(9) we extract the string tension σ:

$$\sigma = \frac{Q^2 M_C^2}{4\pi}\left(1+\frac{8g^2}{M_C^2}\frac{<\bar\chi\chi>}{M_\sigma}+\frac{32g^4}{M_C^4}\frac{<\bar\chi\chi>^2}{M_\sigma^2}\right)\ell n\left(1+\frac{M_\sigma^2}{M_C^2}\right), \tag{10}$$

which is increased in comparison with the string tension calculated in the tree approximation

$$\sigma_0 = \frac{Q^2 M_C^2}{8\pi}\ell n\left(1+\frac{M_\sigma^2}{M_C^2}\right). \tag{11}$$

The string shape fluctuations give a contribution only in the form of a Coulomb–like potential $W_{\rm string-shape} = -\alpha_{\rm string}/L$ with the universal coupling constant $\alpha_{\rm string} = \pi/12$ and $\alpha_{\rm string} = \pi/3$ for open and closed strings, respectively [3, 4].

Conclusion

It is shown that in the MNJL model with dual Dirac strings, the quantum fluctuations of the dual–vector field $C_\mu(x)$ and the scalar field $\sigma(x)$ around the shape of the Abrikosov flux line provide a string tension which exceeds the value obtained in the tree approximation. The string shape fluctuations give a contribution in the form of the Coulomb–like potential with the universal coupling constant $\alpha_{\rm string} = \pi/12$ or $\alpha_{\rm string} = \pi/3$ for open and closed strings, respectively. Unlike the dual Higgs model with dual Dirac strings, the mass of the dual–vector field M_C is not proportional to the order parameter $< \bar{\chi}\chi >$. Due to the independence of the mass of the dual–vector field on the monopole condensate, the string tension σ_0 calculated in the tree–approximation does not depend on the monopole condensate, too. The dependence on the magnetic monopole condensate appears by virtue of the contributions of the quantum field fluctuations of the dual–vector $C_\mu(x)$ and the scalar $\sigma(x)$ fields around the shape of the Abrikosov flux line. Very similar to the dual Higgs model, the quantum field fluctuations increase the value of the string tension. This implies that for the consistent investigation of the superconducting mechanism of quark confinement within the dynamics of magnetic monopoles and dual Dirac strings, one cannot deal with a classical level only, and quantum contributions should be taken into account.

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ENERGIJA NITI U MONOPOLNOM MODELU NAMBUA, JONE I LASINIA

Računa se energija niti u aproksimaciji monopolne petlje u dualnom monopolnom modelu Nanbua, Jone i Lasinia, s dualnim Diracovim nitima, uzimajući u obzir kvantne fluktuacije dualnoa vektorskog polja i oblika niti oko Abrikosove krivulje toka. Pokazuje se da kvantne fluktuacije povećavaju napetost niti. Fluktuacije oblika niti daju doprinose u vidu općeg potencijala Coulombovog oblika. Zbog uvjeta Londonove granice, izmjena skalarnim poljem se uzima u aproksimaciji drveta.