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UNIFICATION OF SPINS AND CHARGES UNIFIES ALL INTERACTIONS

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In a space of d Grassmann coordinates, two types of generators of Lorentz transformations, one of spinorial and the other of vectorial character, define the representations of the group $\mathrm{SO}(1, d-1)$ for fermions and bosons, respectively. The eigenvalues of commuting operators of the subgroups SO(1,3), SU(3), SU(2) and U(1) can be identified with spins and Yang-Mills charges of either fermionic or bosonic fields, which allows the unification of all the internal degrees of freedom, separately for fermions and separately for bosons. When, accordingly, all interactions are unified, the Yang-Mills fields, the Higgs fields and the Yukawa couplings appear as a part of a gravitational field. The theory suggests four families of quarks and leptons. The equal number of fermions and bosons assures supersymmetry.

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1. *Introduction*

Since Newton, the understanding of the laws of Nature has improved from the laws leaving infinitely many parameters free (all possible masses as well as forces to be determined by the experiment), to the unified quantum theory of electromagnetic, weak and colour interactions, known as the electroweak standard model, which only has around 20 free parameters. It has been discovered that we live not only in four dimensional space-time of commuting coordinates, but also in the internal space which defines spins and charges of fermionic and bosonic fields. It is not yet known how the gravitational field enters into the unique quantum theory of laws of Nature, or why ordinary space-time of only four dimensions is experienced, or why metric in this world is not Euclidean but Minkowski, or why the internal space of spins and charges allows fermionic and bosonic degrees of freedom, while in the ordinary space-time only bosonic degrees of freedom exist. Because systems with many degrees of freedom can only be treated in an approximate way, and because searching for solutions for such systems requires very sophisticated calculations, many phenomena in matter are not yet understood. To understand the evolution of the Universe, it has to be discovered also which properties make four-dimensional space-time so special.

Not only has the standard model free parameters, it also has several assumptions: i) There are three families of quarks and leptons, which are massless. ii) Quarks carry colour (SU(3)) and Y- (U(1)) charge. If left-handed, they carry also weak (SU(2)) charge, if right-handed they are weak chargeless. Left-handed leptons carry weak and Y-charge, right-handed leptons carry only Y-charge. iii) Y-charge has to be chosen so that the electromagnetic charge corresponds to physical particles. Accordingly, right-handed chargeless leptons (neutrinos) carry no charge. iv) Charges of fermions are described by the fundamental representations of the groups SU(3), SU(2) and U(1), spins of fermions are described by the fundamental representations of the Lorentz group SO(1,3). Gauge bosons are massless vectors with respect to SO(1,3) and carry either colour or weak charge in the adjoint representations with respect to the corresponding groups. v) There exists a complex massive field, which is a scalar with respect to SO(1,3), it is the weak doublet and carries the Y-charge but no colour charge. Interacting with the gauge weak and Y-fields, the scalar field causes the superpositions of these fields, which manifest as the massless electromagnetic field and the massive weak fields. The massive scalar field makes the fermionic fields massive by the interaction through the Yukawa couplings. Neutrinos don't interact with the scalar field and stay accordingly massless. The (two) parameters of the scalar field, the (two) qauge couplings and the Yukawa couplings are the free parameters of the model, to be determined by the experiment.

The standard model does not say: i) Why SU(3)×SU(2)×U(1) has to be the input symmetry of the model, which due to the Higgs fields breaks into SU(3)×U(1)? ii) Why fermions are left-handed SU(2) doublets and right-handed SU(2) singlets, that is, why at all the weak charge is connected with the spin? iii) Where do the generations come from? Why there exist only three generations of fermions? iv) Where do Yukawa couplings come from?

In the standard model, the Y-charges are free parameters of the model. Embedding SU(3), SU(2) and U(1) in SU(5) fixes the Y-charge uniquely, but leaves the connection between the handedness and charges undetermined. To connect handedness and charges, spins and charges have to unify.

This work presents the approach which unifies spins and charges and might answer to the open questions of the standard model. Assuming that the space has d commuting and d anticommuting Grassmann coordinates, with the same metric in both spaces, all the internal degrees of freedom, spins and charges are described by the generators of the Lorentz transformations in Grassmann space, in which there exist two types of generators of the Lorentz transformations and translations: one is of spinorial character that determines properties of fermions, the other is of vectorial character that determines properties of bosons. Their representations can be expressed as monomials of Grassmann coordinates θ^a. If $d \geq 14$, the generators of the subgroup SO(1,3) of the group SO(1,13) determine spins of fields, while generators of the subgroups SU(3), SU(2), U(1) determine their charges, connecting spins with the charges. The theory predicts four families of fermions.

The Lagrange function describing a particle on a supergeodesic requires the momentum of the particle in Grassmann space to be proportional to the Grassmann coordinate and brings accordingly the Clifford algebra and the spinorial degrees of freedom into the theory.

The supervielbeins, transforming the supergeodesics from the freely falling to the external coordinate system, define all gauge fields – gravitational, Yang-Mills, the Higgs scalars and also the Yukawa couplings [1,2].

2. *Coordinate Grassmann space and internal degrees of freedom of fermions and bosons*

We show in this section that Grassmann space can be used to describe all internal degrees of freedom of all known fermionic and bosonic fields, that is of quarks and leptons, Yang-Mills and Higgs fields and that spins and charges unify.

We define a d-dimensional Grassmann space of real anticommuting coordinates $\{\theta^a\}, a = 0, 1, 2, 3, 5, 6, ..., d$, satisfying the anticommutation relations $\theta^a\theta^b + \theta^b\theta^a := \{\theta^a, \theta^b\} = 0$, called the Grassmann algebra [2,3]. The metric tensor $\eta_{ab} = \mathrm{diag}(1, -1, -1, -1, ..., -1)$ lowers the indices of a vector $\{\theta^a\} = \{\theta^0, \theta^1, ..., \theta^d\}$, $\theta_a = \eta_{ab}\theta^b$. Linear transformation actions on vectors $(\alpha\theta^a + \beta x^a)$, $(\alpha\acute{\theta}^a + \beta\acute{x}^a) = L^a{}_b(\alpha\theta^b + \beta x^b)$, which leave forms $(\alpha\theta^a + \beta x^a)(\alpha\theta^b + \beta x^b)\eta_{ab}$ invariant, are called the Lorentz transformations $L^a{}_c L^b{}_d \eta_{ab} = \eta_{cd}$.

A linear space spanned over a Grassmann coordinate space of d coordinates has the dimension 2^d. If monomials $\theta^{\alpha_1}\theta^{\alpha_2}....\theta^{\alpha_n}$ are taken as a set of basic vectors with $\alpha_i \neq \alpha_j$, half of the vectors have an even (those with an even n) and half of the vectors have an odd (those with an odd n) Grassmann character. Any vector in this space may be represented as a linear superposition of monomials

$$f(\theta) = \alpha_0 + \sum_{i=1}^{d} \alpha_{a_1 a_2 .. a_i} \theta^{a_1}\theta^{a_2}....\theta^{a_i}, \;\; a_k < a_{k+1}, \tag{2.1}$$

where constants $\alpha_0, \alpha_{a_1 a_2 .. a_i}$ are complex numbers.

In Grassmann space, the left derivatives have to be distinguished from the right derivatives, due to the anticommuting nature of the coordinates [2,3]. We make use of left derivatives $\overrightarrow{\partial}^{\theta}_a := \overrightarrow{\partial}/\partial\theta^a$, $\overrightarrow{\partial}^{\theta a} := \eta^{ab}\overrightarrow{\partial}^{\theta}_b$, on vectors of the linear space of monomials $f(\theta)$, defined as follows: $\overrightarrow{\partial}^{\theta}_a\, \theta^b f(\theta) = \delta^b{}_a f(\theta) - \theta^b \overrightarrow{\partial}^{\theta}_a\, f(\theta)$. Here α is a constant of either commuting ($\alpha\theta^a - \theta^a\alpha = 0$) or anticommuting ($\alpha\theta^a + \theta^a\alpha = 0$)

character, and $n_{a\partial}$ is defined as follows

$$n_{AB} = \begin{cases} +1, & \text{if A and B have Grassmann odd character} \\ 0, & \text{otherwise} \end{cases} .$$

We define the following linear operators [1,2].

$$p^\theta{}_a := -i\overrightarrow{\partial^\theta}_a, \;\; \widetilde{a}^a := i(p^{\theta a} - i\theta^a), \;\; \widetilde{\widetilde{a}}^a := -(p^{\theta a} + i\theta^a). \tag{2.2}$$

According to the inner product defined in what follows, the operators $\widetilde{a}^a$ and $\widetilde{\widetilde{a}}^a$ are either hermitian or antihermitian operators. We define the generalized commutation relations (which follow from the corresponding Poisson brackets [1,2]): $\{A, B\} := AB - (-1)^{n_{AB}}BA$, fulfilling the relation $\{A, B\} = (-1)^{n_{AB}+1}\{B, A\}$. We find accordingly

$$\{p^{\theta a}, p^{\theta b}\} = 0 = \{\theta^a, \theta^b\}, \quad \{p^{\theta a}, \theta^b\} = -i\eta^{ab}, \tag{2.3}$$

$$\{\widetilde{a}^a, \widetilde{a}^b\} = 2\eta^{ab} = \{\widetilde{\widetilde{a}}^a, \widetilde{\widetilde{a}}^b\}, \quad \{\widetilde{a}^a, \widetilde{\widetilde{a}}^b\} = 0.$$

We see that θ^a and $p^{\theta a}$ form a Grassmann odd Heisenberg algebra, while $\widetilde{a}^a$ and $\widetilde{\widetilde{a}}^a$ form the Clifford algebra.

We define two kinds of operators [2]. The first ones are binomials of operators forming the Grassmann odd Heisenberg algebra

$$S^{ab} := (\theta^a p^{\theta b} - \theta^b p^{\theta a}). \tag{2.4a}$$

The second ones are binomials of operators forming the Clifford algebra

$$\widetilde{S}^{ab} := -\frac{\mathrm{i}}{4}[\,\widetilde{a}^a, \widetilde{a}^b\,], \;\; \widetilde{\widetilde{S}}^{ab} := -\frac{\mathrm{i}}{4}[\,\widetilde{\widetilde{a}}^a, \widetilde{\widetilde{a}}^b\,], \tag{2.4b}$$

with $[A, B] := AB - BA$ and $S^{ab} = \widetilde{S}^{ab} + \widetilde{\widetilde{S}}^{ab}, \quad \{\widetilde{S}^{ab}, \widetilde{\widetilde{S}}^{cd}\} = 0 = \{\widetilde{S}^{ab}, \widetilde{\widetilde{a}}^c\} = \{\widetilde{a}^a, \widetilde{\widetilde{S}}^{bc}\}$. Either S^{ab} or $\widetilde{S}^{ab}$ or $\widetilde{\widetilde{S}}^{ab}$ fulfil the Lie algebra of the Lorentz group $SO(1, d-1)$ in the d-dimensional Grassmann space: $\{M^{ab}, M^{cd}\} = -\mathrm{i}(M^{ad}\eta^{bc} + M^{bc}\eta^{ab} - M^{ac}\eta^{bd} - M^{bd}\eta^{ac})$, with M^{ab} equal either to S^{ab} or to $\widetilde{S}^{ab}$ or to $\widetilde{\widetilde{S}}^{ab}$ and $M^{ab} = -M^{ba}$.

By solving the eigenvalue problem (see below)*we find that operators* $\widetilde{S}^{ab}$, as well as the operators $\widetilde{\widetilde{S}}^{ab}$, *define* the fundamental or *the spinorial representations of the Lorentz group, while* $S^{ab} = \widetilde{S}^{ab} + \widetilde{\widetilde{S}}^{ab}$ *define the vectorial representations of the Lorentz group* $\mathrm{SO}(1, d-1)$.

Group elements are in any of the three cases defined by: $\mathcal{U}(\omega) = e^{\mathrm{i}\frac{1}{2}\omega_{ab}M^{ab}}$, where ω_{ab} are the parameters of the group. We assume that differentials of

Grassmann coordinates $d\theta^a$ fulfill the Grassmann anticommuting relations [2,3] $\{d\theta^a, d\theta^b\} = 0$ and we introduce a single integral over the whole interval of $d\theta^a$ $\int d\theta^a = 0$, $\int d\theta^a \theta^a = 1, a = 0, 1, 2, 3, 5, .., d$, and the multiple integral over d coordinates $\int d^d\theta^0\theta^1\theta^2\theta^3\theta^4...\theta^d = 1$, with $d^d\theta := d\theta^d...d\theta^3 d\theta^2 d\theta^1 d\theta^0$ in the standard way.

We define [2,3] the inner product of two vectors $< \varphi|\theta >$ and $< \theta|\chi >$, with $< \varphi|\theta >=< \theta|\varphi >^*$ as follows:

$$< \varphi|\chi >= \int d^d\theta(\omega < \varphi|\theta >) < \theta|\chi >, \tag{2.5}$$

with the weight function $\omega = \prod_{k=0,1,2,3,..,d}(\frac{\partial}{\partial\theta^k} + \theta^k)$, which operates on the first function $<\varphi|\theta>$ only, and we define $(\alpha_{a_1a_2...a_k}\theta^{a_1}\theta^{a_2}...\theta^{a_k})^+ = (\theta^{a_k}).....(\theta^{a_2})(\theta^{a_1})(\alpha_{a_1a_2...a_k})^*$. According to the above definition of the inner product, it follows that $\widetilde{a}^{a+} = -\eta^{aa}\widetilde{a}^a$ and $\widetilde{\widetilde{a}}^{a+} = -\eta^{aa}\widetilde{\widetilde{a}}^a$. The generators of the Lorentz transformations (Eqs.(2.4)) are accordingly self adjoint or antiself adjoint operators.

According to Eqs.(2.2) and (2.4), we find

$$S^{ab} = -\mathrm{i}\left(\theta^a \frac{\partial}{\partial\theta_b} - \theta^b\frac{\partial}{\partial\theta_a}\right) \quad \widetilde{a}^a = \left(\frac{\partial}{\partial\theta_a} + \theta^a\right), \ \widetilde{\widetilde{a}}^a = \mathrm{i}\left(\frac{\partial}{\partial\theta_a} - \theta^a\right), \tag{2.6}$$

$$\widetilde{S}^{ab} = \frac{-\mathrm{i}}{2}\left(\frac{\partial}{\partial\theta_a} + \theta^a\right)\left(\frac{\partial}{\partial\theta_b} + \theta^b\right), \quad \widetilde{\widetilde{S}}^{ab} = \frac{\mathrm{i}}{2}\left(\frac{\partial}{\partial\theta_a} - \theta^a\right)\left(\frac{\partial}{\partial\theta_b} - \theta^b\right), \ \text{ if } \ a{\neq}b.$$

To find eigenvectors of any operator A, we solve the eigenvalue problem

$$< \theta|\widetilde{A}_i|\widetilde{\varphi} >= \widetilde{\alpha}_i < \theta|\widetilde{\varphi} >, \ < \theta|A_i|\varphi >= \alpha_i < \theta|\varphi >, \ i = \{1, r\}, \tag{2.7}$$

where $\widetilde{A}_i$ and A_i stand for r commuting operators of spinorial and vectorial character, respectively. To solve equations (2.7), we express the operators in the coordinate representation and write the eigenvectors as polynomials of θ^a. We orthonormalize the vectors according to the inner product, defined in Eq.(2.5).

The algebra of the group SO(1,d-1) *or* SO(d) contains [1] n subalgebras defined by operators $\tau^{Ai}, A = 1, n; i = 1, n_A$, where n_A is the number of elements of each subalgebra, with the properties

$$[\tau^{Ai}, \tau^{Bj}] = \mathrm{i}\delta^{AB} f^{Aijk}\tau^{Ak}, \tag{2.8}$$

if operators τ^{Ai} can be expressed as linear superpositions of operators M^{ab}, $\tau^{Ai} = c^{Ai}{}_{ab}M^{ab}$, $c^{Ai}{}_{ab} = -c^{Ai}{}_{ba}$, $A = 1, n$, $i = 1, n_A$, $a, b = 1, d$. Here f^{Aijk} are structure constants of the (A) subgroup with n_A operators. *According to the two types of operators M^{ab}, one of spinorial and the other of vectorial character, there are two types of operators τ^{Ai} defining subalgebras of spinorial and vectorial* *character, respectively*, those of spinorial types being expressed with either $\tilde{S}^{ab}$ or $\tilde{\tilde{S}}^{ab}$ and those of vectorial type being expressed by S^{ab}. All these operators are, according to Eq.(2.8), defined by the same coefficients $c^{Ai}{}_{ab}$ and the same structure constants f^{Aijk}. From Eq.(2.8) the following relations among constants $c^{Ai}{}_{ab}$ follow [2]: $-4c^{Ai}{}_{ab}c^{Bjb}{}_{c} - \delta^{AB} f^{Aijk} c^{Ak}{}_{ac} = 0$.

Solving [2] the eigenvalue problem for the Casimirs of the subgroups SO(1,3), SU(3),SU(2) and U(1), we find the representations of these subgroups expressed as polynomials of θ^a. In the subspace of $\theta^0, .., \theta^3$, we find four times two spinors ($2^4 = 4 \times 2 \times 2$), suggesting that *there are four families of quarks and leptons*. We also find a scalar, a pseudoscalar and (two) three vectors and (two) four vectors. In the subspace of SU(2), we find in the spinorial sector two doublets and four singlets (of either even or odd Grassmann character), while in the vectorial sector we find five singlets, one triplet (of Grassmann even character) and four doublets (of Grassmann odd character). We find that *the operator* $\tilde{S}^{mh}$*, if* $m = 0, 1, 2, 3$ *and* h *determines the space* of θ^h , defining the representations of the SU(2) group, *transforms the left-handed* SU(2) *doublets to right-handed* SU(2) *singlets*, *connecting left-handed weak doublets and right-handed weak singlets into the same multiplet*. In the space of SU(3) subgroup, we find in the spinorial sector triplets and singlets, in the vectorial sector we find singlets and octets and also triplets. The operator $\tilde{S}^{mk}$, if $m = 0, 1, 2, 3$ and k determines the space of the SU(3) sector, transforms left-handed triplets into right-handed antitriplets or antisinglets, putting left-handed fermions and right-handed antifermions in the same multiplet.

The representations which are the direct product of the representations of the SO(1,3), SU(3) and SU(2) subgroups, with well defined Y-charge, carry all the quantum numbers needed in the standard model to describe four families of quarks and leptons (left-handed weak doublets in the same multiplet with the right-handed weak singlet), all Yang-Mills fields and the Higgs scalar. We also find vectorial representations which are the SU(3) triplets, but we don't find spinorial representations, which are the SU(3) octets, which means that in spite of the fact that the supersymmetry is guaranteed due to equal number of spinorial and vectorial degrees of freedom, the ordinary supersymmetry in this approach is not possible.

3. Lagrange function for a free particle and for a particle in gauge fields

In this section, we derive the Dirac-like equation for a particle which lives in d-dimensional ordinary and Grassmann space. The gravitational field in d dimensions manifests in four-dimensional subspace as the ordinary gravity, the Yang-Mills fields and the Higgs field and takes also care of the Yukawa couplings.

For a free particle which lives in a d-dimensional ordinary space of commuting coordinates and in a d-dimensional Grassmann space of anticommuting coordinates $X^a \equiv \{x^a, \theta^a\}$, and has its geodesics parametrized by an ordinary Grassmann even n parameter (τ) and a Grassmann odd n parameter (ξ), we define the dynamics by choosing the action [1,4] $I = \frac{1}{2}\int \mathrm{d}\tau \mathrm{d}\xi E E^i_A \partial_i X^a E^j_B \partial_j X^b \eta_{ab}\eta^{AB}$, where $\partial_i := (\partial_\tau, \overrightarrow{\partial}_\xi), \tau^i = (\tau, \xi)$, while E^i_A determines a metric on a two dimensional superspace τ^i , $E = \mathrm{Det}(E^i_A)$. We choose $\eta_{AA} = 0, \eta_{12} = 1 = \eta_{21}$, while η_{ab} is the Minkowski metric. The action is invariant under the Lorentz transformations of supercoordinates: $X'^a = L^a{}_b X^b$ and is locally supersymmetric.

Taking into account that either x^a or θ^a depend on an ordinary time parameter τ and that $\xi^2 = 0$, the geodesics can be described as a polynomial of ξ as follows: $X^a = x^a + \varepsilon \xi \theta^a$. We choose ε^2 to be equal either to + i or to - i so that it defines two possible combinations of supercoordinates, and we choose the metric $E^i{}_A$: $E^1{}_1 = 1, E^1{}_2 = -\varepsilon M, E^2{}_1 = \xi, E^2{}_2 = N - \varepsilon\xi M$, with N and M Grassmann even and odd parameters, respectively. We write $\dot{A} = \mathrm{d}A/\mathrm{d}\tau$, for any A.

After integrating the above action over the Grassmann odd coordinate $\mathrm{d}\xi$, the action for a superparticle follows:

$$\int \mathrm{d}\tau (\frac{1}{N}\ \dot{x}^a \dot{x}_a + \varepsilon^2 \dot{\theta}^a \theta_a - \frac{1}{N} 2\varepsilon^2 M\ \dot{x}^a \theta_a). \tag{3.1}$$

Defining the two momenta $p^\theta_a := \overrightarrow{\partial} L/\partial \dot{\theta}^a = \epsilon^2 \theta^a$, $p_a := \partial L/\partial \dot{x}^a = \frac{2}{N}(\dot{x}_a - M p^{\theta a})$, the first *declairing that the coordinate in Grassmann space is proportional to its conjugate momentum*, the two Euler-Lagrange equations follow: $\dot{p}^a = 0, \quad \dot{p}^{\theta a} = \frac{1}{2}\varepsilon^2\ M\ p^a$.

Variation of the action (3.1) with respect to M and N gives two constraints:

$$\chi^1 := p^a a^\theta_a = 0, \quad \chi^2 = p^a p_a = 0, \quad a^\theta_a := i p^\theta_a + \varepsilon^2 \theta_a, \tag{3.2}$$

$$\text{while} \quad \chi^3{}_a := -p^\theta_a + \epsilon^2 \theta_a = 0,$$

and is the third type of constraints of the action (3.1). For $\varepsilon^2 = -$ i, we find (Eq.(2.2)), that $a^\theta{}_a = \tilde{a}^a$, $\chi^3{}_a = \tilde{\tilde{a}}_a = 0$.

We find the generators of the Lorentz transformations for the action (3.1) to be (see also Eq. (2.4)) $M^{ab} = L^{ab} + S^{ab}$, $L^{ab} = x^a p^b - x^b p^a$, $S^{ab} = \theta^a p^{\theta b} - \theta^b p^{\theta a} = \tilde{S}^{ab} + \tilde{\tilde{S}}^{ab}$, which shows that parameters of the Lorentz transformations are the same in both spaces.

Canonical quantization [1,2] of the action (3.1) determines the algebra of Eq. (2.3), while the constraints lead to the Dirac-like and the Klein-Gordon equations

$$p^a \tilde{a}_a |\tilde{\Psi}>= 0\ ,\ \ p^a p_a |\tilde{\Psi}>= 0, \ \ \text{with} \ \ p^a \tilde{a}_a p^b \tilde{a}_b = p^a p_a. \tag{3.3}$$

We further see that although *the operators* $\tilde{a}^a$ fulfill the Clifford algebra, they *cannot be recognized as the Dirac* $\tilde{\gamma}^a$ *operator, since having an odd Grassmann character they transform* Grassmann odd polynomials to Grassmann even polynomials, that means *fermions into bosons*, which is not the case with the Dirac γ^a matrices. We, therefore, define [1] as Dirac γ^m operators: $\tilde{\gamma}^m = -\tilde{\tilde{a}}^0 \tilde{a}^m;\ m = 0, 1, 2, 3.$

Multyplying the first equation in Eqs. (3.3) by $\tilde{\tilde{a}}^0$, we recognize the equation

$$(\tilde{\gamma}^m p_m)|\tilde{\psi}>= 0 \ , \ m = 0,1,2,3. \tag{3.3a}$$

as the Dirac equation. It can be checked that $\tilde{\gamma}^m$ fulfill the Clifford algebra $\{\tilde{\gamma}^m, \tilde{\gamma}^n\} = \eta^{mn}$, while $\tilde{S}^{mn} = -\mathrm{i}\frac{1}{4}[\tilde{\gamma}^m, \tilde{\gamma}^n]_-, m \in \{0,3\}$. We must say, that the constraint $\tilde{\tilde{a}}^a = 0$ can only be taken into account in the expectation value form.

The dynamics of a point particle in gauge fields, the gravitational and the Yang-Mills fields, can be obtained by transforming vectors from a freely falling to an external coordinate system [5]. To do this, supervielbeins $e^a{}_\mu$ have to be introduced, which in our case depend on ordinary and on Grassmann coordinates, as well as on the two parameters $\tau^i = (\tau, \xi)$. The index a refers to a freely falling coordinate system (a Lorentz index), the index μ refers to an external coordinate system (an Einstein index). Vielbeins with a Lorentz index smaller than five will determine ordinary gravitational fields. Those with a Lorentz index higher than three will define, according to what we have said in Sect. 2, the Yang-Mills fields.

We write the transformation of vectors as follows $\partial_i X^a = \mathbf{e}^a{}_\mu \partial_i X^\mu$, $\partial_i X^\mu = \mathbf{f}^\mu{}_a \partial_i X^a$, $\partial_i = (\partial_\tau, \partial_\xi)$. From this, it follows that $\mathbf{e}^a{}_\mu \mathbf{f}^\mu{}_b = \delta^a{}_b$, $\mathbf{f}^\mu{}_a \mathbf{e}^a{}_\nu = \delta^\mu{}_\nu$.

We make a Taylor expansion of vielbeins with respect to ξ: $\mathbf{e}^a{}_\mu = e^a{}_\mu + \varepsilon\xi\theta^b e^a{}_{\mu b}$, $\mathbf{f}^\mu{}_a = f^\mu{}_a - \varepsilon\xi\theta^b f^\mu{}_{ab}$. Both expansion coefficients again depend on ordinary and on Grassmann coordinates. Having an even Grassmann character, $e^a{}_\mu$ will describe the spin 2 part of a gravitational field. The coefficients $\varepsilon\theta^b e^a{}_{\mu b}$ have an odd Grassmann character. They define the spin connections [1,2]. It follows that $e^a{}_\mu f^\mu{}_b = \delta^a{}_b$, $f^\mu{}_a e^a{}_\nu = \delta^\mu{}_\nu$, $e^a{}_{\mu b} f^\mu{}_c = e^a{}_\mu f^\mu{}_{cb}$. We find the metric tensor $\mathbf{g}_{\mu\nu} = \mathbf{e}^a{}_\mu \mathbf{e}_{a\nu}$, $\mathbf{g}^{\mu\nu} = \mathbf{f}^\mu{}_a \mathbf{f}^{\nu a}$.

Rewriting the action for a free particle in terms of an external coordinate system, using the Taylor expansion of supercoordinates X^μ and superfields $\mathbf{e}^a{}_\mu$ and integrating the action over the Grassmann odd parameter ξ, the action

$$\int \mathrm{d}\tau\{\frac{1}{N} g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu \ - \ \frac{1}{N}\ \epsilon^2\ 2M\theta_a e^a{}_\mu \dot{x}^\mu \ + \ \varepsilon^2 \frac{1}{2}(\dot{\theta}^\mu\theta_a - \theta_a\dot{\theta}^\mu)e^a{}_\mu$$

$$+ \varepsilon^2 \frac{1}{2}(\theta^b\theta_a - \theta_a\theta^b)e^a{}_{\mu b}\dot{x}^\mu\}, \tag{3.1a}$$

follows, which defines the two momenta of the system $p_\mu = \partial L/\partial \dot{x}^\mu = p_{0\mu} + \frac{1}{2}\tilde{S}^{ab} e_{a\mu b}$, $p^\theta_\mu = -\mathrm{i}\theta_a e^a{}_\mu = -\mathrm{i}(\theta_\mu + \overrightarrow{e}^a{}_{\nu,\mu_\theta} e_{a\alpha}\theta^\nu\theta^\alpha)$. Here $p_{0\mu}$ are the covariant (canonical) momenta of a particle. For $p^\theta_a = p^\theta_\mu f^\mu{}_a$, it follows that p^θ_a is proportional to θ_a. Then, $\tilde{a}_a = \mathrm{i}(p^\theta_a - i\theta_a)$, while $\tilde{\tilde{a}}_a = 0$. We may further write

$$p_{0\mu} = p_\mu - \frac{1}{2}\tilde{S}^{ab} e_{a\mu b} = p_\mu - \frac{1}{2}\tilde{S}^{ab}\omega_{ab\mu}, \tag{3.4.}$$

$$\omega_{ab\mu} = \frac{1}{2}(e_{a\mu b} - e_{b\mu a}), \ \text{ with } \ e_{a\mu b} = f^\nu{}_{a,\mu} e_{b\nu},$$

which is the usual expression for the covariant momenta in gauge gravitational fields [5]. One can find the two constraints

$$p_0^{\mu} p_{0\mu} = 0 = p_{0\mu} f^{1\mu}{}_a \widetilde{a}^a. \tag{3.5}$$

To see how Yang-Mills fields enter into the theory, the Dirac-like equation (3.5) has to be rewritten in terms of components of fields which determine the ordinary gravitation in the four dimensional subspace and of components, which determine gravitation in higher dimensions, assuming that the coordinates of ordinary space with indices higher than four stay compacted to unmeasurable small dimensions. Since Grassmann space manifests itself through average values of observables only, compactification in the Grassmann part of space has no meaning. However, since parameters of the Lorentz transformations in a freely falling coordinate system for both spaces have to be the same, no transformations to the fifth or higher coordinates should occur at measurable energies. Therefore, the four dimensional subspace of Grassmann space with the generators defining the Lorentz group SO(1,3) is (almost) decomposed from the rest of the Grassmann space with the generators forming the (compact) group SO(d-4).

We shall assume accordingly the case in which only some components of fields differ from zero:

$$\left(\begin{array}{c|c} e^m{}_\alpha & 0 \\ \hline 0 & e^h{}_\sigma \end{array}\right), \quad \alpha, m \in (0,3), \quad \sigma, h \in (5,d), \quad i \in (1,2), \tag{3.6}$$

while vielbeins $e^m{}_\alpha, e^k{}_\sigma$ depend on θ^a and $x^\alpha, \alpha \in \{0,3\}$ only. Accordingly, we have only $\omega_{ab\alpha} \neq 0$. We recognize, as in the freely falling coordinate system, that Grassmann coordinates with indices from 0 to 3 determine spins of fields, while Grassmann coordinates with indices higher than 3 determine charges of the fields. We shall take expectation values of $p^h = 0$, $a \geq 5$. We find

$$\widetilde{\gamma}^a f^\mu{}_a p_{0\mu} = \widetilde{\gamma}^m f^\alpha{}_m (p_\alpha - \frac{1}{2} \widetilde{S}^{mn} \omega_{mn\alpha} + A_\alpha), \;\; \text{where} \;\; A_\alpha = \sum_{A,i} \widetilde{\tau}^{Ai} A^{Ai}_\alpha, \tag{3.7}$$

with $\sum_{A,i} \widetilde{\tau}^{Ai} A^{Ai}_\alpha = -\frac{1}{2} \widetilde{S}^{hk} \omega_{hk\alpha}, \;\; h,k = 5,6,7,..d.$

For $f^\alpha_m = \delta^\alpha_m$ and $f^\sigma_{h,\mu}$ is nonzero for $\mu = \alpha$ and is zero for $\mu = \sigma$, we find the usual Dirac equation in the presence of the gauge fields A^{Ai}_α only. If, however, we assume that $f^{\sigma'}_{h,\sigma}$ is also nonzero, the additional nonzero term $\widetilde{\gamma}^h f^\sigma{}_h p_{0\sigma}$ appears in Eq. (3.7), which according to what was said in Sect. 2, behaves as Yukawa couplings, since the operator $\widetilde{\widetilde{a}}^0 \widetilde{a}^h, h = 5,6,..$ transforms left-handed weak doublets to right-handed weak singlets, if h concerns the SU(2) part of the Grassmann space.

We learn in this section that the Dirac-like equation follows from the action for a particle living in ordinary and Grassmann space. The operators fulfilling the

Clifford algebra appear because spinors "see" the Grassmann coordinates to be proportional to the conjugate momenta. The operators for γ^a matrices have to be defined as a product of two Grassmann odd operators, so that when acting on spinors they do not change their Grassmann character. Supervielbeins determine the ordinary gravity, the Yang-Mills fields, the Higgs field and the Yukawa couplings. Since the dynamical break of symmetries have not yet been treated, it stays as an unsolved problem, why some components of vielbeins and accordingly of spin connections are zero, while the others are not, as well as whether the Yukawa couplings are different from the Planck mass.

4. *Concluding remarks*

We have presented the approach in which space has d ($d \geq 14$) ordinary and d Grassmann coordinates. In Grassmann space, two kinds of generators of the Lorentz transformations, one of spinorial and the other of vectorial character, define spins and charges of fermions and bosons, unifying spins and charges and offering all known degrees of freedom.

The action for a spinning particle leads to the Dirac-like equation, in which gravity in d dimensional space manifests in four-dimensional subspace as ordinary gravity, the Yang-Mills fields, the Higgs field and the Yukawa couplings.

The representations of the group SO(1,13), which contains subgroups SO(1,3), SU(3), SU(2) and U(1) suggest four families of quarks and leptons as well as multiplets with left-handed weak doublets and right-handed weak singlets. Although the number of fermionic and bosonic degrees of freedom is the same, the approach does not support the ordinary supersymmetric models: it predicts bosons which are SU(2) doublets and SU(3) triplets, but not fermions, which are SU(2) triplets and SU(3) octets.

The approach shows the way beyond the standard model offering the answers to the open questions of the standard model. Dynamical breaks of symmetries have not yet been done.

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UJEDINJENJE SPINOVA I NABOJA SJEDINJUJE SVA MEĐUDJELOVANJA

U prostoru Grassmanovih koordinata, dvije vrste generatora Lorentzovih transformacija, jedna spinornog a druga vektorskog značaja, definiraju reprezentacije grupe SO(1,$d-1$) za fermione odnosno bozone. Svojstvene vrijednosti komutirajućih operatora podgrupa SO(1,3), SU(3), SU(2) i U(1) mogu se poistovjetiti sa spinovima i Yang-Mills nabojima bilo fermionskih ili bozonskih polja, što omogućuje sjedinjenje svih unutarnjih stupnjeva slobode, posebno za fermione i za bozone. Kada se, prema tome, ujedine sva međudjelovanja, tada Yang-Millsovi naboji, Higgsova polja i Yukawina vezanja postaju dio gravitacijskog polja. Teorija ukazuje na četiri obitelji kvarkova i leptona. Jednak broj fermiona i bozona osigurava supersimetriju.