ISSN 1330–0016
CODEN FIZBE7

LETTER TO THE EDITOR

LONG VS. SHORT DISTANCE DISPERSIVE TWO-PHOTON $K_L \to \mu^+\mu^-$ AMPLITUDE[1]

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Received 21 January 1999: Accepted 7 June 1999

We report on the calculation of the two-loop electroweak, two-photon mediated short-distance dispersive $K_L \to \mu^+\mu^-$ decay amplitude. QCD corrections change the sign of this contribution and reduce it by an order of magnitude. The resulting amplitude enables us to provide a constraint on the otherwise uncertain long-distance dispersive amplitude.

PACS numbers: 12.15.-y, 12.39.Fe UDC: 539.126

Keywords: kaon decay, muon, dispersive amplitude, QCD corrections

The decay mode $K_L \to \mu^+\mu^-$ is a classical example of the rare flavour changing neutral process that provided valuable insights into the nature of weak interactions. Its non-observation at a rate comparable with that of $K^+ \to \mu^+\nu_\mu$ led to the discovery of the GIM mechanism [1] and to the derivation of the early constraints on the masses of the charmed [2] and top [3] quark.

Also, by studying this mode it was possible to determine the Wolfenstein ρ parameter [4,5], to study the CP violation [6], and even to discover some new physics [7] (e.g., through SUSY-induced FCNC enhancement). Because of this, this decay mode has received sustained theoretical attention over the last three decades.

The lowest-order electroweak amplitude for $K_L \to \mu^+\mu^-$ in a free-quark calculation [2] (Figs. 1a and 1b) is represented by one-loop (1L) W-box and Z-exchange diagrams, respectively, and exhibits a strong GIM cancellation. Therefore, one is addressed to consider the two-loop (2L) diagrams with photons in the intermediate state (Fig. 1c) as a potentially important contribution.

[1] Presented by K. Kumerički.

If we normalize the amplitude $\mathcal{A}$ to the branching ratio:

$$B(\mathrm{K_L} \to \mu^+\mu^-) = |\mathrm{Re}\mathcal{A}|^2 + |\mathrm{Im}\mathcal{A}|^2 \,, \tag{1}$$

then the absorptive ($\mathrm{Im}\mathcal{A}$) part, which is dominated by the process $\mathrm{K_L} \to \gamma\gamma \to \mu^+\mu^-$ (Fig. 1c) with the *real* photons, is easily calculable and gives the so-called unitarity bound [8]

$$B(\mathrm{K_L} \to \mu^+\mu^-) \geq |\mathrm{Im}\mathcal{A}|^2 = (7.1 \pm 0.2) \times 10^{-9} \,, \tag{2}$$

corresponding to $|\mathrm{Im}\mathcal{A}| = (8.4 \pm 0.1) \times 10^{-5}$. If we compare this to the experimental number [9]

$$B(\mathrm{K_L} \to \mu^+\mu^-) = (7.2 \pm 0.5) \times 10^{-9} \,, \tag{3}$$

we see that the absorptive part almost saturates the amplitude, leaving only the small window for the dispersive ($\mathrm{Re}\mathcal{A}$) part

$$\mathrm{Re}\mathcal{A} = \mathcal{A}_{\mathrm{SD}} + \mathcal{A}_{\mathrm{LD}} \,, \qquad |\mathrm{Re}\mathcal{A}|^2 < 5.6 \times 10^{-10} \,. \tag{4}$$

Thus, the total *real* part of the amplitude, being the sum of short-distance (SD) and long-distance (LD) dispersive contributions, must be relatively small compared with the absorptive part of the amplitude. Such a small total dispersive amplitude can be realized either when the SD and LD parts are both small or by partial cancellation between these two parts.

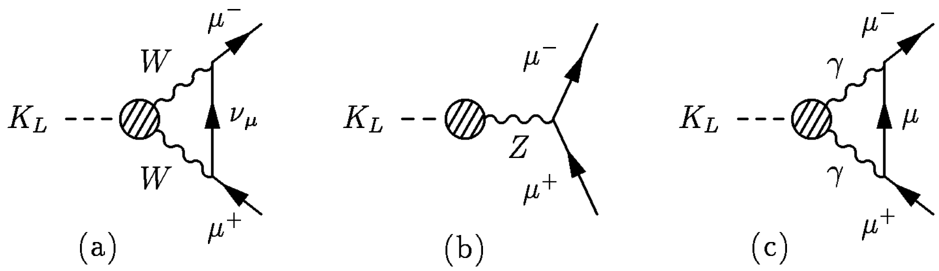

Fig. 1. Possible mechanisms for $\mathrm{K_L} \to \mu^+\mu^-$.

The major obstacle in extracting useful short distance information out of this decay mode is the poor knowledge of $\mathcal{A}_{\mathcal{LD}}$. There are several calculations of this LD part to be found in literature [4,10–13] and, later in this paper, we will try to compare them. To this end it is necessary to have a reliable estimate of the other, theoretically more tractable, SD part $\mathcal{A}_{\mathcal{SD}}$.

Frequently, $\mathcal{A}_{\mathcal{SD}}$ has been identified as the weak contribution represented by the one-loop W-box and Z-exchange diagrams of Figs. 1a and 1b. This one-loop SD contribution $\mathcal{A}_{1L} = \mathcal{A}_{\mathrm{Fig.1a}} + \mathcal{A}_{\mathrm{Fig.1b}}$ is dominated by the t-quark in the loop (proportional to the small KM-factor λ_t), and the inclusion of QCD corrections

[12,13] does not change this amplitude essentially. In this paper, we stress that the diagrams of Fig. 1c, with *virtual* intermediate photons, with relatively high-momentum, lead to the same SD operator. That is, both the 1L diagrams contained in Figs. 1a and 1b, as well as 2L diagrams like those in Fig. 2, lead to the same SD operator of the type

$$K_{SD}\ (\bar{d}\gamma^{\beta}Ls)(\bar{u}\gamma_{\beta}\gamma_5 v)\ , \tag{5}$$

where $s, \bar{d}, u, v$ are the spinors of the s- and $\bar{d}$ quarks in the K-meson, and the μ^+ and μ^-, respectively. The quantity K_{SD} is a constant which contains the result of the SD calculations. The leading contributions from 2L diagrams are proportional to $\alpha_{\rm em}^2 G_{\rm F}\lambda_u$ and dominated by c-quarks in the loop, while the leading 1L is proportional to $G_{\rm F}^2\, m_t^2\, \lambda_t$.

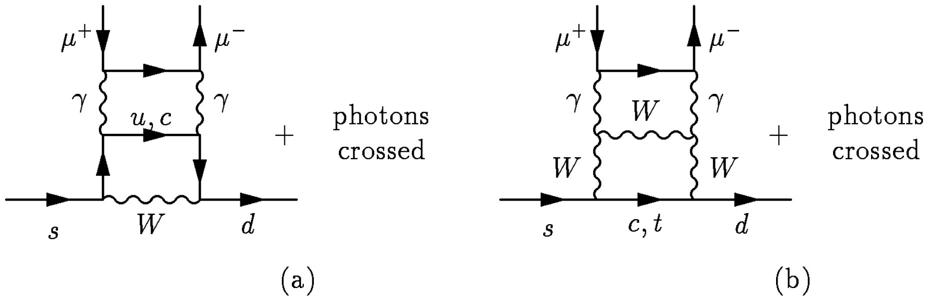

Fig. 2. Typical two-loop diagrams for $\mathrm{K_L} \to \mu^+\mu^-$.

One should note that, as already pointed out in Refs. 16 and 17, the two-loop diagrams with two intermediate virtual photons have a short-distance part $\mathcal{A}_{\rm 2L}$ (contained in $\mathcal{A}_{\rm Fig.1c} = \mathcal{A}_{\rm LD} + \mathcal{A}_{\rm 2L}$) that could pick up a potentially sizable contribution, leading to the total SD amplitude is $\mathcal{A}_{\rm SD} = \mathcal{A}_{\rm 1L} + \mathcal{A}_{\rm 2L}$. By exploring the contribution from Fig. 1c leading to the $\mathcal{A}_{\rm 2L}$ amplitude, we will be able to isolate the strongly model-dependent LD dispersive piece.

A complete treatment of the two-loop SD dispersive amplitude for $\mathrm{K_L} \to \mu^+\mu^-$ was given by us in Ref. 16. There, we used the momenta of the intermediate photons from the diagrams in Fig. 1c to distinguish between SD and LD contributions, SD part being defined by diagrams with photon momenta above some infrared cut-off of the order of some hadronic scale $\Lambda \sim m_\rho$. The fact that the resulting amplitudes depended only mildly on the precise choice of Λ assured us that the procedure was correct.

Our SD calculation in Ref. 18 is dominated by the region $m_\rho < q^2 < m_c^2$ (the high energy ($q^2 > m_c^2$) region is also included). After performing QCD corrections in the leading logarithmic approximation [18], the original electroweak amplitude was considerably suppressed and its sign changed:

$$-0.38\times 10^{-5} \le \mathcal{A}_{\rm 2L} \le -0.001\times 10^{-5}\ , \tag{6}$$

where error bars stem mostly from empirical uncertainty in α_s.

Effectively, the LD calculation of the diagram in Fig. 1c is reduced to the evaluation of the form-factor $F(q_1^2, q_2^2)$ contained in the amplitude

$$A(\mathrm{K_L} \to \gamma^*(q_1, \epsilon_1)\gamma^*(q_2, \epsilon_2)) = \mathrm{i}\varepsilon_{\mu\nu\rho\sigma}\epsilon_1^\mu \epsilon_2^\nu q_1^\rho q_2^\sigma F(q_1^2, q_2^2) \; , \tag{7}$$

where q_1^2 , $q_2^2 \neq 0$ measure the virtuality of the intermediate photons.

The low energy regime $q^2 < \Lambda^2 \sim m_\rho^2$ is explorable by chiral techniques determining $F(0,0)$. In the standard $SU(3)_L \otimes SU(3)_R$ ChPT, where η' is absent, one recovers the cancellation owing to the Gell-Mann-Okubo mass relation, $\sim (3M_\eta^2 + M_\pi^2 - 4M_K^2) \to 0$. Keeping the η' pole contribution in the enlarged $U(3)_L \otimes U(3)_R$ symmetric theory [13], there is a destructive interference between the η and η' contributions, so that the final amplitude is dominated by the pion pole.

If going beyond the ChPT, one faces model calculations, and in particular the calculations based on vector meson dominance (e.g., Refs. 10 and 4). The chiral-quark model may also be used for the LD regime. Some preliminary analysis within the chiral quark model indicates that the dispersive LD amplitude is of the same order of magnitude as the SD.

Combining Eqs. (4) and (6), and $\mathcal{A}_{\mathrm{1L}}$ [15,19], enables us to find the following allowed range for $\mathcal{A}_{\mathrm{LD}}$:

$$-0.1 \times 10^{-5} \leq \mathcal{A}_{\mathrm{LD}} \leq 6.5 \times 10^{-5} \; . \tag{8}$$

Thus, having a dispersive LD part $\mathcal{A}_{\mathrm{LD}}$ of the size comparable with the absorptive part [20] is still not ruled out completely.

The two vector-meson dominance calculations for the LD amplitude considered as the referent calculations in Ref. 19 have basically opposite signs,

$$-2.9 \times 10^{-5} \leq \mathcal{A}_{\mathrm{LD}} \leq 0.5 \times 10^{-5} \quad [10] \; ,$$

$$0.27 \times 10^{-5} \leq \mathcal{A}_{\mathrm{LD}} \leq 4.7 \times 10^{-5} \quad [4] \; ,$$

and the result of [4] seems to be more in agreement with the bound (8). There are also some other, more recent, attempts to calculate the $\mathcal{A}_{\mathrm{LD}}$ 11–13]. The most stringent bound obtained is [12]

$$|\mathrm{Re}\mathcal{A}_{LD}| < 2.9 \times 10^{-5} \; , \tag{9}$$

also well inside the allowed range (8).

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USPOREDBA DUGO- I KRATKODOSEŽNE DISPERZIVNE DVOFOTONSKE AMPLITUDE RASPADA $K_L \to \mu^+\mu^-$

Izlaže se o račun kratkodosežne elektroslabe amplitude raspada $K_L \to \mu^+\mu^-$ na razini dvije petlje, putem dvofotonske izmjene. Uključenje QCD popravki mijenja predznak tog doprinosa i smanjuje ga za red veličine. Dobiven rezultat dozvoljava uvođenje ograničenja na dugodosežnu disperzivnu amplitudu, koja je inače neodređena.