

(HYBRID) BARYONS: SYMMETRIES AND MASSES

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We construct (hybrid) baryons in the flux-tube model of Isgur and Paton. In the limit of adiabatic quark motion, we build proper eigenstates of orbital angular momentum and construct the flavour, spin and J^P of hybrid baryons from the symmetries of the system. The lowest-mass hybrid baryon is estimated at about GeV.

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1. Introduction

Hybrid baryons are bound states of three quarks with an explicit excitation in the gluon field of QCD. In Ref. 1 was detailed the construction of (hybrid) baryons in a model motivated from the strong coupling expansion of the Hamiltonian formulation of lattice QCD, the non-relativistic flux-tube model of Isgur and Paton [1]. This model predicts the adiabatic potentials of (hybrid) mesons at large interquark separations, as well as the mass of the $J^{PC} = 1^{-+}$ hybrid meson, consistent with recent estimates from lattice QCD [2,3]. In Ref. 1, we studied the detailed flux dynamics and built the flux Hamiltonian. We restrict our discussion to cases where the flux settles down in a Mercedes Benz configuration (as motivated by lattice QCD [4]). A minimal amount of quark motion is allowed in response to flux motion, in order to work in the centre of mass frame. Otherwise, we make the so-called “adiabatic” approximation, where the flux motion adjusts itself instantaneously to the motion of the quarks. The main result is that the lowest flux excitation can to a high degree of accuracy (about 5%) be simulated by neglecting all flux-tube motions except the vibration of a junction. This result was obtained within the small oscillation approximation. The junction acquires an effective mass M_{eff} from

¹Work done in collaboration with Simon Capstick.

the motion of the remainder of the flux-tube and the quarks. The model is then simple: a junction is connected via a linear potential to the three quarks. The ground state of the junction motion corresponds to a conventional baryon and the various excited states to hybrid baryons. The junction can move in three directions, and correspondingly be excited in three ways, giving the hybrid baryons H_1 , H_2 and H_3 . The junction motion is depicted in Fig. 1.

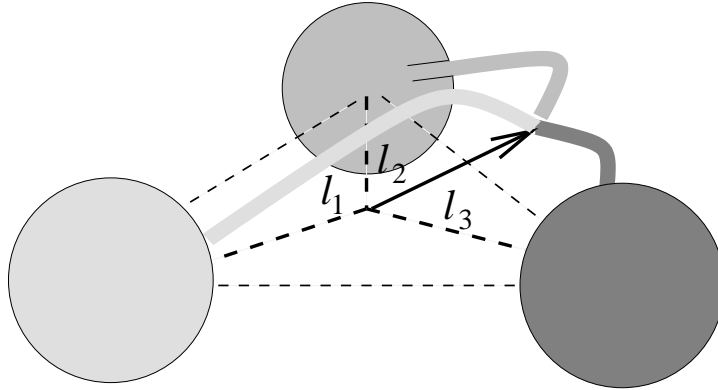


Fig. 1. The junction connects strings coming from the three quarks. The vectors \mathbf{r} and \mathbf{l}_i , respectively, point from the equilibrium position of the junction to its current position and the quark positions.

The Hamiltonian for the junction motion in the Mercedes Benz configuration is simply the kinetic energy of the junction added to the sum of the lengths from the junction to the quarks multiplied by the string tension b ,

$$H_{\text{flux}} = \frac{1}{2} M_{\text{eff}} \dot{\mathbf{r}}^2 + b \sum_{i=1}^3 |\mathbf{l}_i - \mathbf{r}|. \quad (1)$$

We shall be taking ansatz wave functions of the form

$$\hat{\boldsymbol{\eta}}_- \cdot \mathbf{r} \Psi_B(\mathbf{r}) \quad (2)$$

for H_1 hybrid baryons, where $\Psi_B(\mathbf{r})$ is an exponential function. It is not difficult to show that $\hat{\boldsymbol{\eta}}_-$ lies in the plane spanned by the three quarks (the “ qqq plane”).

2. Quantum numbers of low-lying hybrid baryons

2.1. Angular momentum

The Hamiltonian in Eq. (1) is not invariant under rotations in the junction position \mathbf{r} , with fixed quark positions. When the junction wave function, which is hence not an eigenfunction of angular momentum, is combined with the quark

motion wave functions, which are eigenfunctions of angular momentum, it must be done in such a way that the total angular momentum of the junction and quark motion is well-defined. Obtaining a well-defined total angular momentum is a technically challenging problem that is an artifact of the adiabatic approximation, which separates junction and quark motion. We here merely give an intuitive argument why the total angular momentum L of the H_1 baryons is expected to be 1.

The hybrid baryon wave function is proportional to $\hat{\eta}_- \cdot \mathbf{r}$, and since $\hat{\eta}_-$ lies in the qqq plane, it can be regarded as the x -axis, so that $\hat{\eta}_- \cdot \mathbf{r} = \sqrt{2\pi/3} r(-Y_{11}(\hat{\mathbf{r}}) + Y_{1-1}(\hat{\mathbf{r}}))$ in terms of spherical harmonics. If the mathematics of conservation of angular momentum is followed through, it is found that if the angular momentum of the quark motion is $L_q = 0$ (corresponding to the lowest energy quark motion states), then the total angular momentum projection just equals the angular momentum projection of the junction wave functions, which in this case is ± 1 . Hence the total angular momentum projection is ± 1 so that L cannot be zero, and should most likely be 1.

2.2. Exchange symmetry

Exchange symmetry transformations S_{ij} exchange the positions of the quarks $\mathbf{l}_i \leftrightarrow \mathbf{l}_j$. Since the physics does not depend on the quark position labelling convention, the junction Hamiltonian should be exchange symmetric, as can be seen explicitly in Eq. (1), noting that the junction position \mathbf{r} is not determined by the positions of the quarks.

We now argue that the junction wave functions of (hybrid) baryons should transform either totally symmetrically or totally anti-symmetrically under exchange symmetry. Since the Hamiltonian is invariant under exchange symmetry we have the commutation relation $[H_{\text{flux}}, S_{ij}] = 0$. Combining this with the Schrödinger equation

$$H_{\text{flux}}\Psi = V(l_1, l_2, l_3)\Psi \quad \text{gives} \quad H_{\text{flux}}(S_{ij}\Psi) = V(l_1, l_2, l_3)(S_{ij}\Psi), \quad (3)$$

so that $S_{ij}\Psi$ is degenerate in energy with Ψ . Now, since the baryon and each of the hybrid baryons H_i have different energies (except when $l_1 = l_2 = l_3$) it follows that $S_{ij}\Psi$ must be a multiple of Ψ , i.e. that $S_{ij}\Psi = \zeta\Psi$, where ζ is complex number. Now note that the product of two exchange symmetry transformations is the identity, i.e. that

$$S_{ij}S_{ij} = 1, \quad \text{which implies that} \quad \zeta^2 = 1, \quad (4)$$

or $\zeta = \pm 1$. Hence $S_{ij}\Psi = \pm\Psi$.

Assume that $S_{12}\Psi = \zeta\Psi$. We now show² that $S_{23}\Psi = S_{13}\Psi = \zeta\Psi$, i.e. that Ψ is either totally symmetric or totally anti-symmetric under label exchange. This

²This result also follows by noting that $[H_{\text{flux}}, S_{ij}] = 0$ what implies that Ψ must be an irreducible representation of the exchange symmetry group, i.e. totally symmetric, anti-symmetric or of mixed symmetry. But since we already showed that $S_{ij}\Psi = \pm\Psi$, it follows that Ψ is in either the totally symmetric or anti-symmetric irreducible representation.

follows by the two identities

$$\begin{aligned} S_{12}S_{23}S_{13}S_{23} = 1, & \quad \text{which implies that} & \quad S_{12}\Psi = S_{13}\Psi, & \quad (5) \\ S_{23}S_{12}S_{13}S_{12} = 1, & \quad \text{which implies that} & \quad S_{23}\Psi = S_{13}\Psi. \end{aligned}$$

For each of the hybrid baryons H_i , there are hence two varieties: the junction wave function is totally symmetric (S) or totally anti-symmetric (A) under quark label exchange, denoted by H_i^S and H_i^A .

2.3. Parity

The inversion of all coordinates $\mathbf{l}_i \rightarrow -\mathbf{l}_i$ and $\mathbf{r} \rightarrow -\mathbf{r}$, called ‘‘parity’’, is a symmetry of the junction Hamiltonian in Eq. (1).

$\hat{\boldsymbol{\eta}}_-$ is a vector in the qqq plane and is a linear combination of the $\hat{\mathbf{l}}_i$, which span the plane, with coefficients which are functions of l_i . The lengths l_i remain invariant under parity. However, $\hat{\mathbf{l}}_i \rightarrow -\hat{\mathbf{l}}_i$ under parity. It follows that $\hat{\boldsymbol{\eta}}_-$ is *odd under parity*.

The junction wave function in Eq. (2) is thus even under parity, since $\hat{\boldsymbol{\eta}}_- \rightarrow -\hat{\boldsymbol{\eta}}_-$ and $\mathbf{r} \rightarrow -\mathbf{r}$. For a low-lying hybrid the quark motion wave function is even under parity, so that the full hybrid baryon wave function has even parity.

Since quarks are fermions, the wave function should be totally antisymmetric under quark label exchange, called the Pauli principle. Since our philosophy is that (hybrid) baryon dynamics is dominated by (non-perturbative) long-distance physics, we consider the colour structure of the (hybrid) baryon to be motivated from the long-distance limit, i.e. from the strong-coupling limit of the Hamiltonian formulation of lattice QCD [2]. Here, the quarks are sources of triplet colour, which flows along the string connected to the quarks into the junction, where an ϵ_{ijk} neutralizes the colour. The colour wave function ϵ_{ijk} is hence totally antisymmetric under exchange of quarks for *both* the conventional and hybrid baryon.

This imposes constraints on the combination of flavour and non-relativistic spin S of the three quarks that is allowed. For a totally symmetric hybrid baryon junction wave function, the flavour-spin wave functions must be totally symmetric. This is because we are interested in the low-lying hybrid baryons which have the quark motion wave function in the ground state, i.e. totally symmetric. If the flavour is Δ , which is totally symmetric, this implies that the spin must be totally symmetric, i.e. $S = \frac{3}{2}$. Similarly for flavour N the spin must be $\frac{1}{2}$. For a totally antisymmetric junction wave function, the flavour-spin wave function must be totally antisymmetric. For Δ flavour this implies that the spin must be totally antisymmetric, which is not realizable. Hence there are no Δ hybrid baryons with totally antisymmetric junction wave functions. The N flavour is found to have spin $\frac{1}{2}$.

The quantum numbers of the lowest-lying states that can be constructed on the H_1 adiabatic surface are indicated in Table 1.

The total angular momentum $\mathbf{J} = \mathbf{L} + \mathbf{S}$. Since $L = 1$ for the ground state H_1

hybrid baryon, $J = \frac{1}{2}, \frac{3}{2}$ for $S = \frac{1}{2}$, and $J = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}$ for $S = \frac{3}{2}$. These assignments are indicated in Table 1.

One notes from Table 1 that amongst the H_1^S hybrid baryons, there are $N_{\frac{1}{2}}^{1+}$ and $\Delta_{\frac{3}{2}}^{3+}$ states which have identical quantum numbers to the conventional N and Δ baryons.

TABLE 1. Quantum numbers of low-lying hybrid baryons for the adiabatic surface H_1 . In the absence of spin-dependent forces, all these states are degenerate. N, Δ are the flavour structure of the wave function (i.e. those of the conventional baryons N, Δ , respectively) and P the parity.

Hybrid baryon	L	S	$(N, \Delta)^{2S+1} J^P$
H_1^S	1	$\frac{1}{2}, \frac{3}{2}$	$N_{\frac{1}{2}}^{2\frac{1}{2}+}, N_{\frac{3}{2}}^{2\frac{3}{2}+}, \Delta_{\frac{1}{2}}^{4\frac{1}{2}+}, \Delta_{\frac{3}{2}}^{4\frac{3}{2}+}, \Delta_{\frac{5}{2}}^{4\frac{5}{2}+}$
H_1^A	1	$\frac{1}{2}$	$N_{\frac{1}{2}}^{2\frac{1}{2}+}, N_{\frac{3}{2}}^{2\frac{3}{2}+}$

It is interesting to compare our hybrid baryons to the predictions of the bag model. Out of all the states listed under H_1^S and H_1^A in Table 1, only one pair of $N_{\frac{1}{2}}^{2\frac{1}{2}+}, N_{\frac{3}{2}}^{2\frac{3}{2}+}$ states have the same flavour, spin S , total angular momentum and parity as the low-lying hybrid baryons in the bag model [5]. In fact, for the H_1^S hybrid baryons, the bag model swaps the N and Δ flavours from our assignments, keeping other quantum numbers the same. Both our model and the bag model have seven low-lying hybrid baryons [5].

3. Numerical estimate of the hybrid baryon mass

The difference between the hybrid and conventional baryon adiabatic potentials (or junction energies) as a function of quark positions, $V_{H_1}(l_1, l_2, l_3) - V_B(l_1, l_2, l_3)$, was determined numerically from the first part of Eq. (3) by using the Hamiltonian in Eq. (1), and were displayed in Ref. 6.

Now define the hybrid baryon potential as

$$\bar{V}_{H_1}(l_1, l_2, l_3) \equiv \bar{V}_B(l_1, l_2, l_3) + V_{H_1}(l_1, l_2, l_3) - V_B(l_1, l_2, l_3) \quad (6)$$

where $\bar{V}_B(l_1, l_2, l_3)$ is the phenomenologically successful relativized baryon Hamiltonian with Coulomb and linear potential terms of Ref. 7 (with spin-spin, spin-orbit and tensor interactions neglected); and the parameters are also those of Ref. 7. Note that the Coulomb interaction of the conventional and hybrid baryon is assumed to be identical.

We solve the Schrödinger equation for the Hamiltonian in Eq. (6) with 95 spin-space basis states incorporating $L_q = 0, 1, 2$ harmonic oscillator wave functions for the $J = \frac{1}{2}$ case, i.e. construct 95×95 dimensional matrices. These matrices

are subsequently diagonalized. The differences between the energies for the hybrid and the conventional baryon is then added to the experimental mass of the lowest baryon, taken as the spin-averaged mass of the N and Δ , i.e. 1085 MeV [8]. The first three quark orbital excitations $L_q = 0, 1, 2$ of hybrid baryons composed of up and down quarks are found to have masses 1976, 2341 and 2619 MeV, respectively.

Hence, for the lowest hybrid baryon level, with the quantum numbers in Table 1, we obtain that $M_{H_1} - M_B = 891$ MeV, giving a mass estimate of $M_{H_1} = 1976$ MeV.

This mass estimate is substantially higher than other mass estimates in the literature: ≈ 1.5 GeV in the bag model [5] and $1.5 \pm 10\%$ GeV in QCD sum rules [9].

There are two crucial assumptions that were made in the early work on (hybrid) meson masses in the flux-tube model: the adiabatic motion of quarks and the small oscillation approximation for flux motion [2]. It was later shown that when the adiabatic approximation is lifted, the masses go up, and when the small-oscillation approximation is lifted, the masses go down [2]. In our study of (hybrid) baryons, we have partially lifted the adiabatic approximation by working in the centre of mass frame. We have fully lifted the small oscillation approximation. The effects on the masses of (hybrid) baryons when the various approximations are lifted, are the same as those found for (hybrid) mesons.

In our simulation, we obtain the average values $\sqrt{\langle\rho^2\rangle} = \sqrt{\langle\lambda^2\rangle} = 2.12, 2.52$ GeV⁻¹ for the low-lying baryon and H₁ hybrid baryon, respectively. $\langle\rho^2\rangle = \langle\lambda^2\rangle$ is expected since the spatial parts of the wave functions of the low-lying states are totally symmetric under exchange symmetry. The hybrid baryon is 20% larger than the conventional baryon.

4. Phenomenology

The sign of the the Coulomb interaction is expected to be the same for both conventional and hybrid baryons [5]. This means that the hyperfine interaction has the same sign in both situations, so that the Δ hybrid baryons are always heavier than the N hybrids. This implies that only four of the original seven low-lying baryons, the N hybrids, are truly low-lying.

We expect *a priori* the most phenomenologically interesting decay of the low-lying hybrid baryons to be the P-wave decay to $N\rho$ and $N\omega$, simply because the phase space is favourable and ρ and ω are easily isolated experimentally. The $N\rho$ decay would be especially relevant to the electro- and photoproduction of hybrid baryons at TJNAF via the vector-meson-dominated coupling of the photon to the ρ . Indeed, a search for excited N* resonances with mass less than 2.2 GeV is currently underway in Hall B [10]. Given the mass estimate for the low-lying hybrid baryons, the detection of hybrid baryons in $N\rho$ or $N\omega$ is feasible at TJNAF. There are also planned experiments in πN scattering by Crystal Ball E913 at the new D-line at Brookhaven with the capability of searching for states in $N\{\eta, \rho, \omega\}$, which would isolate states in the mass region about 2 GeV [11].

The decay $\psi \rightarrow p\bar{p}\omega$ has been observed with a branching ratio of $(1.30 \pm 0.25) 10^{-3}$ and $\psi \rightarrow p\bar{p}\eta'$ with branching ratio $(9 \pm 4) 10^{-4}$ [8]. Since gluonic hadron production is expected to be enhanced above conventional hadron production in the glue-rich decay of the ψ , it is possible that a partial-wave analysis of the $p\omega$ or $p\eta'$ invariant masses would yield evidence for hybrid baryons. Future work at BEPC and an upgraded τ -charm factory would be critical here.

5. Conclusions

The spin and flavour structure of the low-lying hybrid baryons have been specified, and differ from their structure in the bag model. Exchange symmetry constrains the spin and flavour of the (hybrid) baryon wave function. The orbital angular momentum of the low-lying hybrid baryon is argued to be unity, with the parity even, contrary to conventional baryons where $L = 1$ would imply the parity to be odd. The low-lying hybrid baryon adiabatic potential and mass has been estimated numerically. The mass estimate is considerably higher than bag model and QCD sum-rule estimates.

References

- 1) P. R. Page, *Proc. of "Seventh International Conference on Hadron Spectroscopy" (HADRON '97)*, 25–30 August 1997, Upton, N.Y., U.S.A., eds. S.-U. Chung, H. J. Willutzki, AIP Conf. Proc. No. 432, American Institute of Physics, New York (1998) p. 553;
- 2) N. Isgur and J. Paton, *Phys. Rev. D* **31** (1985) 2910; J. Merlin and J. Paton, *J. Phys. G* **11** (1985) 439; T. Barnes, F. E. Close and E. S. Swanson, *Phys. Rev. D* **52** (1995) 5242; I. Zakout and R. Sever, *Z. Phys. C* **75** (1997) 727;
- 3) C. J. Morningstar, *Proc. of 3rd Int. Conf. in Quark Confinement and Hadron Spectrum (Confinement III)*, June 1998, Newport News, VA, World Scientific, Singapore, hep-lat/9809015;
- 4) G. Bali, *Fizika B (Zagreb)* **8** (1999) 229;
- 5) T. Barnes and F. E. Close, *Phys. Lett. B* **123** (1983) 89; E. Golowich, E. Haqq and G. Karl, *Phys. Rev. D* **28** (1983) 160; C. E. Carlson and T. H. Hansson, *Phys. Lett. B* **128** (1983) 95; I. Duck and E. Umland, *Phys. Lett. B* **128** (1983) 221; C.E. Carlson, *Proc. 7th Int. Conf. on the Structure of Baryons*, October 1995, Santa Fe, NM, eds. B. F. Gibson et al., World Scientific, Singapore (1996) p. 461;
- 6) P. R. Page, *Proc. of 3rd Int. Conf. in Quark Confinement and Hadron Spectrum (Confinement III)*, June 1998, Newport News, VA, World Scientific, Singapore; nucl-th19904642
- 7) S. Capstick and N. Isgur, *Phys. Rev.* **D34** (1986) 2809;
- 8) C. Caso *et al.* (Particle Data Group), *Eur. Phys. J. C* **3** (1998) 1;
- 9) L. S. Kisslinger and Z.-P. Li, *Phys. Letters B* **445** (1999) 271; L. S. Kisslinger, *Nucl. Phys. A* **629** (1998) 30c;
- 10) K. de Jager, *The Hall A program*, talk at the Conference Nuclear and Particle Physics with CEBAF at Jefferson Lab, Dubrovnik, Croatia, 3 – 10 November 1998;

- 11) W. Briscoe, Fizika B (Zagreb) **8** (1999) 11;

SIMETRIJE I MASE (HIBRIDNIH) BARIONA

Na osnovi Isgur–Patonovog modela cijevi toka razvijaju se modeli (hibridnih) bariona. U granici adijabatskog gibanja kvarkova grade se svojstvena stanja impulsnog momenta, te se slažu okus, spin i J^P hibridnih bariona primjenom simetrija sustava. Najniža se masa hibridnog bariona ocjenjuje na oko 2 GeV.