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RELATIVISTIC QUARK MODEL

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A general Lorentz-covariant quark model of mesons, whose nonrelativistic limit correspond to Isgur-Scora-Grinstein-Wise model, is constructed. It possesses the heavy-quark symmetry and can be easily applied to calculation of form factors. Besides it can be engaged in novel tasks, such as the investigation of the two-photon decay of scalar mesons. Its behaviour in the infinite momentum frame and on the light cone is discussed.

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1. *Introduction*

It is useful to develop a fully covariant model that, in nonrelativistic limit (NRL), goes into the ISGW model. Such a covariant model can, to a great extent, retain the simplicity which was endearing and useful feature of the nonrelativistic quark model of Isgur, Scora, Grinstein and Wise (ISGW)[1]. That model has also been employed [2-5] in the investigation of heavy-quark symmetry (HQS). Although the ISGW model helped HQS investigations, this nonrealtivistic model was not capable [4,5] of properly reproducing all of the heavy-quark effective-theory (HQET) relations among semileptonic meson decay form factors. It had to be "relativized" to some extent [2-5]. In the covariant model HQS is produced by free particle Dirac spinors used to describe valence quarks in the correlated quark models [2-7]. This applies also to the models [8,9] in which valence quarks are bound to a center of force.

The $SU(2)$ "spin" symmetry [10,11] which holds in the effective field theory, holds in the covariant quark models also. One obtains HQS relations among form factors

$$F_1 = V = A_2 = \frac{(M_B + M_D)^2}{2M_B M_D(1+w)} A_1 \; ; \qquad w = v_B v_D \tag{1.1}$$

Here $M_{B,D}$ are B, D meson masses and $v_i = P_i/M_i$. The form factor F_1 appears in the $B \to D$ decay and A_1 in the $B \to D^*$ decay for example.

The form factors calculated in a nonrelativistic model satisfy HQS only in the zero recoil limit, where $w = 1$.

The covariant quark model leads to Isgur-Wise functions (IWF) in which relativistic corrections [5] and the Wigner rotation [4] are automatically included. Defining the slope ρ of IWF by

$$\xi(w) = 1 - \rho^2(w-1) + O((w-1)^2) \tag{1.2}$$

Reference 6 found $\rho = 1.17$. In the nonrelativistic model [1] one obtains [5] $\rho = 0.56$ or $\rho = 0.93$ when some relativistic corrections are included. Further increase in ρ is in agreement with Ref. 4 which concluded that Wigner rotation should increase the slope by about 20%.

A relativistic model can provide a qualitatively correct description of the two photon decays of scalar mesons.

Model can be used to illustrate either the infinite momentum frame (IMF), or light cone (LC) behaviour also. In this short review this last point will be only briefly sketched.

2. *Covariant quark model with correlated quarks*

The state of a pseudoscalar meson H is given by

$$|H(E,\vec{P},M)\rangle = N \sum_{c,s_1,s_2} \int 4m_Q m_d \mathrm{d}^4p\delta^4(p^2 - m_Q^2)\Theta(e)\mathrm{d}^4q\delta(q^2 - m_d^2)\Theta(\epsilon)\mathrm{d}^4K$$

$$\cdot F(K)\delta^{(4)}(p+q+K-P)\Theta(E)\phi(l_\perp)\ \overline{u}_Q\gamma_5 v_d\ d_d^+(\vec{q},c,s_2)b_Q^+(\vec{p},c,s_1)|0\rangle \tag{2.1}$$

Here $l^\mu = (p^\mu - q^\mu)/2$, $l_\perp^\mu(P) = l^\mu - P^\mu(P\cdot l)/M^2$, $p^\mu = (e,\vec{p})$, $q^\mu = (\epsilon,\vec{q})$. The valence quarks are represented by the on-mass-shell Dirac spinors $\overline{u}$ and v. The sea function $F(K)$ is for example

$$F(K) = \delta^{(4)}\left[K^\mu - \frac{P^\mu}{M}\left(\frac{P^\nu}{M}\big(P-(p+q)\big)_\nu\right)\right] \tag{2.2}$$

or

$$F(K) = \delta^{(4)}\left[K^\mu - \frac{P^\mu}{M}\left(\frac{P^\nu}{M}\big(P-(p+q)\big)_\nu\right)\right]e^{-\alpha K^2} \tag{2.3}$$

Some discusion of the second possibility can be found in Appendix B. The meson wave function can have the Gaussian form

$$\phi(l_\perp^\mu) = \frac{1}{\pi^{3/4}\beta_S^{3/2}}e^{+(l_\perp^\mu)^2/2\beta_S^2} \tag{2.4}$$

This construction insures that in the NRL model goes in the well known ISGW model [1]. HQS does not depend on a particular form of $F(K)$.

The meson decay constant f_H, for example, is with (2.2) given by

$$\frac{1}{(2\pi)^{\frac{3}{2}}} P^\mu f_H = \langle 0| : \bar{\psi}_2(0)\gamma^\mu \gamma_5 \psi_1(0) : |H(E, \vec{P}, M)\rangle$$

$$= 3N(\vec{P}) \int \mathrm{d}^3 p \frac{m_1 M}{eE} \frac{M m_2}{E\epsilon - \vec{P}\cdot\vec{q}} \phi(l_\perp) [\frac{m_2 p^\mu + m_1 q^\mu}{m_1 m_2}]|_{\vec{q}=-\vec{p}+\frac{\vec{P}}{M}(p_\parallel)T} \tag{2.5}$$

Here $p_\parallel = (Ee - \vec{P}\vec{p})/M$ and $T = 1 + \sqrt{m_2^2 - m_1^2 + (p_\parallel)^2}/p_\parallel$.

One obtains the same numerical value for f_H in any frame, including IMF, where:

$$P^\mu = (E, 0, 0, P) \ , \qquad P \to \infty,$$

$$x = \frac{p_z}{P} \ , \qquad \vec{p}_T = (p_x, p_y),$$

$$f_H = 3N(2\pi)^{\frac{5}{2}} \int_0^\infty p_T \mathrm{d}p_T \int_0^\infty \mathrm{d}x \frac{2m_1}{Mx} \phi_{IMF}(l_\perp), \tag{2.6}$$

$$\phi_{IMF}(l_\perp) = \frac{1}{\pi^{3/4}\beta^{3/2}} exp\{\frac{1}{2\beta^2}[\frac{-\vec{p}_T^2}{2}(1 + \frac{m_1^2}{x^2 M^2} + \frac{\vec{p}_T^2}{2x^2 M^2}) - \frac{M^2}{4}(x - \frac{m_1^2}{xM^2})^2]\}.$$

Due to the presence of the quark-gluon sea, which also carries part K of the total momentum P, the momentum carried by the antiquark, i.e. q_z/P, is not $(1 - x)$.

3. *Two photon decays of scalar mesons*

Here the usage of small quark masses means the avodiance of the weak binding limit approximation in its strictest sense [1]. That might better mimick the real quark fields which should appear in the photon emitting quark loop (Fig. 1) in the first order of QED/QCD expansion. A necessary modification of the model is discussed in Appendix A.

Such reparametrized model is applied to study two photon decays of $a_0(980)$, $f_0(980)$, $f_0(1370)$ and χ_{c0} mesons. The experimental data [12], on strong decays of $a_0(980)$, $f_0(980)$ and $f_0(1370)$ do not lead to the final understanding of the real structure of these mesons [13-26]. Ideas exists that these states are $K\bar{K}$ molecules [13-18]. It seems that $q\bar{q}$ structure is prefered by their decays into light pseudoscalars [18-25].

The analisys was caried out with the quark wave function

$$\phi_f(l_\perp^\mu) = \frac{1}{(1 - \frac{l_\perp^2}{4\beta_{f,H}^2})^2} . \tag{3.1}$$

The dipol form (3.1) was found to be a better choice than the exponential form used earlier [6]. See also some remarks in Appendix A concerning the parameter(s) $\beta_{f,H}$ ($f = u, d, s, c$). For simplicity we have used (2.2). The spinor form in (2.1) is replaced for scalar mesons by

$$\bar{u}_Q v_d \tag{3.2}$$

The scalar meson state is normalized by

$$\langle H(E,\vec{P},M)|H(E,\vec{P},M)\rangle = 2E = 3N(0)^2 \sum_f C_f^2 \int \mathrm{d}^3p \, \frac{\epsilon}{e} \, (\frac{\phi_f(l_\perp)}{q_\parallel})^2 \, (pq - m_1 m_2) \tag{3.3}$$

Such model states are consistent with the very general requirement, i.e.

$$\langle 0|V^\mu|H(E,\vec{P},M)\rangle \equiv 0 \tag{3.4}$$

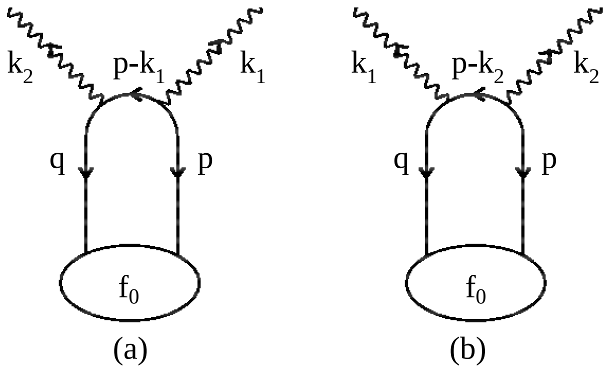

Fig. 1. Two-photon decay. Full lines are valence quarks, wavy lines are emitted photons and blobs symbolize scalar meson states.

The amplitude $\mathcal{M}$ for the transition $f_0 \to 2\gamma$ is determined from the leading diagrams shown in Fig. 1. A loop corresponds to each quark flavor. For example, for the flavor d the amplitude corresponding to the diagram in Fig. 1a is determined by

$$\mathcal{M} = -\frac{3N(0)}{m_d \omega (2\pi)^{\frac{1}{2}}} \int p^2 \mathrm{d}p \, sin\theta d\theta \, \frac{m_d^2}{e^2} \frac{1}{(1 + p^2/(4\beta_{d,f_0}^2))^2} (\vec{\epsilon}_1 \vec{\epsilon}_2) \frac{2p^2(\omega cos^2\theta + e sin^2\theta)}{e^2 - p^2 cos^2\theta}$$

$$\equiv (\vec{\epsilon}_1 \vec{\epsilon}_2) \cdot I_{d,\overline{d}}(f_0) \tag{3.5}$$

The calculation of the decay width

$$\Gamma = \frac{1}{32\pi M_H}\overline{|\mathcal{M}_H|^2} \tag{3.6}$$

requires the summation over photon polarization states, as well as the summation over quark flavors wich are connected with the meson quark structure;

$$|f_0(980)\rangle = \frac{\cos\theta}{\sqrt{2}}(|u\bar{u}\rangle + |d\bar{d}\rangle) + \sin\theta|s\bar{s}\rangle \tag{3.7}$$

$$|a_0(980)\rangle = \frac{1}{\sqrt{2}}(|u\bar{u}\rangle - |d\bar{d}\rangle) \tag{3.8}$$

$$|f_0(1370)\rangle = \frac{-\sin\theta}{\sqrt{2}}(|u\bar{u}\rangle + |d\bar{d}\rangle) + \cos\theta|s\bar{s}\rangle \tag{3.9}$$

$$|\chi_{c_0}(3415)\rangle = |c\bar{c}\rangle \tag{3.10}$$

The gauge invariance of all results can be explicitly tested by showing that the amplitude $\mathcal{M}$ does not change under gauge transformation

$$\epsilon_{i\mu} \to \epsilon_{i\mu} + \Lambda k_{i\mu} \tag{3.11}$$

Using the model parameters detarmined and described in Appendix A, and the states (3.7)-(3.10) corresponding to mixings given [25] given by

$$\cos\theta = 1\ ; \quad \sin\theta = 0 \tag{3.12}$$

or

$$\cos\theta = \frac{1}{3}\ ; \quad \sin\theta = \frac{2\sqrt{2}}{3} \tag{3.13}$$

one ends with Table 1.

The results indicate the importance of the quarkonium structure $q\bar{q}$ [1] in the meson state.The experimental error in $f_0(980) \to 2\gamma$ rate is rather large. Although, the larger theoretical prediction in Table 1, seems to be in better agreement with experiments, the smaller one, which corresponds to the ideal mixing, cannot be ruled out. However, the $f_0(1370)$ decay into pions indicates the presence of the light $q\bar{q}$ combinations [18,21,22]. Our result also agrees with the nonideal mixing as considered by Lanik [24]. Our approach has some analogy with Deakin et al. [24] who used constituent quark masses and concluded that theoretical results depend strongly on the numerical values of those masses.

TABLE 1. Decay widths.

Meson	Mixing	$\Gamma_{theory}(keV)$	$\Gamma_{exp}(keV)$
$a_0(980)$		0.137	0.26 ± 0.08
$f_0(980)$	(3.12)	0.380	0.56 ± 0.11
$f_0(980)$	(3.13)	0.534	0.56 ± 0.11
$f_0(1370)$	(3.12)	0.348	
$f_0(1370)$	(3.13)	0.145	
$\chi_{c_0}(3415)$		4.608	4.0 ± 2.8

4. Covariant quark model and light cone variables

The LC field theory which may allow the derivation of a constituent quark picture [27] has prompted an extensive study of LC quark models [28,29,31].

Any covariant quark model can be written using *LC* variables. The state (2.1) for example becomes ($m_Q = m_d = m$):

$$|H(P^+, \vec{P}_\perp, M)\rangle = N \sum_{c,s_1,s_2} \int \mathrm{d}p^+ \mathrm{d}^2\vec{p}_\perp \frac{m}{p^+} \int \mathrm{d}q^+ \mathrm{d}^2\vec{q}_\perp \frac{m}{q^+} \delta(q^+ + p^+ - 2\frac{P^+}{M}(p_\parallel))$$

$$\cdot\delta(\vec{q}_\perp + \vec{p}_\perp - 2\frac{\vec{P}_\perp}{M}(p_\parallel)) \frac{Mm}{P^+ q_\parallel} \phi(l_\perp) \overline{u}_Q \gamma_5 v_d d_d^+(q^+, \vec{q}_\perp, c, s_2) b_Q^+(p^+, \vec{p}_\perp, c, s_1)|0\rangle. \quad (4.1)$$

Here $p^+ = e + p_3$, $\vec{p}_\perp = (p_1, p_2)$, $p_\parallel = ((p^+ P^- + p^- P^+)/2 - \vec{p}_\perp \vec{P}_\perp)/M$, $p^- = (m^2 + \vec{p}_\perp^2)/p^+$ etc.

Although based on the same physics the models [6] and [28–32] are not identical. Some differences appear in parametrization of the model wave function. The relative coordinate k^2 of Refs. 28, 29, 31 and 32 is in the notation of Ref. 6

$$k^2 = -\frac{1}{4}(p - q)^2 \quad (4.2)$$

In Refs. 27, 28, 29, 31 and 32, a different spinor basis $u_{LC}(p)$ is employed, which is connected to textbook spinors $u(\vec{p}, E)$ [18] through:

$$u_{LC}(p)^s \sim u^s(\vec{p}, E) + v^{-s}(-\vec{p}, -E) \quad (4.3)$$

LC spinors depend only on p^+ and $p_{R,L} = (p_1 \pm ip_2)/2$ combinations.

While an u spinor can be constructed [33] by boosting the Pauli spinor χ, the u_{LC} spinor is produced by a more complex Melosh transformation

$$u_{LC} = \frac{1}{2\sqrt{mp^+}} \begin{pmatrix} U_M & -Z_M \\ Z_M & -U_M \end{pmatrix} \begin{pmatrix} \chi \\ 0 \end{pmatrix} \tag{4.4}$$

$$U_M = \begin{pmatrix} p^+ + m & -p_L \\ p_R & p^+ + m \end{pmatrix}$$

$$Z_M = \begin{pmatrix} p^+ - m & p_L \\ p_R & -p^+ + m \end{pmatrix}$$

This is only a formal difference as model [6] can be easily constructed by using u_{LC}. The results (formfactors, widths) do not change.

Another important difference lies in the model description of sea effects, which in Ref. 6 are contained in $F(K)$ (2.2, 2.3). The δ^4 functions appearing in (2.1) and (2.2) are replaced in Refs. 27, 28, 29, 31 and 32 by a different construct.

Appendix A

Reference 1 has determined the meson wave function parameter β from variational principle assuming the quark-antiquark interaction

$$V(r) = -\frac{4\,\alpha}{3\,r} + b\,r + c \tag{A1}$$

$$\alpha = 0.5, \quad b = 0.18\,GeV^2, \quad c = -0.84\,GeV$$

They had used the nonrelativistic quark kinetic energies e.g. $E_Q = \vec{p}^2/2\overline{m}_Q$ with largish constituent masses $\bar{m}_i$ If procedure is carried out with the same potential but with the relativistic energies $e_Q = \sqrt{\vec{p}^2 + m_Q^2}$ [34] simmilar β's can be found for $m_Q < \overline{m}_Q$. This is ilustrated in Fig 2.

One finds $\beta_{rel} > \beta_{ISGW}$ for $m_Q < 0.25\,GeV$. At about $m_Q = 0.3\,GeV$ β's are comparable. Thus in a relativistic model one might find a wave fnction wich is consistent with ISGW function, while model quark masses are small.

For a scalar meson in the rest frame the model (mock meson) mass M_0 is

$$M_0 = \frac{\sum_f C_f^2 \int\limits_0^\infty 4\pi p^2\,\mathrm{d}p \frac{2p^2\phi_f^2}{p^2+m_f^2}(2\epsilon_f + V(p))}{\sum_f C_f^2 \int\limits_0^\infty 4\pi p^2\,\mathrm{d}p \frac{2p^2\phi_f^2}{p^2+m_f^2}} \tag{A2}$$

Here summation goes over quark flavors, and the model wave functions has dipole form (3.1). Masses are:

$$m_u = m_d = 0.015\,GeV;\;\; m_s = 0.120\,GeV;\;\; and\; m_c = 1.4\,GeV$$

If one omits the potential and requires that $M = M_0$, where M is the real meson mass (Table 1), one finds for β parameters

$$\beta = 0.288;\ 0.288;\ 0.262;\ 0.380;\ 0.399;\ 0.346 \qquad \text{(A3)}$$

[These values correspond to mesons listed in the first column of the Table 1.]

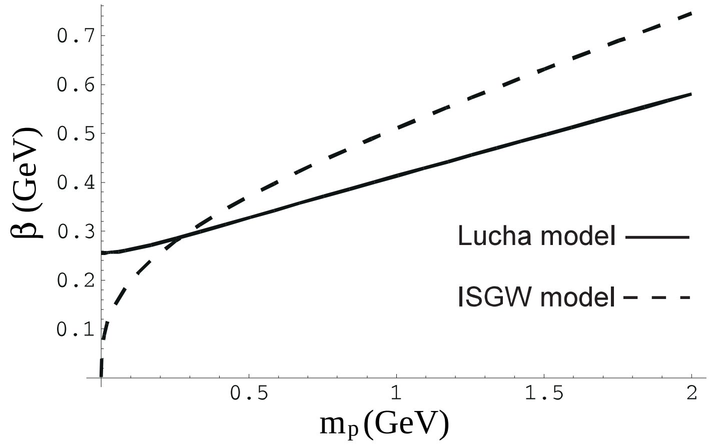

Fig. 2. ISGW [1] and Lucha [34] model β's for $|q\bar{q}\rangle$

Alternatively one can use the potential (A1) and find β's which minimase the expresion (A2) [1,34]. The results, obtained for simplicity for $m_f = 0$, are in resonable agreement with (A3). For the first two two cases one finds $\beta = 0.256$ which agrees within 12%. The choice describes mock meson states for which $P^2 = M^2$.

Appendix B

Some more realistic forms of the sea function, as for example (2.3) would also influence model predictions. This possibilities have not yet been fully exploited. Some interesting preliminary results, concerning the Isgur-Wise function (IWF) and the meson decay constant f_M are listed below.

Using parameters from [1] and (2.2) one obtains for the slope ρ (1.2)

$$\rho \cong 1.07$$

With (2.3) and $\alpha = 1/2\beta^2$ one finds

$$\rho \cong 1.17$$

Obviously the sea is noticable.

The sea contribution could lead to better description of meson states. That can be illustrated in the calculation of the meson decay constants. With (2.2) one finds $f_\pi = 461MeV$ ($f_{\pi_{exp}} = 131.7MeV$) and $f_K = 412MeV$ ($f_{K_{exp}} = 160.6MeV$). However by using (2.3) with $\alpha = \alpha_s/2\Lambda^2$, $\Lambda = 0.25GeV$, and $\alpha_s = 0.6(0.55)$ for $\pi(K)$ meson, one finds $f_\pi = 132MeV$ and $f_K = 160MeV$.

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RELATIVISTIČKI MODEL KVARKOVA

Izveli smo poopćeni Lorentz-kovarijantni model mezona čiji nerelativistički limes odgovara Isgur-Scora-Grinstein-Wiseovom modelu. Model ima teško-kvarkovsku simetriju i može se primijeniti za računanje faktora oblika. Osim toga u ovom se modelu mogu rješavati nove zadaće kao što je dvofotonski raspad skalarnih mezona. Raspravljamo svojstva modela u sustavu beskonačnog impulsa i na svjetlosnom stošcu.