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NEW RESULTS ON PERTURBATIVE COLOUR TRANSPARENCY IN QUASI-EXCLUSIVE ELECTROPRODUCTION

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We review the perturbative QCD formalism of hadronic electromagnetic form factors and the colour transparency ratio for quasi-exclusive electroproduction of the proton and pion from nuclear targets. We have completed the first full calculations including all leading-order quark subprocesses and integrations over distribution amplitudes, including Sudakov effects. For the case of the proton, the calculated result shows scaling beyond $Q^2 = 10$ GeV^2. The calculation incorporating filtering due to the nuclear medium is cleaner than the corresponding calculation in free space because of attenuation of large-distance amplitudes. We find that the colour transparency ratio is rather insensitive to theoretical uncertainties inherent in the perturbative formalism, such as the choice of the hadron distribution amplitude.

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1. *Introduction*

The practical applicability of perturbative QCD to exclusive processes [1–3], such as hadronic electromagnetic form factors, cannot yet be considered to be settled. It has been argued that even at the highest momenta explored so far in the laboratory, the dominant contribution to form factors comes from the end-point regions of the wave function, where the perturbative treatment fails [4,5]. In the case of hadron-hadron scattering, there exist further difficulties, such as the common failure of the helicity conservation selection rules to agree with experimental data. However, in nearly all experiments one finds that the naive prediction for quark counting scaling laws tend to agree very well with data. In view of the problems listed, this is quite mysterious, since so far there does not exist any alternate mechanism which can explain these scaling laws.

An interesting prediction of perturbative QCD is colour transparency [6]. At large momentum transfers, only the short-distance components of the hadron wave function can contribute to exclusive processes. Since the total cross section of hadrons σ is proportional to their area b^2, the strong interactions of these hadrons are expected to be reduced. If we consider quasi-exclusive electron-nucleus scattering, eA→ e'p(A-1), where A denotes a nucleus with A nucleons, then the nucleus is predicted to be transparent to all protons participating in this process. This is an asymptotic argument applicable for fixed A as $Q^2 \to \infty$. Experimentally, however, we can only take the limit of large A and moderately large Q^2, in which such processes appear to be more complicated and much more interesting. One picture that is emerging [7] is that exclusive processes in free space get significant contribution from perturbatively calculable hard amplitudes but also have non-negligible soft contamination. The corresponding nuclear processes, however, may be much cleaner [8,9] because the large quark separations will be strongly attenuated in nuclear medium. This phenomenon, called nuclear filtering, has some experimental support. Experimentally, one finds that the fixed-angle free-space process pp' → p"p"' [10] shows significant oscillations at 90° as a function of energy. These oscillations are not a small effect, but roughly 50% of the $1/s^{10}$ behaviour, and are interpreted as coming from interference of long- and short-distance amplitudes. The corresponding process in a nuclear environment pA→ p'p"(A-1) shows no oscillations, and obeys the pQCD scaling-power law better than the free-space data [8,9]. The A dependence, when analyzed at fixed Q^2, shows statistically significant evidence of reduced attenuation [11].

2. *Formalism*

We briefly review the framework for the calculation of hadronic form factors following Li and Sterman [12]. It has long been known that the transverse separation of quarks in free-space reactions is controlled by effects known as the Sudakov form factor. The pion form factor is the simplest example. Li and Sterman included Sudakov effects here, arguing that a perturbative treatment becomes fairly reliable at momenta of the order of 5 GeV. As low as 2 GeV, it was found that less than 50% of the contribution comes from the soft region.

Let b_{ij} be the transverse separation between quarks i and j, or b the corresponding quantity for a single pair of quarks. An essential feature is the inclusion of $\exp(-S)$, a Sudakov form factor which suppresses the large b region. Including the b dependence, the pion electromagnetic form factor can be written as,

$$F_\pi(Q^2) = \int \mathrm{d}x_1 \mathrm{d}x_2 \frac{\mathrm{d}\vec{b}}{(2\pi)^2} \mathcal{P}(x_2, \vec{b}, P', \mu) T_H(x_1, x_2, \vec{b}, Q, \mu) \mathcal{P}(x_2, \vec{b}, P, \mu) \ , \tag{1}$$

where

$$\mathcal{P}(x, b, P, \mu) = \exp(-S) \times \phi(x, 1/b) + O(\alpha_s(1/b))$$

plays the role of the hadron wave function, $\phi(x, 1/b)$ is the meson-distribution amplitude, P and P' are the incident- and outgoing-pion momenta, respectively, and S is the Sudakov form factor. The improved factorization used in Ref. 12 retains the intrinsic transverse-momentum k_T dependence in the gluon propagator, since k_T need not be small compared to $\sqrt{x_1 x_2}\,Q$, if one of the x_i gets close to zero. The variable b in Eq. (1) is conjugate to $k_{T1} - k_{T2}$, where k_{T1} and k_{T2} are the transverse momenta of the incident and outgoing pions, respectively. As long as x_1 and x_2 are not close to their endpoints, the dominant scale in the scattering is $\sqrt{x_1 x_2}\,Q$, and the small b region dominates the amplitude. Close to the end points of x_1 or x_2, $\sqrt{x_1 x_2} Q$ may become very small. However, the dominant scale in this region is $1/b$, which is again not too small since the large b region is strongly damped by the Sudakov form factor. The results for the free-space form factor for pion using this procedure are given in Ref. 12. The authors show that at $Q^2 = 5$ GeV^2, something like 90% of the contribution comes from a region where α_s/π is less than 0.7 and hence could be regarded as perturbative.

The nuclear medium modifies the quark wave function such that [13,14]

$$\mathcal{P}_A(x, b, P, \mu) = f_A(Q^2, b)\mathcal{P}(x, b, P, \mu), \tag{2}$$

where $\mathcal{P}_A$ is the wave function inside the medium and f_A is the nuclear filtering amplitude. We use a simple model for f_A,

$$f_A = \exp(-\int \mathrm{d}z\ \sigma\rho).$$

The effective inelastic cross section σ is known to scale like b^2 in QCD, where b is the size of the hadron. We parametrize it as kb^2 and adjust the value of k to find a reasonable fit to the experimental data.

The situation for the proton form factor [15] is somewhat more complicated than that of the pion; we do not have the space for all details here which are given in Refs. 16 and 17. There has been some controversy regarding the proper choice of the infrared cutoff in the Sudakov exponent. In the case of pion, this was simply the quark-antiquark separation b. The choice proposed in Ref. 18 uses the largest distance between the three quarks as the cutoff. It was found that this gave results about 50% smaller than experiments. Perhaps this is the right direction, if indeed other wave functions (and in particular, non-zero quark angular momentum) contribute heavily in free space. On the other hand, in Ref. 16, it was observed that the largest distance does not correspond to a physical size of the three quark system. A more appropriate choice might be obtained by considering the triplet of valence quarks as a quark-diquark system. This choice takes the maximum value of the distance between quark and diquark as the effective cutoff in the Sudakov exponent. This essentially amounts to using a scale cw for infrared cutoff, with $c \approx 1.14$, where w is the inverse of the largest distance between any two valence quarks in the proton. Remarkably, this small modification leads to results in good agreement with the experiment [16].

From investigations of the proton form factor in free space, it seems that the Sudakov effects eliminate about 50% of the contribution from the soft region. The Sudakov filtering in free space does something useful, but does not seem to be sufficient to make present free-space calculations totally reliable. The same diagrams for Sudakov effects, of course, occur in a nuclear environment. In addition, there are much stronger interactions with the nuclear target, when one goes from pure "vacuum filtering" by Sudakov to nuclear filtering. We find that nuclear medium eliminates much of the remaining 50% of the soft region. These are the first full calculations of these ideas within perturbative QCD. We find that the main uncertainty in the nuclear calculation arises from uncertainties in nuclear medium itself, in particular, in uncertainties in the nuclear spectral functions and correlations. With standard assumptions, one can proceed with the calculation essentially using zero parameters and no model dependence. However, we find that numerical differences between models of nuclear matter are large enough to cause significant uncertainties. Indeed, comparison with data shows that the uncertainties in the nuclear spectral functions and the nuclear correlations now dominate the theoretical uncertainties, and are larger effects than, for example, the dependence on the hadron distribution amplitude.

3. *Results and discussion*

The results for free-space proton form factor are shown in Fig. 1. An important

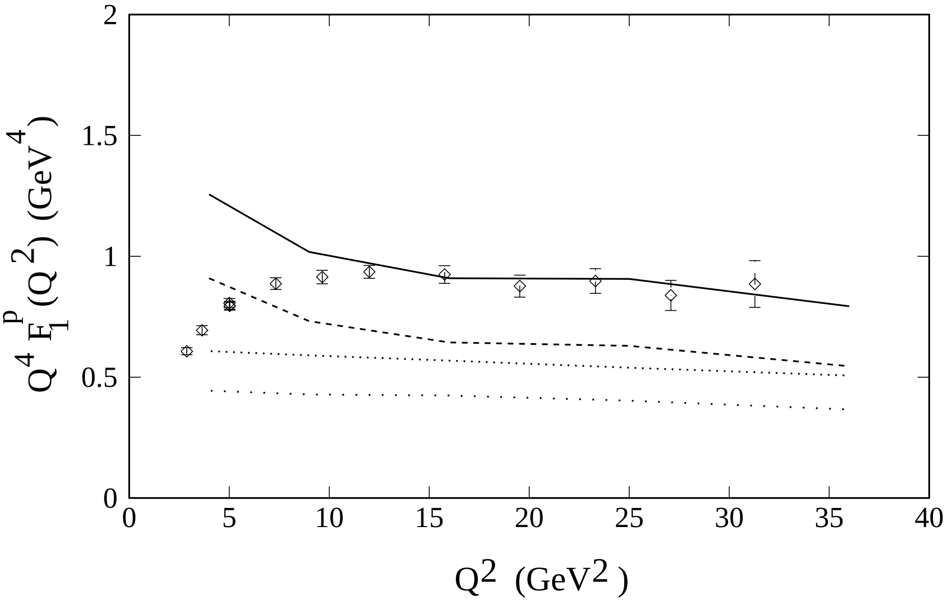

Fig. 1. The Q^2 dependence of the proton form factor Q^4F_1 using the KS wave function ($c = 1.14$, solid line; $c = 1$, dense-dot line) and for the CZ wave function ($c = 1.14$, dashed line; $c = 1$, dotted line). The experimental data are also shown.

feature of this result, which is independent of the details of the wave function, is that it shows scaling for Q^2 larger than about 10 GeV^2. This is a nontrivial confirmation that Q^2 indeed dominates over the intrinsic momentum k_T^2.

In Fig. 2, we show results for colour transparency for electroproduction of pions for different nuclei using the CZ wave function. Here, we adjust the value of k, corresponding to the pion attenuation cross-section of 25–30 mb for a pion size of about 0.8 fm. The predicted results are shown for $k = 4$. The precise value of k might best be obtained by making a fit to the data for colour transparency after they become available, or perhaps by detailed comparison with diffractive calculations. Compared to the asymptotic wave functions, the results for T change by less than 3% for Q^2 larger than 10 GeV^2.

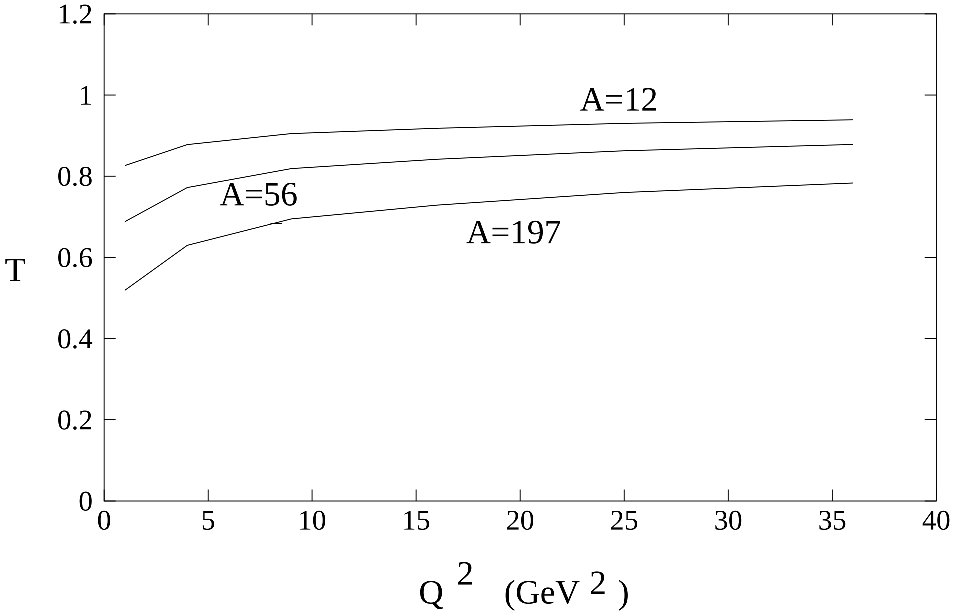

Fig. 2. The calculated pion transparency ratio for different nuclei.

The results for the proton transparency ratio are given in Fig. 3. The parameter k in the attenuation cross section $\sigma = kb^2$ was chosen so as to provide a reasonable fit to the experimental data [19,20]. We find that a value of $k = 6$ gives a reasonable fit. Taking the attenuation cross section of normal protons to be 36 mb, this corresponds to a typical b of about 0.77 fm, which is a reasonable estimate of the proton size. Since the data for T are available only in the region where the calculated free space form factor is in disagreement with the experimental result, the value of k obtained by this procedure cannot be taken too seriously. In fact, parameter k would be best obtained by fitting to the experimental value of T after it is measured at higher energies. A reasonable range of k values, which we take to be $k = 5$ and $k = 6$, corresponds to b values of 0.85 fm and 0.77 fm, respectively, and has been used in the figure.

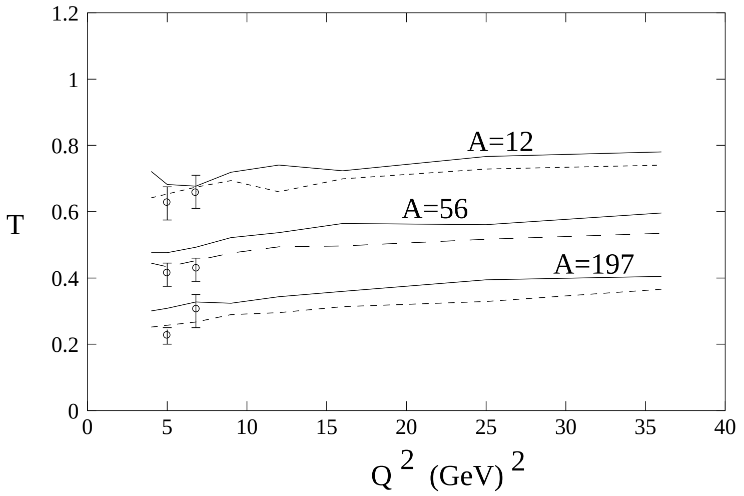

Fig. 3. The calculated transparency ratio for the proton for different nuclei. The experimental points are taken from Refs. 19 and 20. The solid curves are calculated with $k = 5$ and the dashed curves with $k = 6$.

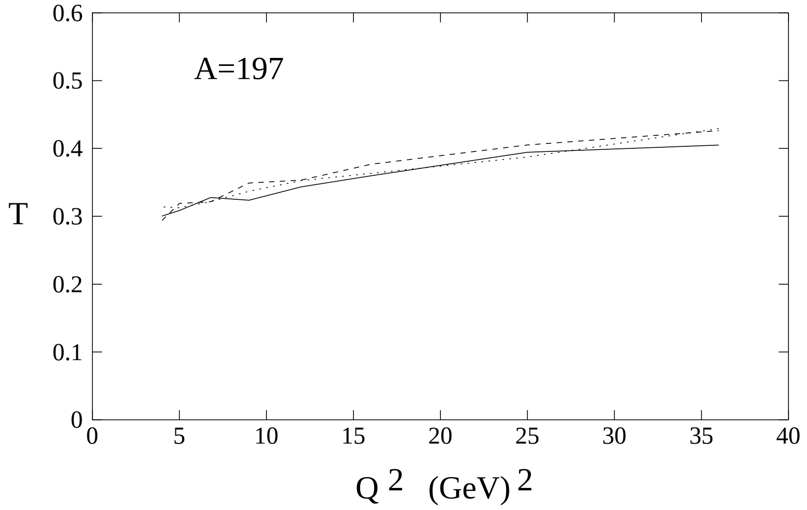

Fig. 4. The sensitivity of the calculated transparency ratio to different proton wave functions. Slight oscillations are an artifact of the Monte Carlo integrations. The solid curve is calculated with the KS wave function, as in Fig. 3, and the dotted curve is calculated with the CZ wave function; both curves use the infrared cutoff parameter $c = 1.14$. For reference, the dashed curve shows the result for the cutoff proposed in Ref. 18, which amounts to setting $c = 1.0$, using the KS wave function. The calculations are shown for $A = 197$.

We have also checked the dependence of our result on the infrared cutoff parameter c and the choice of the wave function. We find in Fig. 4 that the results for transparency ratio change very little if we use the CZ wave function instead of the KS. This is a surprising result, and one of the bases of our claim that the dominant uncertainty in transparency ratio may be due to the nuclear model itself.

4. *Conclusion*

We have reviewed the calculation of hadronic electromagnetic form factors and colour transparency using perturbative QCD. We find a slow rise of the transparency ratio at energies that can be probed in the future at CEBAF and ELFE. As discussed elsewhere [11,7], precision experiments can discover colour transparency even with a slow rise in Q^2 by measuring the A dependence at fixed moderately large Q^2. Due to filtering of long-distance components in the medium, the nuclear calculation is considerably cleaner compared to the free-space calculation. We also find rather remarkable insensitivity of the transparency ratio to the present theoretical uncertainties in the perturbative QCD treatment, such as the choice of the distribution amplitude. To further improve the accuracy of predictions for colour transparency ratio, it is necessary to improve the modelling of nuclear medium which now appears to be the dominant source of error.

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NOVI REZULTATI ZA PERTURBATIVNU PROVIDNOST BOJE U POLUISKLJUČIVOJ ELEKTROTVORBI

Opisuje se formalizam perturbativne QCD za hadronske elektromagnetske faktore oblika i omjer providnosti boje poluisključive elektrotvorbe protona i piona u nuklearnim metama. Završili smo prvi potpun račun, uključivši sve vodeće kvarkovske potprocese i integracije preko raspodjelnih amplituda, uključivši i Sudakove učinke. Rezultat za proton pokazuje sličnost dalje od $Q^2 = 10$ GeV^2. Račun s filtriranjem uzrokovanim nuklearnom sredinom je pouzdaniji od odgovarajućeg računa u slobodnom prostoru zbog slabljenja amplituda na velikim udaljenostima. Nalazimo da je omjer providnosti boje vrlo neosjetljiv na teorijske neodređenosti perturbativnog formalizma, kao što je odabir amplitude hadronske raspodjele.