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# MESON ELECTROWEAK FORM FACTORS IN THE LIGHT-CONE APPROACH

CHUENG-RYONG JI [1] and HO-MEOYNG CHOI

*Department of Physics, North Carolina State University Raleigh, NC 27695-8202, USA*

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We investigate the electroweak form factors and semileptonic decay rates of pseudoscalar mesons using the constituent quark model based on the light-cone degrees of freedom. Our results demonstrate the broader applicability of the light-cone approach including the time-like region of exclusive processes.

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## 1. *Introduction*

While the well-known successful examples of light-cone approach are often referred to inclusive processes, such as the parton model and the PQCD corrections in deep inelastic lepton hadron scattering, etc., the quark counting rule in exclusive processes, rigorously predicted from the light-cone formulation of PQCD, is still under severe debate [1]. “How high is really high?” is the main debate; $Q^2 \approx 100?$, 10? or 1? $\mathrm{GeV}^2/c^2$ as the relevant minimum momentum scale for the PQCD applicability to exclusive processes. While the debate is still going on, we recently applied our light-cone quark model [2] to the exclusive meson reactions and decay processes at small $Q^2$ region (less than a few $\mathrm{GeV}^2/c^2$) and found that the model works fairly well for various mass spectra and wave-function-related observables. In this paper, we mainly discuss our light-cone constituent quark model predictions for the meson electroweak form factors at small momentum transfers (i.e. $Q^2 \lesssim$ few $\mathrm{GeV}^2/c^2$). Especially, we focus on the calculations of observables in time-like $|Q^2|$ region overcoming the diffculties associated with the quark-antiquark pair creation(“Z-graph”) in the light-cone approach. As an explicit example of the

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[1] E-mail address: crji@ncsu.edu

application to the time-like $|Q^2|$ region, we present our recent analysis of semileptonic weak decay processes [3]. In Section 2, we discuss the distinguished features of light-cone degrees of freedom. In Section 3, we describe our QCD-motivated light-cone quark model. In Section 4, we present our model calculations of semileptonic pseudoscalar meson decay processes as an example of application to the time-like $|Q^2|$ region. Conclusions and discussions follow in Section 5.

## *2. Light-cone degrees of freedom*

In the light-cone approach, a hadron is characterized by a set of Fock-state wave functions, the probability amplitudes for finding different combinations of bare quarks and gluons in the hadron at a given light-cone time $\tau = t + z/c$. These wave functions provide the essential link between hadronic phenomena at short distances (perturbative) and at long distances (non-perturbative) [4]. The distinguished features in the light-cone approach are the simplicity of the vacuum, except the zero-modes and the dynamical property of rotation operators. The vacuum at equal $\tau$ presents a dramatic difference compared to the vacuum at equal $t$. For a particle which has the mass $m$ and the four-momentum $k = (k^0, k^1, k^2, k^3)$, the relativistic energy-momentum relation of the particle at equal $\tau$ is given by

$$k^- = (\vec{k}_\perp^2 + m^2)/k^+, \tag{1}$$

where the light-cone energy conjugate to $\tau$ is given by $k^- = k^0 - k^3$ and the light-cone momenta $k^+ = k^0 + k^3$ and $\vec{k}_\perp = (k^1, k^2)$ are orthogonal to $k^-$ and form the light-cone three-momentum $\underline{k} = (k^+, \vec{k}_\perp)$. The rational relation given by Eq. (1) is drastically different from the irrational energy-momentum relation at equal $t$, $k^0 = \sqrt{\vec{k}^2 + m^2}$, where the energy $k^0$ is conjugate to $t$ and the three-momentum vector $\vec{k}$ is given by $\vec{k} = (k^1, k^2, k^3)$. The important point here is that the signs of $k^+$ and $k^-$ are correlated while at equal $t$ the signs of $k^0$ and $\vec{k}$ are not correlated. Thus the momentum $k^+$ is always positive because only the positive energy $k^-$ makes the system evolve to the future direction, i.e., positive $\tau$, while the momentum $k^3$ can be either positive or negative even though $k^0$ is positive to evolve the system in the future direction (i.e. positive $t$). This provides a remarkable feature to the light-cone vacuum; i.e., the Fock-state vacuum is an eigenstate of the full Hamiltonian. Consequently, all bare quanta in hadronic Fock states are associated with the hadron and none are disconnected elements of the vacuum. Furthermore, the problem of boost operators at equal $t$ changing particle numbers can be cured by this framework since the quantization surface $\tau = 0$ is invariant under both longitudinal and transverse boosts defined at equal $\tau$. However, the quantization surface $\tau = 0$ is not invariant under the transverse rotation whose direction is perpendicular to the direction of the quantization axis $z$ at equal $\tau$. Thus, the transverse angular momentum operator involves the interaction that changes the particle number and it is not easy to specify the total angular momentum of a particular hadronic state. Also, $\tau$ is not invariant under parity. We avoid these problems by using the

Melosh transformation [5] of each constituent from equal $t$ to equal $\tau$. The Melosh transformation uniquely determines the assignment of angular momentum, parity and charge conjugation for the light-cone wave function of hadrons.

## 3. *QCD motivated light-cone quark model*

The key idea in our light-cone quark model for mesons is to saturate the Fock-state expansion by the constituent quark and anti-quark and use the variational principle for a QCD motivated Hamiltonian to fix all model parameters by comparing with the meson mass spectra. The QCD motivated effective Hamiltonian for the description of the meson mass spectra is given by

$$H_{q\bar{q}} = H_0 + V_{q\bar{q}} = \sqrt{m_q^2 + k^2} + \sqrt{m_{\bar{q}}^2 + k^2} + V_{q\bar{q}}. \tag{2}$$

We use the usual confining interaction potential $V_{q\bar{q}} = V_0(r) + V_{\rm hyp}(r)$ given by

$$V_{q\bar{q}} \quad = \quad a + b\mathcal{V}_{\rm conf.}(r) - \frac{4\kappa}{3r} + \frac{2\vec{S}_q \cdot \vec{S}_{\bar{q}}}{3m_q m_{\bar{q}}} \nabla^2 V_{\rm Coul}, \tag{3}$$

where $\mathcal{V}_{\rm conf.}(r) = r[r^2]$ is linear (harmonic oscillator) type potential. Our variational method first evaluates $< \phi|[H_0 + V_0]|\phi >$ with a trial function $\phi(k^2) = Ne^{-k^2/2\beta^2}$ that depends on the parameters $(m, \beta)$, and varies these parameters until the expectation value of $H_0 + V_0$ is a minumum.

*TABLE 1. Optimized quark masses* $m_q$ [GeV] *and the Gaussian parameters* $\beta$ [GeV] *for both harmonic oscillator and linear potentials obtained from the variational principle. q=u and d.*

| Potential | $m_q$ | $m_s$ | $m_c$ | $m_b$ | $\beta_{q\bar{q}}$ | $\beta_{s\bar{s}}$ | $\beta_{q\bar{s}}$ | $\beta_{q\bar{c}}$ | $\beta_{q\bar{b}}$ |
|---|---|---|---|---|---|---|---|---|---|
| H.O. | 0.25 | 0.48 | 1.8 | 5.2 | 0.32 | 0.37 | 0.34 | 0.42 | 0.50 |
| Linear | 0.22 | 0.45 | 1.8 | 5.2 | 0.37 | 0.41 | 0.39 | 0.47 | 0.53 |

*TABLE 2. Decay constants* [MeV] *and charge radii* [fm$^2$] *for various heavy pseudoscalar and vector mesons.*

| Potential | $f_D$ | $f_{D^*}$ | $f_B$ | $f_{B^*}$ | $r^2_{D^+}$ | $r^2_{D^0}$ | $r^2_{B^+}$ | $r^2_{B^0}$ |
|---|---|---|---|---|---|---|---|---|
| H.O. | 127.1 | 149.6 | 113.8 | 122.3 | 0.182 | −0.309 | 0.420 | −0.208 |
| Linear | 139.2 | 168.9 | 121.2 | 131.4 | 0.176 | −0.301 | 0.438 | −0.217 |

Once these model parameters are fixed, then, the mass eigenvalue of each meson is obtained by $M_{q\bar{q}} =< \phi|[H_0+V_0]|\phi > + < \phi|H_{\rm hyp}|\phi >$. Our model parameters are summarized in Table 1. Both the mass spectra and the wave functions of the light pseudoscalar $(\pi, K, \eta, \eta')$ and vector$(\rho, K^*, \omega, \phi)$ mesons were analyzed using our light-cone constituent quark model [2]. The mixing angles of $\omega - \phi$ and $\eta - \eta'$ were predicted and various physical observables such as decay constants, charge radii, and radiative decay rates etc., were calculated in Ref. 2. Our numerical results in Ref. 2 are in a good agreement with the available experimental data. In Table 2, the decay constants and charge radii of heavy mesons are summarized.

## *4. Semileptonic pseudoscalar meson decays*

The semileptonic decays of a pseudoscalar meson $Q_1\bar{q}$ into another pseudoscalar meson $Q_2\bar{q}$ are governed by the weak vector current as follows

$$< P_2|\bar{Q}_2\gamma^\mu Q_1|P_1 > \;=\; f_+(q^2)(P_1+P_2)^\mu + f_-(q^2)(P_1-P_2)^\mu, \qquad (4)$$

where $q^\mu = (P_1 - P_2)^\mu$ is the four-momentum transfer to the leptons. In the LCQM calculations presented in Ref. 6, $q^+\neq 0$ frame has been used to calculate the weak decays in the time-like region $m_l^2 \leq q^2 \leq (M_1 - M_2)^2$, with $M_{1[2]}$ and $m_l$ being the initial [final] meson mass and the lepton ($l$) mass, respectively. However, when the $q^+\neq 0$ frame is used, the inclusion of the non-valence contributions arising from quark-antiquark pair creation ("Z-graph") is inevitable and this inclusion may be very important for heavy-to-light and light-to-light decays. Nevertheless, the previous analyses [6] in the $q^+\neq 0$ frame considered only valence contributions neglecting non-valence contributions.

*TABLE 3. Branching ratio $Br(A \to X l \nu_l)$ for various semileptonic decays.*

| Process | | $Br^{\rm Th.}$ | $Br^{\rm exp.}$ |
|---|---|---|---|
| $K \to \pi$ | H.O. | $(37.60 \pm 0.60)\%$ | $(38.78 \pm 0.27)\%$ |
| | Linear | $(37.90 \pm 0.61)\%$ | |
| $B \to \pi$ | H.O. | $(1.20 \pm 0.14^{+0.24+0.63}_{-0.25-0.36}) \times 10^{-4}$ | $(1.8 \pm 0.4 \pm 0.3 \pm 0.2)$ $\times 10^{-4}$ |
| | Linear | $(1.33 \pm 0.16^{+0.27+0.71}_{-0.28-0.40}) \times 10^{-4}$ | |
| $D \to K$ | H.O. | $(3.43 \pm 1.33)\%$ | $(3.64 \pm 0.20)\%$ |
| | Linear | $(3.46 \pm 1.34)\%$ | |
| $D \to \pi$ | H.O. | $(2.24 \pm 0.33) \times 10^{-3}$ | $(3.8^{+1.2}_{-1.0}) \times 10^{-3}$ |
| | Linear | $(2.28 \pm 0.34) \times 10^{-3}$ | |
| $B \to D$ | H.O. | $(2.36 \pm 0.36)\%$ | $(2.35 \pm 0.2 \pm 0.44)\%$ |
| | Linear | $(2.47 \pm 0.37)\%$ | |

In this work, we circumvent this problem by calculating the processes in the $q^+ = 0$ frame and analytically continuing to the time-like region. The $q^+ = 0$

frame is useful because only valence contributions are needed. However, one needs to calculate the component of the current other than $J^+$ to obtain the form factor $f_-(q^2)$. Since $J^-$ is not free from the zero-mode contributions even in the $q^+ = 0$ frame [7], we use $J_\perp$ instead of $J^-$ to obtain $f_-$. In the standard $q^+ = 0$ frame, we obain the form factors $f_+(q^2)$ and $f_-(q^2)$ using the matrix element of the "+" and "$\perp$"-components of the current, $J^\mu$, respectively, and then continued to the time-like $q^2 > 0$ region by changing $q_\perp$ to $\mathrm{i}q_\perp$ in the form factors. We note that our analytic continuation method is equivalent to that of Ref. 8 where the form factors are obtained by the dispersion representations through the (Gaussian) wave functions of the initial and final mesons. The rates for the semileptonic decays $K \to \pi$, $B \to \pi(D)$, and $D \to \pi(K)$ are presented in Table 3 and compared with the available experimental data [9]. Our numerical results are in a good agreement with the available experimental data.

## 5. *Conclusions and discussions*

The space-like form factors [10] are well described by our light-cone quark model. Analyzing the semileptonic decays of a pseudoscalar meson into another pseudoscalar meson using the same model, we demonstrated that our model can also be applied to the time-like exclusive processes. The form factors $f_\pm$ are obtained in the $q^+ = 0$ frame and then analytically continued to the time-like region by changing $q_\perp$ to $\mathrm{i}q_\perp$ in the form factors. The matrix element of the "$\perp$" component of the current $J^\mu$ is used to obtain the form factor $f_-$. Our numerical results are in a good agreement with the available experimental data. We also confirmed that our analytic continuation method is equivalent to that of Ref. 8. For the estimation of the zero-mode contribution, we calculated the "$-$" component of the current. To the extent that the zero-modes have a significant contribution to some physical observables [3], it seems conceivable that the condensation of zero-modes could lead to the nontrivial realization of chiral symmetry breaking in the light-cone quantization approach. However, the zero-mode contribution seems highly suppressed as the quark mass increases [3,7]. The detailed analysis of heavy-to-heavy and heavy-to-light semileptonic decays will be presented elsewhere. It seems to us that the success of our model calculations reveals the effectiveness of light-cone degrees of freedom in exclusive processes.

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## MEZONSKI ELEKTROSLABI FAKTORI OBLIKA U PRISTUPU SVJETLOSNOG STOŠCA

Ispituju se mezonski elektroslabi faktori oblika i poluleptonski raspadi primjenom kvarkovskog modela zasnovanog na stupnjevima slobode svjetlosnog stošca. Rezultati pokazuju širu primjenljivost svjetlosno-konusnog pristupa, uključivši i vremensko područje ekskluzivnih procesa.