ISSN 1330–0016
CODEN FIZBE7

ELECTROMAGNETIC FORM FACTORS OF MESON TRANSITIONS[1]

PETER C. TANDY

Center for Nuclear Research, Department of Physics Kent State University, Kent, OH 44242, USA

Received 23 February 1999; Accepted 12 July 1999

Within the Dyson-Schwinger-equation approach to modeling QCD for meson physics, we present new results for $g_{\rho\pi\pi}$ and the coupling constants and form factors for the transitions $\gamma^*\pi\rho$ and $\gamma^*\pi^0\gamma$. We discuss the role of the sub-dominant covariants of the π Bethe-Salpeter amplitude and investigate the asymptotic behavior of the $\gamma^*\pi^0\gamma$ form factor.

PACS numbers: 12.38.Lg, 13.40.Gp, 14.40.Aq, 24.85.+p UDC 535.217, 539.21

Keywords: nonperturbative modeling of QCD, Dyson-Schwinger equations, confinement, meson coupling, rho and pion electromagnetic transitions, form factors

1. QCD modeling of mesons

The Dyson-Schwinger-equation (DSE) approach [1] to non-perturbative QCD modeling of hadrons and their interactions [2] combines truncated QCD equations of motion for propagators and vertices with infrared phenomenology fitted to a few key low-energy observables. Parameter-free predictions for other hadron properties and observables are then used to develop and test our understanding of hadron physics at the quark-gluon level. The DSE for the fully-dressed and renormalized quark propagator in Euclidean metric is

$$S^{-1}(p) = Z_2[\mathrm{i}\gamma\cdot p + m_0(\Lambda)] + Z_1\,\frac{4}{3}\int^{\Lambda}\frac{\mathrm{d}^4k}{(2\pi)^4}g^2D_{\mu\nu}(p-k)\gamma_\mu S(k)\Gamma^g_\nu(k,p), \quad (1)$$

where $m_0(\Lambda)$ is the bare-mass parameter and Λ characterizes the regularization mass scale. In the DSE approach, the renormalized dressed gluon propagator $D_{\mu\nu}(q)$ and dressed quark-gluon vertex $\Gamma^g_\mu(k,p)$ are constrained in the UV by perturbative results and are representated by phenomenological IR forms with parameters fitted to selected pion and kaon observables.

Mesons are generated as bound states of a quark of flavor f_1 and an antiquark of flavor $\bar{f}_2$ via the Bethe-Salpeter (BS) equation

[1]Presented at the Int. Conf. on Nuclear and Particle Physics with CEBAF at Jefferson Lab, November 1998, Dubrovnik, Croatia.

$$\Gamma(p\,;P) = \int \frac{\mathrm{d}^4 q}{(2\pi)^4} K(p,q;P) S_{f_1}(q+\xi P)\Gamma(q;P)S_{f_2}(q-\bar{\xi}P) \;, \tag{2}$$

where $\xi + \bar{\xi} = 1$ describes momentum sharing. The kernel K is the renormalized, amputated $\bar{q}q$ scattering kernel that is irreducible with respect to a pair of $\bar{q}q$ lines. The present stage of QCD modeling truncates $K(p,q;P)$ at the ladder approximation and couples quark color currents with bare vertices and an effective gluon 2-point function in Landau gauge. The latter involves $\alpha_{\rm eff}(q^2)$ for interpolation between the 1-loop pQCD result in the UV and a phenomenological enhancement in the IR. The treatment of the quark DSE that is dynamically matched to this is the bare-vertex or rainbow approximation; the axial vector Ward-Takahashi identity is then preserved, Goldstone's theorem is manifest, and realistic pion and kaon solutions are ensured [3].

The general form of the pion BS amplitude is

$$\hat{\Gamma}^j_\pi(k;P) = \tau^j\gamma_5[\mathrm{i}E_\pi(k;P) + \not{P}F_\pi(k;P) + \not{k}\,k\cdot P\,G_\pi(k;P) + \sigma_{\mu\nu}\,k_\mu P_\nu\,H_\pi(k;P)]\,, \tag{3}$$

and the first three terms are significant in realistic solutions [3]. With $S^{-1}(p) = \mathrm{i}\not{k}A(k^2) + B(k^2) + m$, the axial Ward-Takahashi identity in the chiral limit specifies the dominant amplitude as $E_\pi(k;P=0) = B_0(k^2)/f_\pi$; the amplitudes F_π and G_π are related to $A(k^2)$ and its derivative in a less direct manner [4].

In Euclidean metric, the BS meson mass-shell condition requires that $S(p)$ be evaluated in a certain domain of complex p^2. The required domain for meson decays and form factors can be quite demanding. To facilitate such studies, we make use of an analytic parametrization of the numerical solutions of the quark DSE to represent $S(p)$ as an entire function in the complex p^2-plane describing absolutely confined [5] dressed quarks. Typically five parameters are used to achieve a good description of pion and kaon observables: $f_{\pi/K}$; $m_{\pi/K}$; $\langle\bar{q}q\rangle$; the π-π scattering lengths; the charge radii r_π, r_{K^0} and $r_{K^\pm}$; and the pion charge form factor [6,7]. Current efforts along this line are concentrated upon vector and axial vector mesons and hadronic and semi-leptonic decays. Here we outline recent studies of several electromagnetic transitions and also the $\rho\pi\pi$ coupling.

2. The $\rho\pi\pi$ coupling constant

The first term in a skeleton-graph expansion of the $\rho\pi\pi$ vertex [2,8] is

$$\begin{aligned}\Lambda_\nu(P,Q) &= -P_\nu g_{\rho\pi\pi}\,F(Q^2) \\ &= 2N_c\,\mathrm{Tr}_s \int \frac{\mathrm{d}^4 k}{(2\pi)^4}\Gamma_\pi(k''';-P_+)S(q')\Gamma^\rho_\nu(k';Q)S(q'')\Gamma_\pi(k'';P_-)S(q''')\;,\end{aligned} \tag{4}$$

where $P_\pm = P \pm Q/2$ and both pions are on the mass-shell. The first argument of each BS amplitude is a relative $\bar{q}q$ momentum (we choose equal partitioning), and the second argument is the incoming meson momentum. After the loop momentum

k is specified in terms of one internal momentum, the others are easily deduced. By definition, we have $F(Q^2 = -m_\rho^2) = 1$.

For this $\rho\pi\pi$ study, we employ approximate Γ_π and Γ_ν^ρ obtained from a rank-2 separable ansatz [9] for the ladder/rainbow kernel of the DSE and BSE. The π is properly massless in the chiral limit as required by Goldstone's theorem; but the full consequences of the axial Ward-Takahashi identity [4] are not preserved. Parameters are fit to $m_{\pi/K}$ and $f_{\pi/K}$. The results have the form [9]

$$\Gamma_\pi(k;P) = \mathrm{i}\gamma_5\, f(k^2)\,\lambda_1^\pi - \gamma_5 \not{P}\, f(k^2)\,\lambda_2^\pi \;, \tag{5}$$

$$\Gamma_\nu^\rho(k;P) = k_\nu^T\, g(k^2)\lambda_1^\rho + \mathrm{i}\gamma_\nu^T\, f(k^2)\lambda_2^\rho + \mathrm{i}\gamma_5\epsilon_{\mu\nu\lambda\rho}\,\gamma_\mu k_\lambda P_\rho\, g(k^2)\lambda_3^\rho \;. \tag{6}$$

Here $g(k^2) = [A(k^2) - 1]/a$ and $f(k^2) = B(k^2)/b$ with a and b given in Ref. 9 from the quark DSE solution. The relative strength of the λ_i is given by the separable BSE solution which requires a phenomenological UV suppression. In contrast to previous work [9], here we normalize the λ_i in the canonical way [4] without such a suppression in any of the momentum dependent quantities.

The results for $g_{\rho\pi\pi}$ are given in Table 1, where an error in our previous work [10] is corrected. The empirical value associated with the $\rho \to \pi\pi$ decay width is overestimated by 21%. The integral for $g_{\rho\pi\pi}$ is significantly influenced by the normalization of Γ_π; the form of the separable amplitude Eq. (5) may be too primitive to generate a realistic norm. Improved BS amplitudes [3,11] are available for future work. The pseudovector π component is much more important here (-84%) than it is for m_π and f_π (25-35%). The sub-dominant ρ amplitudes make only a minor correction.

TABLE 1. $g_{\rho\pi\pi}$ calculation and contributions from meson covariants.

$g_{\rho\pi\pi} = 7.32$ [expt 6.05]			
π Covariants		ρ Covariants	
γ_5	184%	γ_μ	98.5%
$\gamma_5\gamma\cdot Q$	-84%	$\gamma_5\epsilon_\mu\,\gamma k Q$	1.4%
		k_μ	0.1%

3. The $\gamma^*\pi\rho$ form factor

The isoscalar $\gamma^*\pi\rho$ meson-exchange current contributes significantly to electron scattering from light nuclei. Our understanding of the deuteron EM structure functions for $Q^2 \approx 2-6$ GeV^2 requires knowledge of this form factor [12]. The general form of the vertex, and the explicit quark loop that arises in the impulse approximation generalized to include the dressing of the propagators and vertices, is

$$\Lambda_{\mu\nu}(P,Q) = -\mathrm{i}\frac{e}{m_\rho}\,\epsilon_{\mu\nu\alpha\beta}\,P_\alpha Q_\beta\, g_{\gamma\pi\rho}\, F(Q^2) \tag{7}$$

$$= \frac{2N_c}{3}\,\mathrm{Tr}_s \int \frac{\mathrm{d}^4k}{(2\pi)^4}\,\bar{\Gamma}_\pi(k''';-P_+)\,S(q')\,\Gamma_\nu(k';Q)\,S(q'')\,\Gamma_\mu^\rho(k'';P_-)\,S(q''') \;.$$

The momentum notation is the same as for the previous $\rho\pi\pi$ case. The dressed photon-quark vertex is taken to be the Ball-Chiu [13] ansatz

$$\Gamma_\nu(k;Q) = -\frac{\mathrm{i}\gamma_\nu}{2}(A_+ + A_-) + \frac{k_\nu}{k\cdot Q}\Big[\mathrm{i}\not{k}(A_- - A_+) + B_- - B_+\Big] , \tag{8}$$

where $f_\pm = f(k_\pm)$ and $k_\pm = k \pm \frac{Q}{2}$. This form obeys the Ward-Takahashi identity (WTI) and the relevant symmetries and is conveniently determined completely in terms of the quark propagator. It then follows that $Q_\nu\, \Lambda_{\mu\nu} = 0$; the $\gamma\pi\rho$ current is conserved. With the above separable model π and ρ BS amplitudes, along with the associated quark-propagator parameterization [7], we obtain $g_{\gamma\pi\rho} = 0.45$ in reasonable agreement with the empirical value $g^{\rm expt}_{\gamma\pi\rho} = 0.54 \pm 0.03$ from ρ decay.

The pseudovector π amplitudes F_π and G_π defined in Eq. (3) generate the correct asymptotic behavior of the pion charge form factor [14]. Also, their UV relationship $k^2 G_\pi(k^2) \to 2F_\pi(k^2)$ implements convergence for the f_π integral [14]. It is impossible to generate these properties in the previous separable-ansatz approach; we use here the approximating forms [14] of a numerical solution [3]

$$E_\pi(k^2) \approx \frac{B_0(k^2)}{N_\pi}\ ,\ \ F_\pi(k^2) \approx \frac{E_\pi(k^2)}{110 f_\pi}\ ,\ \ G_\pi(k^2) \approx \frac{2F_\pi(k^2)}{k^2 + M^2_{UV}}\ ,\ \ H_\pi(k^2) \approx 0, \tag{9}$$

where N_π is fixed by the standard BS normalization condition [4]. We use the quark-propagator parameterization (set A) [14] associated with Eq. (9). There is no dynamically matched Γ^ρ_μ available, so we simply take Eq. (6). We then obtain $g_{\gamma\pi\rho} = 0.708$. Since the canonical ρ covariant is so dominant, we expect that the

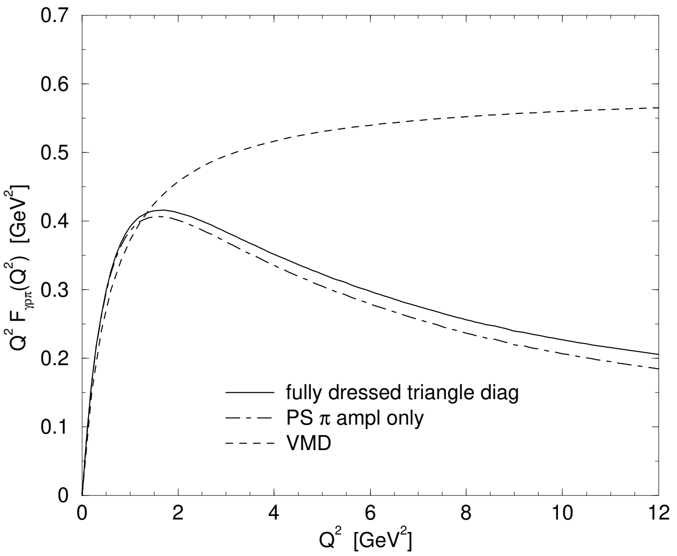

Fig. 1. The $\gamma^*\pi\rho$ transition form factor with π and ρ on shell.

mismatch is simply one of normalization and that the produced $F(Q^2)$ should be quite realistic. The result in Fig. 1 is much softer than the vector meson dominance (VMD) prediction. The available data for elastic EM deuteron form factors $A(Q^2)$ and $B(Q^2)$ in the range $2-6$ GeV^2 have been shown [12] to strongly favor our previous $\gamma^*\pi\rho$ vertex result [8]. The present work is less phenomenological, employs more realistic representations of the π and ρ, and produces a harder form factor. The influence on the deuteron form factors remains to be determined.

4. *The $\gamma^*\pi^0 \to \gamma$ transition*

The coupling constant for the $\pi^0 \to \gamma\gamma$ decay is given by the axial anomaly and its value is a consequence of only gauge invariance and chiral symmetry in quantum field theory. The form factor of this anomalous transition is not dictated by symmetries; it is of interest as a test of our ability to model nonperturbative QCD because of the relatively simple hadronic dynamics that is involved. In the asymptotic UV region, one expects a simple result dictated by the known electromagnetic coupling to current quarks and the intrinsic properties of the pion.

The general form of the vertex allowed by CPT symmetry, and the explicit quark loop that arises in the impulse approximation generalized to include the dressing of the propagators and vertices, are

$$\begin{aligned}\Lambda_{\mu\nu}(P,Q) &= \mathrm{i}\frac{\alpha}{\pi f_\pi}\,\epsilon_{\mu\nu\alpha\beta}\,P_\alpha Q_\beta\, g_{\pi\gamma\gamma}\, F(Q^2) \qquad (10)\\ &= \frac{N_c}{3}\,\mathrm{Tr}_s \int \frac{\mathrm{d}^4 k}{(2\pi)^4}\, S(q')\Gamma_\nu(k';Q)\, S(q'')\Gamma_\mu(k'';-P-Q)\, S(q''')\Gamma_\pi(k''';P).\end{aligned}$$

The γ^* momentum is Q, and the other photon and the pion are on the mass-shell. With the convenient choice $k'''=k$, the other internal momenta can be determined. The Ball-Chiu ansatz Eq. (8) is used for the dressed-quark-photon vertices Γ_ν and Γ_μ. The chiral limit anomalous decay $\pi^0 \to \gamma\gamma$ gives $g^0_{\pi\gamma\gamma} = 1/2$ providing an excellent account of the 7.7 eV width. The form factor defined by Eq. (10) satisfies $F(0) = 1$; it should not be confused with a different quantity $\tilde{F}(Q^2) = F(Q^2)/4\pi^2 f_\pi$ which contains the non-tensor strength of the transition matrix element $M_{\mu\nu} = 2\Lambda_{\mu\nu}$ and in terms of which the CLEO [16] data and some theoretical works are expressed.

Here we update an earlier study [15] by using the more realistic Γ_π in Eq. (9) along with the associated quark propagator (set A) [14]. We obtain $g_{\pi\gamma\gamma} = 0.4996$ at the physical m_π value. The form factor is displayed in Fig. 2. Although the anomalous coupling strength is correct, the calculation does not fall off fast enough in the infrared and significantly overestimates the data. It is unlikely that our description of the pion is responsible for this; the sub-dominant amplitudes F_π, G_π give a negligible contribution here while the closely related dynamical quantities, r_π and f_π, are well described [14]. The removal of dressing at the virtual photon vertex produces the dashed line. The better agreement with the data is rather fortuitous since the coupling constant is reduced to 70% of the former correct value. The pion transition radius from the impulse approximation (≈ 0.48 fm) is clearly less than that suggested by the data (≈ 0.65 fm); a similar underestimate also occurs for the pion charge radius in this approach.

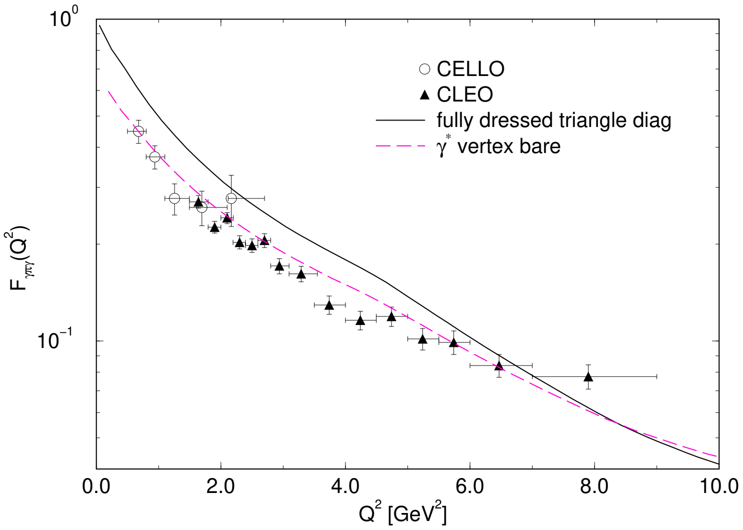

Fig. 2. The $\gamma^*\pi^0\gamma$ transition form factor. The data are taken from Ref. 17 (CELLO) and Ref. 16 (CLEO).

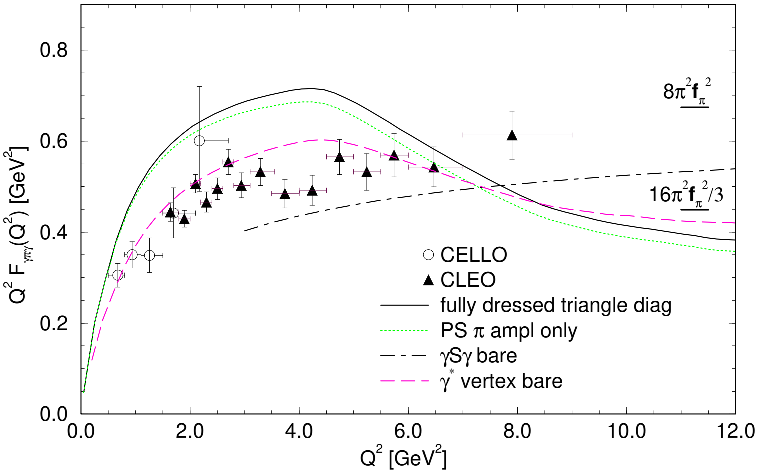

Fig. 3. The $\gamma^*\pi^0\gamma$ transition form factor times Q^2. The asymptotic limits marked are described in the text.

Our results for $Q^2F(Q^2)$ are displayed in Fig. 3. The approach to the asymptotic limit of the present dressed quark loop Eq. (10) is quite slow and is governed by the following considerations. After the mass-shell conditions for the pion and the real photon are realized, one finds that in the domain $k^2 < 1$ GeV$^2 \ll Q^2$ of integral support dictated by $\Gamma_\pi(k^2)$, the leading behavior of the momenta for each quark propagator is: $(q')^2$, $(q''')^2 \sim \mathcal{O}(kQ)$, and $(q'')^2 = \frac{Q^2}{2}[1 + \mathcal{O}(k/Q)]$. Using $A(p^2) = 1 + \mathcal{O}(1/p^2)$ and $B(p^2) \sim \mathcal{O}(1/p^2)$, one finds from Eq. (8) that the leading behavior of each of the photon vertices is $\Gamma_\nu = -\mathrm{i}\gamma_\nu[1 + \mathcal{O}(1/kQ)]$. The photon insertions of the diagram effectively collapse to a point axial vector with the $\Gamma_\nu S(q'')\Gamma_\mu$ leg of the loop having the leading form

$$\frac{2\mathrm{i}}{\mathrm{Q}^2}\,\gamma_\nu \not{q}''\gamma_\mu = \frac{2\mathrm{i}}{\mathrm{Q}^2}\,\epsilon_{\mu\nu\alpha\beta}\,\gamma_5\gamma_\alpha Q_\beta + \cdots\ , \tag{11}$$

where the current mass is ignored and the terms not shown do not survive the spin trace. The loop integral coupling the π to $\gamma_5\gamma_\alpha$ gives exactly $f_\pi P_\alpha/3$ by definition [4]. This gives the asymptotic limit $\Lambda_{\mu\nu}(Q^2) \to \frac{2f_\pi}{Q^2}\,\mathrm{i}\epsilon_{\mu\nu\alpha\beta}\,P_\alpha Q_\beta[1 + \mathcal{O}(1/Q)]$ or the form factor limit $\frac{16}{3}\pi^2 f_\pi^2 + \mathcal{O}(1/Q)$, in agreement with Refs. 19 and 20. The limit marked by $8\pi^2 f_\pi^2$ is from pQCD factorization [18]. It is evident that the sub-dominant amplitudes F_π and G_π make a minor contribution, although the integrated effect via the produced f_π value is some 30%.

With the hard leg $\Gamma_\nu S(q'')\Gamma_\mu$ taken to be bare, the result is the dot-dashed line in Fig. 3. This is consistent with the evident sub-leading correction in the propagator denominator being $\langle k/Q\rangle \approx 30\%$ at $Q^2 \approx 10$ GeV^2. The dressing of the hard propagator $S(q'')$ contributes little to the dot-dashed curve; thus the difference between the three relevant curves illustrates the persistent contribution from photon vertex dressing. Since the Ball-Chiu vertex is exact at both $Q^2 = 0$ and the UV limit, and only the longitudinal component is correct for all Q^2, it is possibly the deficiencies of this ansatz at infrared and intermediate momenta that are being exposed in the present study. This is also the preliminary finding from a study of the ladder Bethe-Salpeter solution for the vector vertex [21].

5. *Summary*

The results presented here suggest that the present approach to modeling low-energy QCD can capture the mechanisms that dominate infrared physics. The sub-dominant pseudovector terms in the pion BS amplitude make a much stronger contribution to $g_{\rho\pi\pi}$ than they do to m_π, f_π; the contributions to the $\gamma^*\pi^0\gamma$ and $\gamma^*\pi\rho$ transition form factors are minor at low and intermediate momenta. The new result here for the $\gamma^*\pi\rho$ form factor should be applied to the deuteron electromagnetic form factors where this meson exchange current is still a serious ambiguity. Our examination of the pion axial anomaly form factor with the Ball-Chiu ansatz for the dressed photon-quark vertex clarifies the slow approach to asymptotic behavior and suggests that deficiencies in this ansatz at low momenta may be evident. A study of electromagnetic radii should be made from this perspective.

Acknowledgements

Thanks are due to the conference organizers, especially Dubravko Klabučar, for the excellent hospitality and enjoyable atmosphere provided at this conference.

Helpful discussions with D. Klabučar, C. D. Roberts, P. Maris and M. Pichowsky are gratefuly acknowledged. This work is partly supported by the US National Science Foundation under grants No. PHY97-22429 and INT96-03385.

References

1) C. D. Roberts and A. G. Williams, Prog. Part. Nucl. Phys. **33** (1994) 477;

2) P. C. Tandy, Prog. Part. Nucl. Phys. **39** (1997) 117;

3) P. Maris and C. D. Roberts, Phys. Rev. C **56** (1997) 3369;

4) P. Maris, C. D. Roberts and P. C. Tandy, Phys. Lett. B **420** (1998) 267;

5) C. D. Roberts, A. G. Williams and G. Krein, Int. J. Mod. Phys. A **7** (1992) 5607;

6) C. D. Roberts, Nucl. Phys. A **605** (1996) 475;

7) C. J. Burden, C. D. Roberts and M. J. Thomson, Phys. Lett. B **371** (1996) 163;

8) P. C. Tandy, Prog. Part. Nucl. Phys. **36** (1996) 97; K. L. Mitchell, PhD Thesis, Kent State University, unpublished, (1995);

9) C. J. Burden, Lu Qian, C. D. Roberts, P. C. Tandy and M. J. Thomson, Phys. Rev. C **55** (1997) 2649;

10) P. C. Tandy, *Modeling nonperturbative QCD for mesons and couplings, Proceedings, IV^{th} Workshop on Quantum Chromodynamics*, June, 1998, nucl-th/9812005, to be published;

11) P. Maris and P. C. Tandy, *Vector Meson Masses and Decay Constants*, preprint KSUCNR-104-99 (1999); nucl-th9905056; to be published in Phys. Rev. C;

12) J. W. Van Orden, N. Devine and F. Gross, Phys. Rev. Lett. **75** (1995) 4369;

13) J. S. Ball and T. W. Chiu, Phys. Rev. D **22** (1980) 2542;

14) P. Maris and C. D. Roberts, Phys. Rev. C **58** (1998) 3659;

15) M. R. Frank, K. L. Mitchell, C. D. Roberts and P. C. Tandy, Phys. Lett. B **359** (1995) 17;

16) J. Gronberg et al., (CLEO Collaboration), Phys. Rev. D **57** (1998) 33;

17) H.-J. Behrend et al., (CELLO Collaboration), Z. Phys. C **49** (1991) 401;

18) G. P. Lepage and S. J. Brodsky, Phys. Rev. D **22** (1980) 2157;

19) D. Kekez and D. Klabučar, $\gamma^* \gamma \to \pi^0$ *transition and asymptotics of* $\gamma^*\gamma$ *and* $\gamma^*\gamma^*$ *transitions of other unflavored pseudoscalar mesons*, hep-ph/9812495; Fizika B (Zagreb) **8** (1999) 303;

20) C. D. Roberts, *Dyson Schwinger equations: connecting small and large length-scales*, hep-ph/9901091; Fizika B (Zagreb) **8** (1999) 285;

21) P. Maris, private communication, (1999).

ELEKTROMAGNETSKI FAKTORI OBLIKA MEZONSKIH PRIJELAZA

Polazeći od Dyson-Schwingerove jednadžbe za modeliranje QCD mezonske fizike, predstavljaju se novi ishodi za $g_{\rho\pi\pi}$, stalnice vezanja i faktore oblika prijelaza $\gamma^*\pi\rho$ i $\gamma^*\pi^0\gamma$. Raspravlja se uloga pod-dominantne kovarijante π Bethe-Salpeterove amplitude i istražuje asimptotsko ponašanje faktora oblika $\gamma^*\pi^0\gamma$.