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LATEST RESULTS FROM LATTICE NRQCD

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We review the current status of lattice calculations of properties of hadrons containing b quarks with the focus on the NRQCD method. In particular, we discuss the latest results for the Υ spectrum. These indicate the lower-lying spin-averaged spectrum is reproduced well to within the statistical accuracy of a few percent by an $O(Mv^4)$ NRQCD action, even without the presence of sea quarks. Extensive calculations have also been performed of the spectrum and (in some cases) decay constants of mesons and baryons containing a single heavy quark: the B and B_s mesons, including radial and orbital excitations and the Λ_b, Σ_b and Ω_b baryons. The properties of these particles are less well determined experimentally, and we show reliable predictions are now being provided using lattice calculations.

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1. *Introduction*

Hadrons containing b quarks, Υ, B, Λ_b ..., have been studied intensively, both theoretically and in experiment during the last 20 years. The non-relativistic nature of b quarks in these states lends these particles to theoretical methods such as non-relativistic quantum chromodynamics (NRQCD) [1]. The wealth of experimental results for the spectrum of heavy quarkonia make it the ideal place to test these approaches before applying them to heavy-light particles. In contrast, properties of heavy-light hadrons, those containing a single b quark and one or two light quarks, are less well determined experimentally. However, an order of magnitude of improvement in accuracy is expected with results from several B-factories in the next few years. The central question is whether CP violation in the weak decays of these particles can be fully described by a single phase in the CKM matrix. The challenge for the theoretical community is to calculate the low-energy QCD factors, needed to extract the CKM matrix elements from these experiments, to a similar accuracy.

Both for testing QCD in the low-energy regime and for computing form factors and decay constants of weak decays, lattice calculations are essential. Lattice gauge theory is the only nonperturbative, first-principles approach to QCD available. Similar to experiment, the systematic errors which arise can be explored and, at least in principle, reduced to a desired accuracy. In addition, since the parameters of the theory can be freely varied, the validity of different models and approximations can easily be investigated on the lattice. The computing power available is now measured in tens of Gflops, and 'state of the art' calculations are reaching the stage where the leading systematic errors are under control to around a few percent. In this paper, we review the progress made in this direction in heavy-quark physics with the focus on the NRQCD method.

2. *Motivation for NRQCD*

Naively, simulating b quarks on the lattice presents a problem: at present, typical lattice spacings lie in the range, $a^{-1} = 1{-}3$ GeV $\ll M_b$. This gives rise to very large discretisation errors in a simulation when using the standard (relativistic) lattice quark action which has leading errors of $O(aM)$. However, since, for example, the $b\bar{b}$ system is clearly nonrelativistic, $v_b^2 \approx 0.1$ [2], one can explicitly remove the mass scale M_b, which is irrelevant for the quark dynamics, by setting the cut-off a such that $aM_b > 1$. Thus, heavy quark simulations are immediately within the reach of present calculations. The usual lattice analogue of the Dirac action is replaced by a nonrelativistic Schrödinger action, which is an expansion in v^2. On the lattice, Lorentz invariance is broken. However, in principle, it can be restored order by order to the desired accuracy.

In order to formulate a quark action in this nonrelativistic theory (NRQCD), all terms of a given dimension which satisfy the remaining symmetries - rotational symmetry, gauge invariance etc, must be considered. Truncating the action at $O(v^4)$, we find, up to radiative corrections,

$$S_{NRQCD} = \psi^\dagger (D_t + H_0 + \delta H)\psi, \tag{1}$$

where

$$H_0 = -\frac{\Delta^{(2)}}{2M_0}, \tag{2}$$

$$\delta H_{h.o.} = -c_B \frac{\sigma \cdot B}{2M_0} \tag{3}$$

$$-c_3 \frac{g}{8(M_0)^2}\sigma \cdot (\nabla \times E - E \times \nabla) + c_2 \frac{\mathrm{i}g}{8(M_0)^2}(\nabla \cdot E - E \cdot \nabla) \tag{4}$$

$$-c_1 \frac{(\Delta^{(2)})^2}{8(M_0)^3} + c_4 \frac{a^2 \Delta^{(4)}}{24 M_0} - c_5 \frac{a(\Delta^{(2)})^2}{16n(M_0)^2}. \tag{5}$$

The form of the action is familiar from atomic physics: (2) represents the lattice version of the kinetic energy (order v^2), while (3) and (4) represent the hyperfine interaction and the spin-orbit and Darwin interaction, respectively (all order v^4). Also at $O(v^4)$ is the correction to the kinetic energy and two lattice discretisation terms (5). Note that the power counting of terms is different in heavy-light hadrons. This is discussed in Section 5.

With each new term in the action, a coefficient, $c_i(M_0 a, g^2)$, is introduced, which must be tuned to compensate for the effect of omitting relativistic momenta on low-energy physics. However, in principle the coefficients can be computed in perturbation theory, as long as $a^{-1} \gg \Lambda_{QCD}$, and the predictability of the theory is retained. In practice, most of the coefficients are estimated using tadpole improvement [3], in which it is assumed that a large part of the renormalisation comes from tadpole diagrams, and where this contribution is estimated using mean-field theory. So far only c_1 and c_4 have been calculated to 1-loop in lattice regularisation [4]; the 1-loop contribution is found to be small, about 10%, roughly independent of aM_0. Attempts to compute c_B are in progress [5].

Since NRQCD is an effective, non-renormalisable theory, which breaks down for $aM_b < 1$, extrapolating results to $a \to 0$ is not possible, and the systematic errors inherent in the calculation must be reduced to the level of statistical accuracy at finite a. The main sources of error are (1) finite lattice spacing (from the gauge action and the light quark action when simulating heavy-light hadrons), (2) the truncation of the NRQCD series, (3) the coefficients, c_i, and the matching factors needed when calculating matrix elements, (4) finite volume and (5) the quenched approximation, i.e. the omission of sea quarks, which is often introduced to reduce the amount of computer time required. In the following we will discuss how well these errors are controlled.

3. *Results for bottomonium*

With the $O(v^4)$ action, one (naively) expects to be able to calculate the spin-averaged Υ spectrum to roughly 1%, and the fine structure to 10%. In Fig. 1, the results for the $b\bar{b}$ spectrum from the NRQCD collaboration [6] and the Cambridge group [7], in the quenched approximation[1], are shown in comparison with experiment; the lower-lying states have been fully calculated, including the first and second radial excitations of the S-states.

Note that the lattice spacing and the bare b quark mass, M_b^0, must be fixed post-simulation using experiment. Normally, a is fixed from the spin-averaged P$-{}^3$S$_1$ splitting, since this is expected to only weakly depend on the quark mass [8]. M_b^0 is then found by tuning the ^{3}S$_1$ state mass until it agrees with experiment. Of the remaining 5 predictions for spin-averaged splittings, experimental results exist for 2S$-$1S, 3S$-$1S and 2P$-$1S. We see that the S-state splittings are reproduced well, within errors. The 2P$-$1S splitting is rather high and this may be due to finite

[1] All presented results are obtained with this approximation unless stated otherwise.

volume effects (the lattice size is roughly 1.6 fm), although the statistical error may have been underestimated since radial excitations are much more difficult to extract reliably.

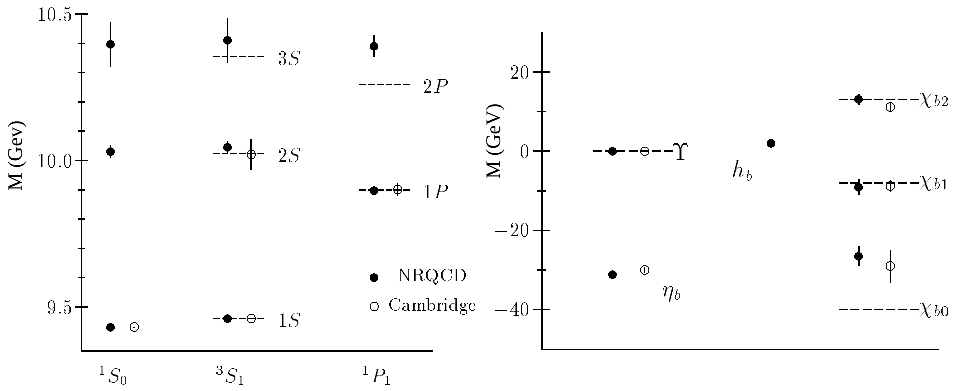

Fig. 1. Results for the spin-averaged and fine structure splittings of Υ.

Another possibility is the quenched approximation. At a certain accuracy, the quenched theory will not reproduce the real world (indeed this is physically interesting to determine), and inconsistencies with experiment are expected. Since the quenched strong coupling runs faster than that in the real world, using a lattice spacing fixed by a quantity characterised by a particular physical scale will only lead to predictions in agreement with experiment for quantities characterised by a similar scale. However, it is difficult to identify which physical scale dominates for any given quantity and thus predict which mass ratios are expected to agree with experiment for a given lattice spacing. In general, all other systematic errors must be controlled before quenching errors can be uncovered.

Significant discrepancies with experiment are also found in the results for the P-state fine structure. Figure 1 shows that the results do not agree with experiment within the 10% error expected using the $O(v^4)$ action. The possible sources of error are discussed in the next section.

3.1. Scaling

As mentioned previously, we must demonstrate that the predictions are independent of the lattice details, in particular the lattice spacing, within the range in a for which NRQCD is valid. Figure 2 presents the results from the NRQCD collaboration [6] for the ratio of spin-averaged splittings, $R_{SP} = (2\mathrm{S}-1\mathrm{S})/(1\mathrm{P}-1\mathrm{S})$ as a function of a. The ratio is clearly consistent with scaling (and experiment - indicated by the dashed line), as a varies by a factor of roughly 2. The results for the ratios of the higher excited states, (3S−1S)/(1P−1S) and (2P−1S)/(1P−1S) (not shown), also suggest scaling. However, more work is needed as the statistical errors are much larger at the bigger values of a.

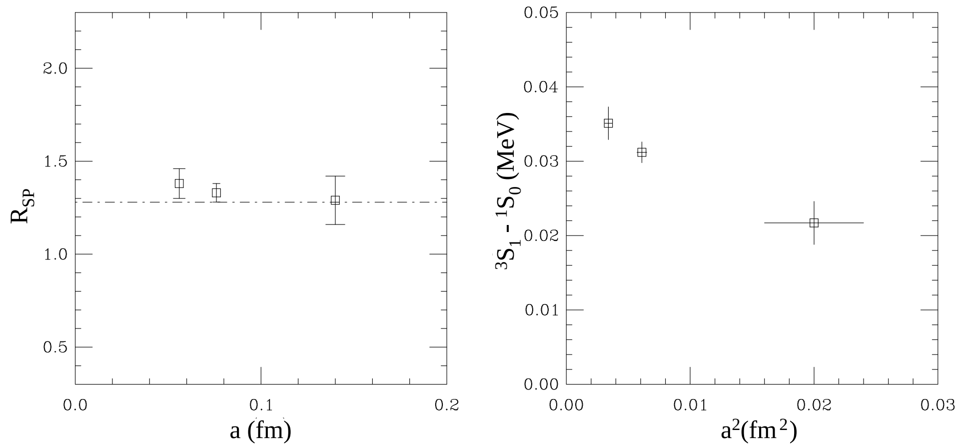

Fig. 2. The scaling behaviour of the spin-averaged ratio, R_{SP}, and the hyperfine splitting, ${}^3\mathrm{S}_1-{}^1\mathrm{S}_0$.

Further effort is also required for the fine structure splittings, for which scaling violations have been seen. The hyperfine splitting, a very short distance quantity, shown in Fig. 2, is the worst case. Several improvements can be made: the next order, $(O(v^6))$, spin-dependent term can be included, the coefficients, c_i, should be more accurately calculated, and the quark mass should be more carefully tuned to the b quark mass. All three effects are important and the initial indications are that scaling is improved once they are implemented [6].

3.2. n_f dependence

Since the systematic errors (1)−(4) appear to be under control for the ratio R_{SP}, it is reasonable to begin to investigate the errors due to quenching. Figure 3 shows the ratio calculated by the NRQCD collaboration [9] and the SESAM collaboration [10] in the quenched case, and when two degenerate flavours of sea quark ($n_f = 2$) with $m_q \approx m_s$ ($m_s/3$ for the SESAM results) are introduced. $n_f = 3$ is considered the relevant number of sea quark flavours for this quantity [11]. Clearly, a big increase in statistics is needed to probe sea quark effects in this ratio.

Instead, we consider two quantities from very different systems, which are dominated by different physical scales. Figure 3 also presents the results for the ratio $(1\mathrm{P}-1\mathrm{S})/m_\rho$ at $n_f = 0$, using the values for m_ρ and 1P−1S from the UKQCD [12] and NRQCD [6] collaborations, respectively. This ratio is not significantly dependent on a, but it is clearly inconsistent with experiment. Results will soon be available for m_ρ at $n_f = 2$ and it will be interesting to see how much this ratio changes.

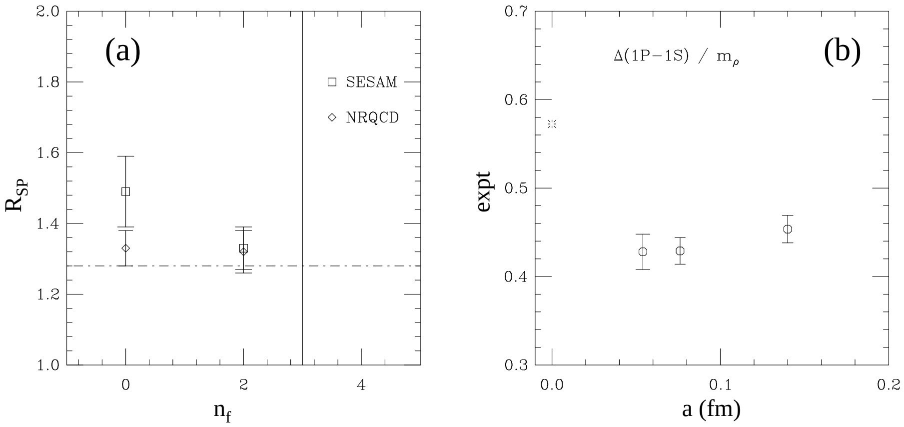

Fig. 3. (a) The n_f dependence of the ratio R_{SP}. (b) The ratio (1P−1S)$/m_\rho$ *as a function of the lattice spacing.*

4. Results for hybrid mesons

With the efficacy of NRQCD demonstrated in the Υ system, this approach has been applied to predicting properties of experimentally less well explored systems. Bottom hybrid mesons, $b\bar{b}g$, in which the glue contributes quantum numbers to the state, are particles which are unlikely to be observed experimentally in the near future (although the best possibility for a hybrid signal may be in the charmonium system). However, the relatively few states below the B$\bar{\text{B}}$ threshold and the small computational cost of the NRQCD simulations (compared to states containing light quarks) mean that this is a good system in which to explore lattice methods of calculating hybrids.

Since the statistical accuracy is much lower than that for $b\bar{b}$, it is sufficient to use the $O(v^2)$ NRQCD action. Lattice operators have been constructed from gauge links corresponding to the T_1^{-+} and T_1^{+-} lattice representations, which, once coupled to quarks, have an overlap with the following continuum quantum numbers:

$$T_1^{+-}: \quad 1^{--}, 0^{-+}, \mathbf{1}^{-+}, 2^{-+} \qquad T_1^{-+}: \quad 1^{++}, \mathbf{0}^{-+}, 1^{+-}, \mathbf{2}^{-+}, \tag{6}$$

where the exotic quantum numbers, i.e. those states which are not found in the quark model, are shown in bold. Spin-dependent interactions are needed to lift the degeneracy within each representation and, in the case of non-exotic J^{PC}, mixing with quark model states.

Figure 4 displays the results from the Glasgow [14] and Cambridge groups [15]. We see that T_1^{+-} is lighter than T_1^{-+} and therefore the lightest exotic is 1^{-+}, in agreement with light hybrid mesons [13]. An important question is whether stable hybrids exist. The results lie above the B$\bar{\text{B}}$ threshold, but close to that for

B$\bar{\mathrm{B}}^{**}$, where B** is a P-state B meson. This confirms results derived from hybrid potentials [14]. Flux tube models suggest B$\bar{\mathrm{B}}^{**}$ to be the relevant threshold for hybrids. Simulations are underway to confirm the position of the hybrid states relative to the thresholds and to study the main sources of systematic error, such as finite volume effects.

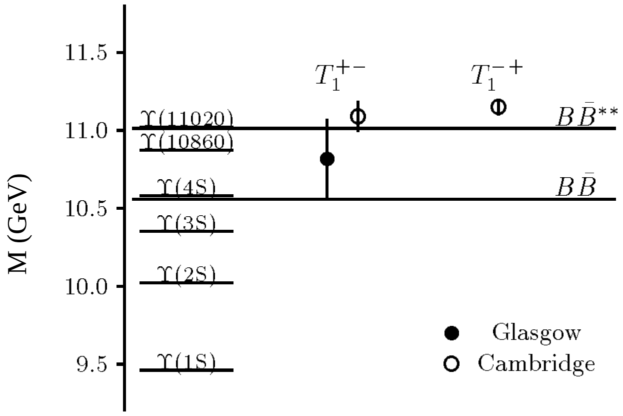

Fig. 4. The results for the T_1^{-+} and T_1^{+-} hybrid mesons.

5. NRQCD applied to heavy-light systems

Theoretical understanding of heavy-light hadrons has been revolutionised through the discovery of heavy quark symmetry (HQS) [16] and the subsequent development of a theoretical framework, heavy quark effective theory (HQET) [17]. In the infinite mass limit, the heavy quark acts as a static colour source for the light quark in, for example, a meson bound state. The light quark is insensitive to the flavour or spin of the heavy quark and spin-flavour symmetry is established. Away from the static limit, the violation of these symmetries is expected to be small for the B meson since the binding energy is small compared to the b quark mass.

When applying NRQCD to simulate a heavy-light hadron on the lattice, the action in equation (1) must be re-interpreted. The importance of each term is counted simply by the order in v (rather than v^2 for quarkonium) or, equivalently, $1/M_0$. Thus, the kinetic energy and hyperfine interactions are $O(1/M_0)$ or 10% contributions, while the spin-orbit and Darwin terms are at higher order, $O(1/M_0^2)$ or 1%. In order to calculate form factors and decay constants within this approach, one must also express the corresponding matrix element of full (relativistic) QCD in terms of NRQCD matrix elements weighted by powers of $1/M_0$ and matching factors (needed to ensure the effective theory matches the full theory at intermediate energies).

At present, the most important matrix element to calculate, with high precision, is the axial-vector current $< 0|Q^\dagger\gamma_0\gamma_5 q|B >$, from which the decay constant,

f_B is defined. The current 20% theoretical error in f_B represents the dominant uncertainty in constraining the size of CP violation in the weak decays $b \to u$ and $\mathrm{B}^0{-}\bar{\mathrm{B}}^0$ mixing. In NRQCD the decay constant has been calculated consistently to $O(1/M_0)$:

$$f_B M_B =< 0|\bar{q}\gamma_0\gamma_5 h|B >= \sum_{j=0}^{3} C_j(aM_0, \alpha) < J_j > +O(1/M_0^2, \alpha^2, a^2, \alpha a/M_0), \quad (7)$$

where

$$J_0 =< 0|\bar{q}\gamma_0\gamma_5 Q|B >= O(1), \quad J_1 =< 0|\bar{q} - \gamma_0\gamma_5\frac{\gamma\cdot D}{2M_0}Q|B >= O(1/M_0) \quad (8)$$

$$J_2 =< 0|\bar{q}\frac{\overleftarrow{\gamma}\cdot\overleftarrow{D}}{2M_0}\gamma_0\gamma_5 Q|B >= O(\alpha/M_0), \quad J_3 =< 0|a\bar{q}\overleftarrow{\gamma}\cdot\overleftarrow{D}\gamma_0\gamma_5 Q|B >= O(a\alpha). \quad (9)$$

The matching factors C_j have been calculated to 1-loop by Morningstar and Shigemitsu [18]. To this order a calculation of f_B to $\approx 10\%$ accuracy should be possible.

5.1. Results for the spectrum

The most comprehensive calculation of the spectrum of heavy-light mesons and baryons has been performed by the GLOK collaboration [19]. Figure 5 shows that the overall spectrum is dominated by the light-quark d.o.f.. The states appear in doublets specified by the light-quark quantum numbers, e.g. B and B^*, $j_l = \frac{1}{2}$, and Σ and Σ_b^*, $J_l = 1$, and are only split by the small hyperfine interaction.

In general, agreement with experiment, where it exists, is reasonable. However, the hyperfine splittings are rather low; $\mathrm{B}^*{-}\mathrm{B} = 23(5)$ MeV compared to 45.7(4) MeV. This discrepancy may be due to underestimating the coefficient c_B, or may indicate quenching effects. So far the scaling behaviour of these predictions has been checked using two values of the lattice spacings. The results are consistent with no scaling violations [20]. However, more work is needed to confirm this. Preliminary studies indicate that the spectrum is not significantly affected if two flavours of sea quarks are introduced [21].

Since M_0 can be freely varied on the lattice, the heavy quark mass dependence of level splittings can easily be studied, and HQS tested: those splittings that are dominated by the light quark are expected to be almost independent of M_0 while hyperfine splittings are expected to be linear in $1/M_0$. So far the results are in agreement with this picture. Figure 6 shows that the 2S$-$1S and 1P$-$1S splittings are almost independent of M_0. The $^3\mathrm{P}_2{-}^3\mathrm{P}_0$ splitting, that is related to the spin-orbit interaction of the light quark, depends slightly on M_0. However, it tends to a constant of $O(\Lambda_{QCD})$ in the static limit as expected. In contrast the much smaller P-state hyperfine splitting, $^3\mathrm{P}_2{-}^3\mathrm{P}_1$ tends to zero for large M. The intercepts and slopes of these splittings can be extracted and used in HQET.

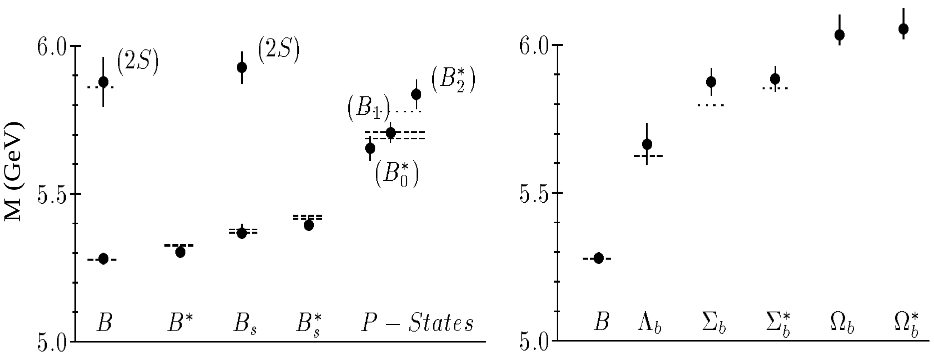

Fig. 5. The results for the lower lying spectrum of mesons and baryons containing a single b *quark. The dashed lines indicate the experimental lower and upper bounds. The dotted lines indicate the first experimental signal.* a *is fixed from* m_ρ *and* aM_b^0 *is fixed using the* B *meson.*

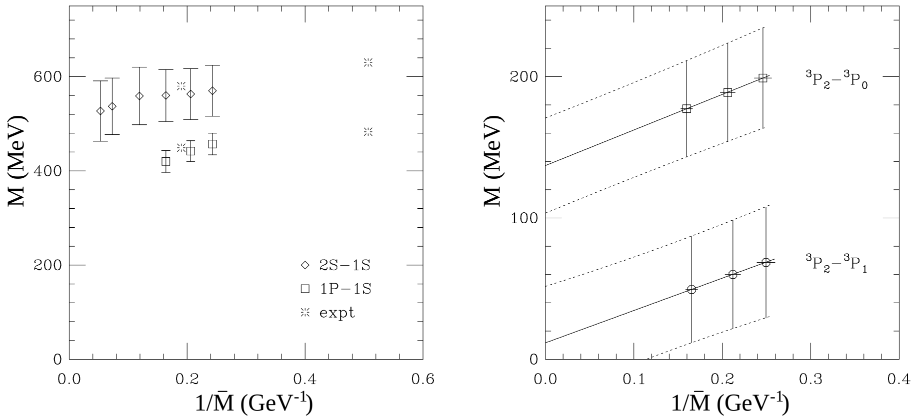

Fig. 6. The heavy quark mass dependence of various spin-average and spin-dependent heavy-light meson mass splittings.

5.2. Results for the decay constants

f_B has been calculated using NRQCD by the GLOK [22] and JLQCD [23] groups. The GLOK results from a single lattice spacing corresponding to $a^{-1} \approx$ 2 GeV are

$$f_B = 147(11)(stat)(^{+8}_{-12})(a^{-1})(9)(pert)(6)(disc)\ \text{MeV}, \tag{10}$$

$$f_{B_s} = 175(8)(stat)(^{+7}_{-10})(a^{-1})(11)(pert)(7)(disc)(^{+7}_{-0})(\kappa_s)\ \text{MeV}, \tag{11}$$

where the tree-level $O(1/M_0^2)$ terms in the action and in the matrix elements have also been included in the calculation. In accord with the expectation, these contribute only a few percent. A detailed analysis has been made of the systematic errors arising from the uncertainty in the lattice spacing, the perturbative matching factors, C_j, the discretisation errors arising from the light quark action and, in the case of f_{B_s}, fixing the strange quark mass. The combined uncertainty is around 14%. Note that most systematic errors cancel for the ratio, $\frac{f_{B_s}}{f_B} = 1.20(4)(stat)(^{+4}_{-0})(\kappa_s)$. Unfortunately, however, it is not clear whether resolving the mixing within the B_s^0–$\bar{\mathrm{B}}_s^0$ system is within the reach of the B factories.

The JLQCD results are consistent with those presented above. JLQCD finds no indication of scaling violations for the decay constant over a range of lattice spacings $a^{-1} = 1-2.3$ GeV. In addition, agreement is found with results obtained by use of an alternative approach for simulating heavy quarks on the lattice developed by the FERMILAB group [24].

The vector decay constant, $f_{B_s^*}$, has also been calculated by GLOK; $f_{B_s^*} = 188(20)$ MeV. This quantity, along with the heavy quark mass dependence of the decay constants, is interesting for studying spin-flavour symmetry. However, the appropriate matching factors are not yet available and only a tree-level study is possible at present.

In a preliminary calculation with one lattice spacing, the SGO collaboration [21] has found f_B to be very sensitive to the presence of sea quarks. The decay constant increases by 20−25% at $n_f = 2$, compared to the quenched results, and f_B in the real world may lie significantly higher than the value quoted in equation 10. Further work is needed to investigate the dependence on the sea quark mass and n_f.

6. *Conclusions*

NRQCD has been successfully applied to a wide variety of systems involving b quarks. Comprehensive calculations have been made of the Υ and heavy-light meson and baryon spectra, where the systematic errors, at least for spin-averaged splittings, appear to be under control, and preliminary studies of quenching effects have been attempted. Predictions have been made for exotic systems such as heavy hybrids. Precision calculations of B decay constants, needed to convert experimental results to CKM matrix elements, are in progress. However, for spin-dependent splittings in both quarkonium and heavy-light systems more work needs to be done. In particular, the matching coefficients, c_i, need to be known more accurately.

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References

1) W. Caswell and G. P. Lepage, Phys. Lett. B **167** (1986) 437; B. A. Thacker and G. P. Lepage, Phys. Rev. D **43** (1991) 196; G. P. Lepage et al., Phys. Rev. D **46** (1992) 4052;
2) C. Quigg and J. L. Rosner, Phys. Rept. **56** (1979) 1670;
3) G. P. Lepage and P. Mackenzie, Phys. Rev D **48** (1992) 2250;
4) C. Morningstar, Phys. Rev. D **50** (1994) 5902;
5) H. Trottier and G. P. Lepage, Nucl. Phys. B (Proc. Suppl.) **63** (1998) 865;
6) C. T. H. Davies et al., Phys. Rev. D **58** (1998) 54505;
7) T. Manke et al., Phys. Lett. B **408** (1997) 308;
8) G. Bali and P. Boyle, HUB-EP-98-58, hep-lat/9809180;
9) C. Davies et al, Phys. Rev. D **56** (1997) 2755;
10) N. Eicker et al., Phys. Rev. D **57** (1998) 4080;
11) J. Shigemitsu, Nucl. Phys. (Proc.Suppl.) **53** (1997) 16;
12) R. Kenway, Nucl. Phys. (Proc.Suppl.) **53** (1997) 206;
13) P. Lacock et al., Phys. Rev. D **54** (1996) 6997;
14) S. Collins et al., Nucl. Phys. B (Proc. Suppl.) **63** (1998) 335;
15) T. Manke et al., Phys. Rev. D **57** (1998) 3829;
16) N. Isgur and M. Wise, in *Proceedings of Hadron '91*, (1992) 549;
17) M. Neubert, Phys. Rept. **245** (1994) 259; A Falk, JHU-TIPAC-96017, hep-ph/9610363;
18) C. Morningstar and J Shigemitsu, Phys. Rev. D **57** (1998) 6741;
19) A. Ali Khan et al, Talk given at *16th Int. Symp. on Lattice Field Theory*, Boulder CO, 13-28th July 1998; hep-lat/9809140; in preparation;
20) J. Hein et al., Nucl. Phys. (Proc. Suppl.) B **63** (1998) 347;
21) S. Collins et al., GUTPA-98-11-01; hep-lat/9901001;
22) A. Ali Khan et al., Phys. Lett. B **427** (1998) 132;
23) K-I. Ishikawa et al., Talk given at *16th Int. Symp. on Lattice Field Theory (LATTICE 98)*, Boulder, CO, 13-18 July 1998, KEK-CP-075; hep-lat/9809152;
24) A. El-Khadra et al., Phys. Rev. D **58** (1998) 014506; S. Aoki et al, Nucl. Phys. B (Proc.Suppl.) **63** (1998) 356; C. Bernard et al., Nucl. Phys. A (Proc.Suppl.) **60** (1998) 106.

NOVI ISHODI NERELATIVISTIČKE TEORIJE QCD NA REŠETCI

Daje se pregled računa na rešetci za svojstva hadrona koji sadrže b kvarkove, s naglaskom na nerelativističku QCD (NRQCD). Posebno, raspravljaju se posljednji ishodi računa za Υ-spektar. Oni pokazuju da se niskoležeći spektar uz prosjek po spinovima može postići dobro do na točnost od par posto djelovanjem $O(Mv^4)$ NRQCD, čak i bez prisustva kvarkovskog mora. Načinili smo opsežne račune spektra i (za par slučajeva) konstanti raspada mezona i bariona koji sadrže težak kvark: B i B_s mezona, uključivo radijalne i stazne uzbude i Λ_b, Σ_b i Ω_b bariona. Svojstva tih čestica još se nisu dobro utvrdila eksperimentalno, a ovdje se daju pouzdana predvidanja primjenom računa na rešetci.