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OVERVIEW FROM LATTICE QCD

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A review is given of recent lattice results. In particular, the confinement mechanism and string breaking, glueballs and hybrid mesons as well as light hadron spectroscopy are discussed.

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1. *Introduction*

The lattice approach to QCD facilitates the numerical evaluation of expectation values without recourse to perturbative techniques. Although the lattice formulation is almost as old as QCD itself and first simulations of the path integral have been performed as early as in 1979 [1], only recently computers have become powerful enough to allow for a determination of the infinite volume continuum light hadron spectrum in the quenched approximation to QCD within uncertainties of a few per cent. To this accuracy the quenched spectrum deviates from experiment. Some collaborations have started to systematically explore QCD with two flavours of light sea quarks and the first precision results indeed indicate such differences.

Lattice QCD is a *first principles* approach; no parameters apart from those that are inherent to QCD, i.e. a strong coupling constant at a certain scale and n quark masses, have to be introduced. In order to fit these $n + 1$ parameters, $n + 1$ low energy quantities are matched to their experimental values: the lattice spacing $a(g, m_i)$, that results from given values of the bare lattice coupling g and (in unquenched QCD) quark masses m_i, can be obtained by fixing m_ρ as determined on the Lattice to the experimental value. The lattice parameters that correspond to physical $m_u \approx m_d$ can then be obtained by adjusting m_π/m_ρ; the right m_s can be reproduced by adjusting m_K/m_ρ or m_ϕ/m_ρ to experiment etc.. Once the scale and quark masses have been set, everything else becomes a prediction. Due to the evaluation of path integrals by use of a stochastic process, lattice predictions carry statistical errors which can in principle be made arbitrarily small by increasing the statistics, i.e. the amount of computer time spent. In this sense, it is an exact approach.

Lattice results have in general to be extrapolated to the (continuum) limit $a \to 0$ at fixed physical volume. The functional form of this extrapolation is theoretically well understood and under control. This claim is substantiated by the fact that simulations with different lattice discretisations of the continuum QCD action yield compatible results after the continuum extrapolation. For high energies, an overlap between certain quenched Lattice computations and perturbative QCD has been confirmed too [2], excluding the possibility of fixed points of the β-function at finite values of the coupling, other than $g = 0$. After taking the continuum limit an infinite volume extrapolation is performed. Results on hadron masses from quenched evaluations on lattices with spatial extent $La > 2$ fm are virtually indistinguishable from the infinite volume limit within typical statistical errors in most cases. However, for QCD with sea quarks the available information is not yet sufficient for definite conclusions, in particular as one might expect a substantial dependence of the on-set of finite size effects on the sea quark mass(es).

The effective infinite volume limit of realistically light pions cannot be realised at a reasonable computational cost, neither in quenched nor in full QCD. Therefore, in practice another extrapolation, in the quark mass, is required. This extrapolation to the correct light quark mass limit is theoretically less well under control than those to the continuum and infinite volume limits. The parametrisations used are in general motivated by chiral perturbation theory and the theoretical uncertainties are the dominant source of error in latest state-of-the-art spectrum calculations.

Many important questions are posed in low-energy QCD: is the same set of fundamental parameters (QCD coupling and quark masses) that describes for instance the hadron spectrum consistent with high energy QCD or is there place for new physics? Are all hadronic states correctly classified by the naïve quark model or do glueballs, hybrid states and molecules play a rôle? At what temperatures/densities does the transition to a quark-gluon plasma occur? What are the experimental signatures of quark-gluon matter? Can we solve nuclear physics on the quark and gluon level? Clearly, complex systems like iron nuclei are unlikely ever to be solved from *first principles* alone but *modelling* and certain *approximations* will always be required.

It is desirable to test model assumptions, to gain control over approximations and, eventually, to derive low-energy effective Lagrangians from QCD. Lattice simulations are a very promising tool for this purpose and in the first part of this article a flavour is given of such more theoretically motivated studies before reviewing recent results on glueballs and exotic hybrid mesons as well as discussing the light hadron spectrum.

2. *The confinement scenario*

Two prominent features of QCD, confinement of colour sources and spontaneous breaking of chiral symmetry, are both lacking a proof. They appear to be related, however: in the low temperature phase of broken chiral symmetry, colour sources are effectively confined. Clearly, an understanding of what is going on should help us in developing the methods required to tackle a huge class of non-perturbative problems.

It is worthwhile to consider the simpler pure SU(N) gauge theory. In this case *confinement* can be rigorously defined since the Polyakov line is an order parameter for the de-confinement phase transition that is related to spontaneously breaking a global Z_N symmetry. Some results will be presented that have been obtained in the computationally cheaper SU(2) gauge theory whose spectrum shares most qualitative features with that of SU(3).

In the past decades, many explanations of the confinement mechanism have been proposed, most of which share the feature that topological excitations of the vacuum play a major rôle. These pictures include, among others, the dual superconductor picture of confinement [3,4] and the centre vortex model [5]. Depending on the underlying scenario, the excitations giving rise to confinement are thought to be magnetic monopoles, instantons, dyons, centre vortices, etc.. Different ideas are not necessarily exclusive. For instance, all mentioned excitations are found to be correlated with each other in numerical as well as in some analytical studies, such that at present it seems to be rather a matter of personal preference which one to consider as more *fundamental*.

Recently, the centre vortex model has enjoyed renewed attention [6]. In this picture, excitations that can be classified in accord with the centre group provide the disorder required to produce an area law of the Wegner-Wilson loop and, therefore, confinement. One striking feature is that — unlike monopole currents — centre vortices form gauge invariant two-dimensional objects, such that in four space-time dimensions, a linking number between a Wegner-Wilson loop and centre vortices can unambiguously be defined, providing a geometric interpretation of the confinement mechanism [7].

Here the discussion is restricted to the superconductor picture which is based on the concept of electro-magnetic duality after an Abelian gauge projection and has originally been proposed by 't Hooft and Mandelstam [3]. The QCD vacuum is thought to behave analogously to an electrodynamic superconductor but with the rôles of electric and magnetic fields being interchanged: a condensate of magnetic monopoles expels electric fields from the vacuum. If one now puts electric charge and anti-charge into this medium, the electric flux that forms between them will be squeezed into a thin, eventually string-like, Abrikosov-Nielsen-Oleson vortex which results in linear confinement.

In all quantum field theories in which confinement has been proven, namely in compact U(1) gauge theory, the Georgi-Glashow model and SUSY Yang-Mills theories, this scenario is indeed realised. However, before one can apply this simple picture to QCD or SU(N) chromodynamics one has to identify the relevant dynamical variables: it is not straight forward to generalise the electro-magnetic duality of a U(1) gauge theory to SU(N) where gluons carry colour charges. How can one define electric fields and dual fields in a gauge invariant way?

In the Georgi-Glashow model, the $SO(3)$ gauge symmetry is broken down to a residual U(1) symmetry as the vacuum expectation value of the Higgs field becomes finite. It is currently unknown whether QCD provides a similar mechanism and various reductions of the SU(N) symmetry have been conjectured. In this spirit, it has been proposed [4] to identify the monopoles in a $\mathrm{U}(1)^{N-1}$ Cartan subgroup of SU(N) gauge theory after gauge fixing with respect to the off-diagonal $\mathrm{SU}(N)/\mathrm{U}(1)^{N-1}$ degrees of freedom. After such an Abelian gauge fixing QCD can be regarded as a theory of interacting photons, monopoles and matter fields (i.e. off-diagonal gluons and quarks). One might assume that the off-diagonal gluons do not affect long range interactions. This conjecture is known as *Abelian dominance* [8]. Abelian as well as monopole dominance are qualitatively realised in Lattice studies of SU(2) gauge theory [9] in the maximally Abelian (MA) gauge projection, which appears to be the most suitable gauge fixing condition.

Figure 1 displays the electric field distribution between SU(2) quarks, separated by a distance $r = 15a \approx 1.2$ fm, that has been obtained within the MA gauge projection. Everything is measured in lattice units $a \approx 0.081$ fm where the physical scale derived from the value $\sqrt{\kappa} = 440$ MeV for the string tension is intended to serve as a guide to what one might expect in "real" QCD. Indeed an elongated Abrikosov-Nielsen-Oleson vortex forms between the charges. Figure 2 displays a cross section through the centre plane of this vortex. While the electric field strength decreases with the distance from the core, the modulus of the dual Ginzburg-Landau (GL) wave function, f, i.e. the density of superconducting magnetic monopoles decreases towards the centre of the vortex where superconductivity breaks down. In this study the values $\lambda = 0.15(2)$ fm and $\xi = 0.25(3)$ fm have been obtained

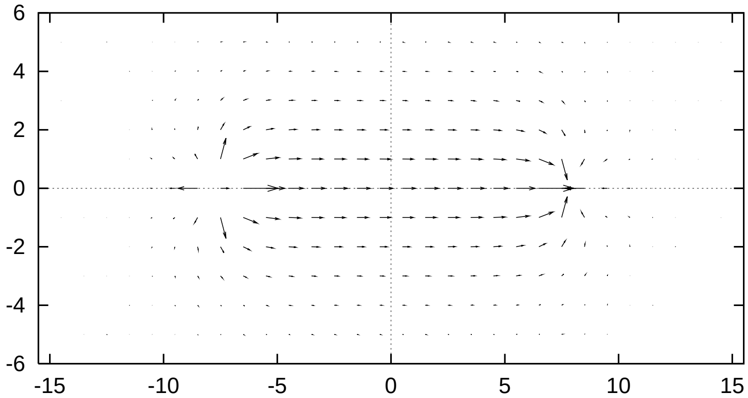

Fig. 1. Electric field distribution between two static SU*(2) sources in the MA projection. Everything is in lattice units* $a \approx 0.081$ fm*. The sources are located at the coordinates* $(-7.5, 0)$ *and* $(7.5, 0)$*.*

for penetration depth and GL coherence length, respectively. The ratio $\lambda/\xi = 0.59(13) < 1/\sqrt{2}$ corresponds to a (weak) type I superconductor, i.e. QCD flux tubes appear to attract each other. For details see Ref. 10.

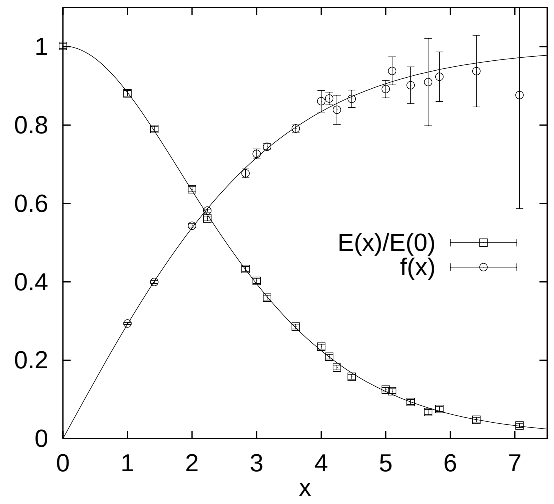

Fig. 2. Electric field and GL wave function density in the centre plane between the sources.

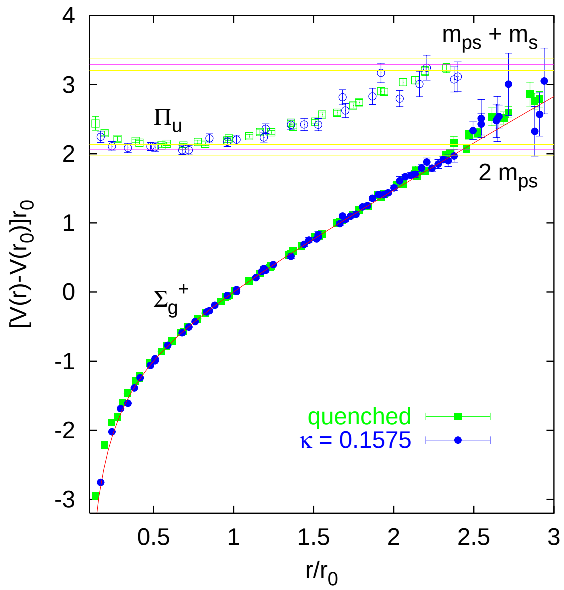

Fig. 3. The QCD static Σ_g^+ and Π_u potentials, together with masses of pairs of static-light mesons.

3. String breaking

In the pure gauge theory results presented above, the energy stored in the vortex between charges increases in proportion to their distance *ad infinitum*. In full QCD with sea quarks, however, the string will break into two parts as soon as this energy exceeds the energy required to create a quark-antiquark pair from the vacuum: inter-quark forces at large separation will be completely screened by sea quarks and excited Υ states can decay into a $B\overline{B}$ meson pair. Figure 3 shows a recent comparison between the quenched and $n_f = 2$ static potential by the $T\chi L$ collaboration at a sea quark mass $m_{ud} \approx m_s/3$ [11]. Estimates of masses of pairs of static-light mesons into which the static heavy-heavy systems can decay are also included into the figure. The potentials have been matched to each other at a distance $r = r_0 \approx 0.5$ fm. In presence of sea quarks anti-screening is weakened and, therefore, starting from the same infrared value, the effective QCD coupling runs slower towards the $\alpha_s = 0$ ultraviolet limit. This effect explains why at small r the unquenched data points are somewhat below their quenched counterparts: the effective Coulomb force remains stronger. Around $r = 1.2$ fm, the unquenched potential is expected to become unstable. However, the data are not yet precise enough to resolve this effect.

Motivated by such QCD simulations, the dynamics of string breaking has recently been analysed in some toy models [12]. First results on interactions between two B mesons in quenched QCD have been reported by Pennanen and Michael [13].

4. Glueballs and quark-gluon hybrid states

In Fig. 3, not only the ground state potential but also a so-called hybrid potential is displayed in which the gluonic component contributes to the angular momentum. Recently, the spectrum of such potentials has been accurately determined by Juge, Kuti and Morningstar [14]. The presence of gluons in bound states should also affect light meson and baryon spectra: one would expect additional excitations that cannot be classified in accord with the naïve constituent quark model. On the Lattice and in experiment it should be most easy to discriminate states with exotic, i.e. quark model forbidden, quantum numbers from “standard” hadrons. Spin-exotic baryons cannot be constructed but only mesons and glueballs. First results on light hybrid mesons have been reported by two groups [15]. The lightest spin-exotic particle has quantum numbers $J^{PC} = 1^{-+}$ and a mass between 1.8 and 2 GeV. Recent investigations incorporating sea quarks [16] confirm these findings. However, at present all experimental candidates have masses smaller than 1.6 GeV. Therefore, in the interpretation of experiment mixing between spin exotic mesons and four-quark molecules, such as a πf_1, should be considered. It is certainly worthwhile to investigate this possibility on the Lattice too.

Recent results by Morningstar and Peardon [17] have revolutionised our knowledge on the quenched glueball spectrum. Only in case of the scalar glueball they fail to reach the precision of the 1993 state-of-the-art Lattice predictions [18]: finer lattices are required for a safer continuum limit extrapolation. As can be seen from Fig. 4, the ordering of glueball states has become fairly well established. The fact that the lightest spin-exotic state 2^{+-} lies well above 4 GeV explains why such states have escaped observation so far. Glueballs are quite heavy and spatially rather extended, due to the lack of quarks that tie the flux together. Therefore, these states lend themselves to the use of anisotropic lattices: the size of the temporal lattice spacing is dictated by the heavy mass that one wishes to resolve while a much coarser spatial spacing can be adapted to resolve the glueball wave function.

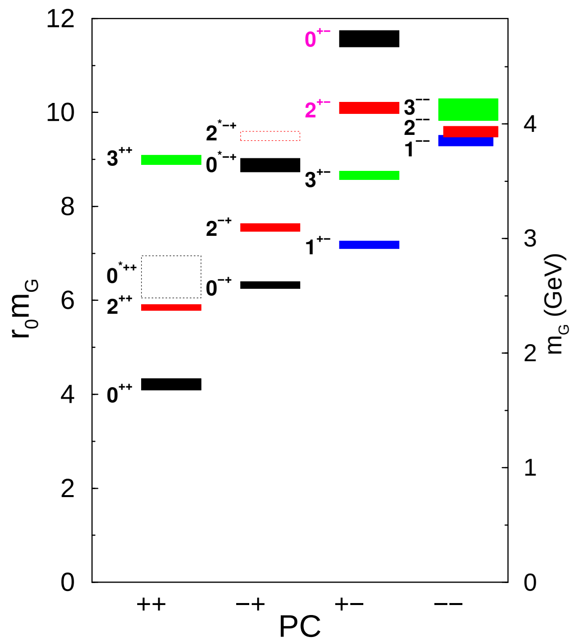

Fig. 4. The quenched glueballs spectrum (from Ref. 17).

Introducing this anisotropy was vital for the improvement achieved. Recent results in QCD with sea quarks on the scalar and tensor glueballs are consistent with quenched findings [19,11]. Beyond the quenched approximation, glueballs will mix with standard quark model states. Investigations of such mixing and decay rates of the mixed states are challenging questions [20] that are waiting to be approached by Lattice studies in the near future.

5. Light hadrons

In addition to quantities that theorists or experimentalists are interested in, well-known observables can be computed on the Lattice too. The motivation is two-fold: testing QCD and gauging the Lattice methodology. Experimental low energy input like the hadron spectrum is required in the first place to fix the lattice spacing and quark masses. Subsequently, among other predictions, the fundamental parameters α_s and quark masses [2] can be converted to, for instance, the $\overline{MS}$ scheme that is convenient for perturbative continuum calculations. It is not *a priori* clear whether the low energy results are compatible with values required to explain high energy QCD phenomenology.

Assuming that QCD is the right theory, the observed states can serve as a guideline to judge the viability of approximations, such as quenching in the absence of high precision full QCD results. Last but not least, quark masses and other parameters can be varied and Lattice results can be confronted with predictions of, for instance, chiral perturbation theory. Indeed, evidence for quenched chiral logarithms has been reported [21].

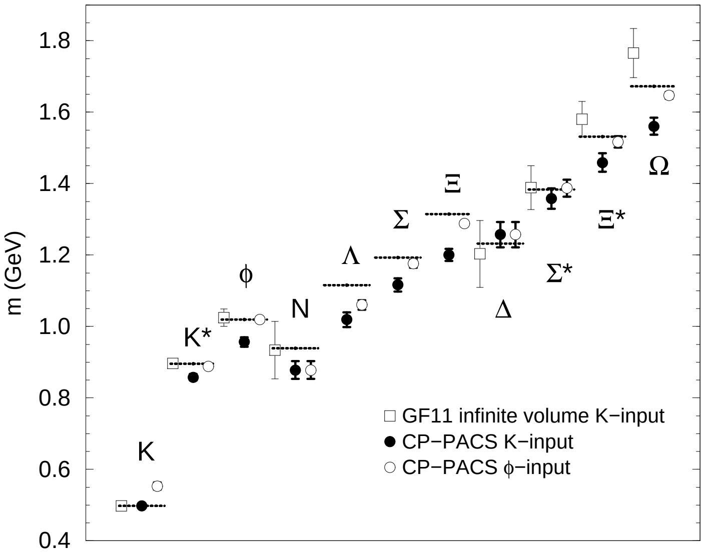

Fig. 5. The quenched light hadron spectrum (from Ref. 21).

Figure 5 displays the results from a recent state-of-the-art calculation of the quenched light hadron spectrum by the CP-PACS collaboration [21]. For comparison the results from the GF11 collaboration [22] that have set the standard back in 1994 are included (squares). The π and ρ masses that have been used as input values for the lattice spacing a and quark mass $m_u = m_d$ are omitted from the plot. m_s has been set by two methods: forcing the K mass to agree with experiment (full circles) and forcing the ϕ mass to agree (open circles). Neither of the methods can bring the spectrum completely in line with experiment. However, no mass comes out to be wrong by more than 10%, indicating that the main effect of sea quarks is to renormalise the over-all value of the coupling, rather than altering mass ratios, despite the fact that all particles displayed, with the exception of the nucleon, become unstable in full QCD. First un-quenched results by the same collaboration indicate an improvement in the direction of the experimental values. Many groups are at present studying quantities which one might expect to be more sensitive towards quenching like the η' mass, quark masses and the $\pi_{N\sigma}$ term [23].

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PREGLED QCD NA REŠETKI

Daje se pregled ishoda računa na rešetki. Posebice se raspravljaju mehanizam zatočenja, lom struna, glounske lopte, hibridni mezoni i spektroskopija lakih hadrona.