ISSN 1330–0016
CODEN FIZBE7

CLUSTER REDUCTION AND SOLUTION OF LOW-ENERGY SCATTERING PROBLEM FOR $N > 3$ NUCLEON SYSTEMS

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Received 11 January 1999; revised manuscript received 16 July 1998
Accepted 19 July 1999

Cluster reduction method for solution of low-energy scattering problem in few-nucleon system is described. The method reduces the Yakubovsky differential equations to effective equations describing relative motion of clusters. Application of the method to numerical solution of low-energy scattering problem in n – ^{3}H, p – ^{3}He, n – ^{3}He, p – ^{3}H and ^{2}H – ^{2}H systems are presented.

PACS numbers: 21.45.+v, 25.10.+s UDC 539.128, 539.17
Keywords: few-body nuclei, nuclear reactions, scattering, low energy

1. Introduction

The great variety of processes in systems consisting of $N > 3$ nucleons makes the solution of the scattering problem extremely difficult. Already in the four nucleon case, the list of two-fragment reactions looks rather impressive:

$$\mathrm{n} + {}^3\mathrm{H} \to \mathrm{n} + {}^3\mathrm{H}, \qquad \mathrm{n} + {}^3\mathrm{He} \to \begin{cases} \mathrm{n} + {}^3\mathrm{He}, \\ \mathrm{p} + {}^3\mathrm{H}, \\ {}^2\mathrm{H} + {}^2\mathrm{H}, \end{cases}$$

$$\mathrm{p} + {}^3\mathrm{He} \to \mathrm{p} + {}^3\mathrm{He},$$

$$\mathrm{p} + {}^3\mathrm{H} \to \begin{cases} \mathrm{p} + {}^3\mathrm{H}, \\ \mathrm{n} + {}^3\mathrm{He}, \\ {}^2\mathrm{H} + {}^2\mathrm{H}, \end{cases} \qquad {}^2\mathrm{H} + {}^2\mathrm{H} \to \begin{cases} {}^2\mathrm{H} + {}^2\mathrm{H}, \\ \mathrm{n} + {}^3\mathrm{He}, \\ \mathrm{p} + {}^3\mathrm{H}. \end{cases}$$

The most widely used techniques are based on the Kohn-Hulthén variational principles and the Faddeev-Yakubovsky equations (FYE, AGS). The latter seem to be the most adequate for the complete description of multichannel scattering, but require huge computer facilities for numerical calculation. This paper describes a method for the solution of FYE for two-fragment reactions which allows to reduce numerical difficulties drastically. It uses the ideas which are somewhat similar to the resonating group method ansatz and presents the results of calculations of low-energy scattering characteristics of the above reactions.

2. *Cluster reduction of YDE*

The elastic and rearrangement processes in the four-particle system with two clusters in the initial and final states can be treated adequately on the base of the Yakubovsky differential equations (YDE) [1–3]

$$(H_0 + V_{a_3} - E)\Psi_{a_3a_2} + V_{a_3} \sum_{(c_3\neq a_3)\subset a_2} \Psi_{c_3a_2} = -V_{a_3} \sum_{d_2\neq a_2} \sum_{(d_3\neq a_3)\subset a_2} \Psi_{d_3d_2}.$$

For the two-cluster collisions, the YDE admit a further reduction. Let $H_0 = T_{a_2} + T^{a_2}$ be the separation of the kinetic energy operator into the intrinsic kinetic energy, T_{a_2}, of partition a_2 and the kinetic energy T^{a_2} of the relative motion of a_2 clusters. The cluster reduction procedure consists in the expansion of the components $\Psi_{a_3a_2}$ along the basis of the solutions to the Faddeev equations (FE) for subsystems of partition a_2

$$(T_{a_2} + V_{a_3})\psi^{a_3}_{a_2,k} + V_{a_3} \sum_{(c_3\neq a_3)\subset a_2} \psi^{c_3}_{a_2,k} = \varepsilon^k_{a_2}\psi^{a_3}_{a_2,k}.$$

The expansion has the form

$$\Psi_{a_3a_2}(\mathbf{X}) = \sum_{k=0}^{\infty} \psi^{a_3}_{a_2,k}(\mathbf{x}_{a_2})F^k_{a_2}(\mathbf{z}_{a_2}). \tag{1}$$

Here, the unknown amplitudes $F^k_{a_2}(\mathbf{z}_{a_2})$ depend only on the relative position vector $\mathbf{z}_{a_2}$ between the clusters of the partition a_2, and by $\mathbf{x}_{a_2}$ are denoted coordinates that are intrinsic to clusters of partition a_2. The solutions of FE form a complete set but not an orthogonal basis [4,5] due to the not-Hermitness of FE. The biorthogonal basis is formed by the solutions of equations conjugated to FE

$$(T^{a_2} + V_{a_3})\phi^{a_3}_{a_2,k} + \sum_{(c_3\neq a_3)\subset a_2} V_{c_3}\phi^{c_3}_{a_2,k} = \varepsilon^k_{a_2}\phi^{a_3}_{a_2,k}.$$

Introducing the expansion for $\Psi_{a_3a_2}(\mathbf{X})$ into YDE and projecting onto the elements of biorthogonal basis $\{\phi^{a_3}_{a_2,k}(\mathbf{x}_{a_2})\}$ leads to a reduced YDE (RYDE) [6,7] for $F^k_{a_2}(\mathbf{z}_{a_2})$

$$(T^{a_2} - E + \varepsilon^k_{a_2})F^k_{a_2} = -\sum_{a_3\subset a_2} \langle\phi^{a_3}_{a_2,k}|V_{a_3} \sum_{d_2\neq a_2} \sum_{(d_3\neq a_3)\subset a_2} \sum_{l\geq 0} \psi^{d_3}_{d_2,l} F^l_{d_2}\rangle, \qquad (2)$$

where the brackets $\langle.|.\rangle$ mean the integration over $\mathbf{x}_{a_2}$. The boundary conditions for $F^k_{a_2}(\mathbf{z}_{a_2})$ have the following *two body* form as $|\mathbf{z}_{a_2}| \to \infty$

$$F^k_{a_2}(\mathbf{z}_{a_2}) \sim \delta_{k0}[\delta_{a_2 b_2} \exp \mathrm{i}(\mathbf{p}_{a_2}, \mathbf{z}_{a_2}) + \mathcal{A}_{a_2 b_2} \frac{\exp \mathrm{i}\sqrt{E - \varepsilon^0_{a_2}}|\mathbf{z}_{a_2}|}{|\mathbf{z}_{a_2}|}],$$

where the index b_2 corresponds to the initial state, and $\mathbf{p}_{a_2}$ is the conjugated momentum to $\mathbf{z}_{a_2}$. The charged-particles case can be treated within the framework of YDE formalism by adding the Coulomb potentials to the kinetic energy operator H_0 and by replacing the plane and spherical waves in the asymptotics by respective Coulomb modifications [3]. Coulomb modification consists in replacing the left hand side of Eqs. (2) by

$$(-\delta_{a_2}\Delta_{\mathbf{z}_{a_2}} - E + \varepsilon^k_{a_2})F^k_{a_2}(\mathbf{z}_{a_2}) + \sum_{a_3,l} \langle\phi^{a_3}_{a_2,k}|V^c|\psi^{a_3}_{a_2,l}\rangle F^l_{a_2}(\mathbf{z}_{a_2}) = ...,$$

and in replacing the asymptotic boundary conditions by

$$F^0_{a_2}(\mathbf{z}_{a_2}) \sim \delta_{a_2 l_2}\Gamma(1 + \mathrm{i}\eta_{a_2})e^{-\pi\eta_{a_2}/2}{}_1F_1(-\mathrm{i}\eta_{a_2}, 1, \mathrm{i}(|\mathbf{p}_{a_2}||\mathbf{z}_{a_2}| - (\mathbf{p}_{a_2}, \mathbf{z}_{a_2}))$$

$$+\mathcal{A}_{a_2 l_2} \frac{\exp\{\mathrm{i}\sqrt{E - \varepsilon_{a_2}}|\mathbf{z}_{a_2}| - \mathrm{i}\eta_{a_2} \log 2\sqrt{E - \varepsilon_{a_2}}|\mathbf{z}_{a_2}|\}}{|\mathbf{z}_{a_2}|}.$$

3. *Application to low-energy scattering in the four nucleon system*

RYDE (2), after suitable partial wave decomposition, become one dimensional in the variable $|\mathbf{z}_{a_2}|$. We solve numerically these equations by means of finite-difference approximation in the $|\mathbf{z}_{a_2}|$ variable, spline expansion of the integrand on the right-hand side and truncation of summation over l by to finite number N. In all the cases, a satisfactory convergence was observed with the parameter N not exceeding 20, which confirms the efficiency of the expansion (1). The maximal size of the linear system to be solved was of the order 10^4, so that the calculations were performed on a standard workstation. We have used MT I-III model with the parameters from Ref. 8 for the NN forces.

TABLE 1. S-T channel N – NNN *scattering lengths (in* fm*).*

S	T	Ref. 9	Our results
0	0	14.75	14.7
1	0	3.25	3.2
0	1	4.13	4.0
1	1	3.73	3.6

The first group of results we are presenting concerns the isospin approximation (i.e., neglecting the Coulomb interaction). The values for the channel scattering lengths for nucleon scattered off a three-nucleon cluster, presented in Table 1, are in agreement with the results of Grenoble group obtained by a direct discretization of YDE. Note that the $T = 1$ channels correspond to singlet and triplet $\mathrm{n} - {}^3\mathrm{H}$ scattering.

The second group of results is more realistic in view of taking into account the Coulomb interaction between protons. In Table 2, we collect our results for $\mathrm{p} - {}^3\mathrm{H}$ (${}^{2S+1}\mathrm{A}_{pt}$) and $\mathrm{p} - {}^3\mathrm{He}$ (${}^{2S+1}\mathrm{A}_{ph}$) elastic scattering lengths with data obtained on the basis of the resonating-group-method (RGM) [10] calculations and experimental values from Ref. 11. Last two rows of Table 2 show the position E_r and weight Γ in MeV of ${}^4\mathrm{He}$ – nucleus 0^+ resonance (measured relatively to $\mathrm{p} - {}^3\mathrm{H}$ threshold), extracted from the low-energy behaviour of calculated $\mathrm{p} - {}^3\mathrm{H}$ phase-shift.

TABLE 2. Singlet and triplet $\mathrm{p} - {}^3\mathrm{H}$ *and* $\mathrm{p} - {}^3\mathrm{He}$ *scattering lengths (in* fm*).*

	${}^1\mathrm{A}_{pt}$	${}^3\mathrm{A}_{pt}$	${}^1\mathrm{A}_{ph}$	${}^3\mathrm{A}_{ph}$	E_r	Γ
Our results	-22.6	4.6	8.2	7.7	0.15	0.3
Ref. 10	-21.46				0.12	0.26
Ref. 11			10.8± 2.6	8.1±0.5		
Ref. 12					0.3±0.05	0.27±0.05

TABLE 3. Singlet and triplet $\mathrm{n} - {}^3\mathrm{He}$ *and singlet and pentaplet* ${}^2\mathrm{H} - {}^2\mathrm{H}$ *scattering lengths (in* fm*).*

	${}^1\mathrm{A}_{nh}$	${}^3\mathrm{A}_{nh}$	${}^1\mathrm{A}_{dd}$	${}^5\mathrm{A}_{dd}$
Our results	7.5-4.2i	3.0+0.0i	10.2-0.2i	7.5
Ref. 10	7.25-3.92i			
Ref. 13				6.68-0.135i

Table 3 shows the results of calculations of the scattering lengths for $\mathrm{n} - {}^3\mathrm{He}$ (${}^{2S+1}\mathrm{A}_{nh}$) and ${}^2\mathrm{H} - {}^2\mathrm{H}$ (${}^{2S+1}\mathrm{A}_{dd}$) scattering. Due to the open rearrangement channels, the scattering lengths in these cases have nontrivial imaginary part.

4. *Conclusions*

The cluster reduction procedure allows to obtain the effective *two body* equations for the two-cluster collisions in a four-nucleon system, keeping completeness of the description of few-body dynamics. RDYE are efficient for calculating scattering parameters of interest. The CR method can in principle be extended to realistic NN forces without problems. The CR method can be extended to systems consisting of more than four particles.

Acknowledgements

This work was partially supported by Russian Foundation for Basic Research grant No. 98-02-18190. The author would like to thank G. M. Halle for providing his preliminary result on R-matrix analysis of experimental data for pentaplet $^2\mathrm{H} - {}^2\mathrm{H}$ scattering. Author is thankful to Organizing Committee of the Conference on Particle and Nuclear Physics with CEBAF at Jefferson Lab for financial support for his participation in the Conference.

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NAKUPINSKA REDUKCIJA I RJEŠENJE ZADAĆE NISKOENERGIJSKOG RASPRŠENJA ZA NUKLEONSKE SUSTAVE S $N > 3$

Opisuje se metoda nakupinskog svodenja za rješavanje problema raspršenja sustava s malo nukleona. Metoda svodi Yakubovkove diferencijalne jednadžbe na djelotvorne jednadžbe koje opisuju relativno gibanje nakupina. Predstavljaju se ishodi numeričkog rješavanja zadaće niskoenergijskog raspršenja u sustavima n – ^{3}H, p – ^{3}He, n – ^{3}He, p – ^{3}H i ^{2}H – ^{2}H.