ISSN 1330–0016
CODEN FIZBE7

USE OF REALISTIC INTERACTIONS TO CALCULATE nt − nt,dd − pt and dd − dd SCATTERING OBSERVABLES

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Received 5 January 1999; Accepted 7 June 1999

The four-body equations of Alt, Grassberger and Sandhas are solved for a system of four nucleons, using realistic NN interactions in channels ${}^1S_0$, ${}^3S_1{-}{}^3D_1$, ${}^1P_1$, ${}^3P_0$, ${}^3P_1$ and ${}^3P_2$. The results of the calculation are compared with data for the reactions dd → dd, dd → p^{3}H and n^{3}H → n^{3}H. The calculations indicate that the nucleon-nucleon p-waves have a strong effect on 4N observables, but one finds some disagreement with data that indicates the need for a 3N force or new 2N+3N force models.

PACS numbers: 21.45.+v, 25.10.+s UDC 539.128, 539.142, 539.17

Keywords: nuclear system of four nucleons, realistic NN interactions, channels ${}^1S_0$, ${}^3S_1{-}{}^3D_1$, ${}^1P_1$, ${}^3P_0$, ${}^3P_1$ and ${}^3P_2$, reactions dd → dd, dd → p^{3}H and n^{3}H → n^{3}H

1. *Introduction*

In a recent review article on the three-nucleon continuum [1], one finds that n–d elastic observables (cross sections, vector and tensor polarizations) are insensitive to the choice of realistic NN potential. Beyond the persistent Ay discrepancy at low energy, the agreement between calculations and data is excellent in the energy range up to 65 MeV in the elastic channel. In the present work, we extend our understanding of realistic NN interactions by testing them in the four-nucleon continuum where one expects the observables to be more sensitive to the spin structure of the NN force and its off-shell dependence. Our main goal is to calculate both isospin $\mathcal{I} = 0$ and $\mathcal{I} = 1$ four-nucleon observables in order to understand the low-energy spectrum of such systems, and identify possible failures that may shed light on the NN interaction one uses.

The starting point involves the solution of the equations of Alt, Grassberger and Sandhas [2] for the transition operators involving all $(2)+(2)$ and $(3)+1$ channels. For the local NN potentials, such equations are three-vector-variable integral equations which, after partial wave decomposition, reduce to a set of coupled equations in three continuous variables. Similar equations were recently solved for ^{4}He [3] and n^{3}H elastic scattering [4] below three-body breakup threshold, requiring more than one hundred hours of single processor super-computing time to handle large-dimension matrices ($n > 10^6$). Since scattering calculations require a great number of channels, we follow an approach based on the separable representation of subsystem amplitudes in order to reduce the equations to two or one continuous variable. Although the number of effective $1 + (3)$ channels increases by a factor of three or four, there is a net gain due to the reduction in the dimensionality of the equations and the internal sum of two- and three-body subsystem channels in the kernel of the equations, leading to matrices that are three orders of magnitude smaller. The integral equations we use are the same as in Ref. 5 and result from the modified AGS equations [6] after one has: (a) represented the original NN t-matrix by an operator of rank one; (b) represented the resulting 3N t-matrix by a finite-rank operator and taken as many terms as needed for convergence. Since in the modified AGS equations, the $2 + 2$ subamplitudes are expressed in terms of a convolution integral involving two non-interacting pair-propagators, as first proposed by Fonseca [7], the sole approximation in this approach involves a rank one representation of the 2N t-matrix which may be obtained from the well-known method of Ernest, Shakin and Taylor [8]. The multi-term representation of the 3N t-matrix is done using the EDPE method developed by Sofianos, McGurk and Fiedeldey [9]. This latter approximation for the 3N t-matrix is well under control since one may compare the finite-rank approximation with the original t-matrix results for the 3N observables (cross sections and analyzing powers), and check the convergence rate of 4N observables for increasing rank of the 3N representation.

This method was first used in Ref. 5 to calculate the binding energy of ^{4}He and later confirmed to be accurate by the exact work of Kamada and Glöckle [10]. More recently [11], the results of our calculations for n^{3}H elastic scattering were shown to agree with the results of the Grenoble groups [4], for both Malfliet-Tjon and Argonne V14 potentials taken in 2N partial waves with $j \leq 1^+$ (^{1}S$_0$, ^{3}S$_1-^3$D$_1$).

2. Results

The four-nucleon calculations, we present here, make use of the Paris, Bonn-A, Bonn-B and Argonne V14 potentials in channels ^{1}S$_0$,^{3}S$_1-^3$D$_1$, ^{1}P$_1$, ^{3}P$_0$, ^{3}P$_1$ and ^{3}P$_2$. The first two channels correspond to including all 2N partial waves with $j \leq 1^+$, while the first five channels to those with $j < 1$. As mentioned before, the sole approximation is the use of a rank-one EST expansion of the respective 2N t-matrix in each partial wave. Independently of the number of 2N partial waves that are included for a given NN interaction, one needs to set upper limits for a given subsystem of quantum numbers in order to reach a converged 4N result. In particular, one has to decide the largest 3N total angular momentum J to be included and the rank of the corresponding EDPE expansion. Since preliminary work was first presented in Ref. 12, we do not show here how the 4N results converge with the rank "r" in the EDPE expansion of the 3N t-matrix. Instead, we show in Fig. 1 how dd $\rightarrow$ p^{3}H and dd $\rightarrow$ dd observables change with the number of 3N subamplitudes for increasing 3N total angular momentum J. The N-N potential is Bonn-B in channels $j \leq 1^+$ and all 4N observables are calculated using four-nucleon amplitudes with the total angular momentum up to $\mathcal{J} = 6$, in all corresponding $1 + 3$ and $2 + 2$ channels with the relative orbital angular momentum $\mathcal{L} \leq 5$. The 3N subamplitudes for a given J are calculated using all underlying 3N channels with N − (2N) orbital angular momentum $L \leq 3$ and fixed rank "r" equal six for $J \leq 3/2$, four for $J = 5/2$ and two for $J \geq 7/2$.

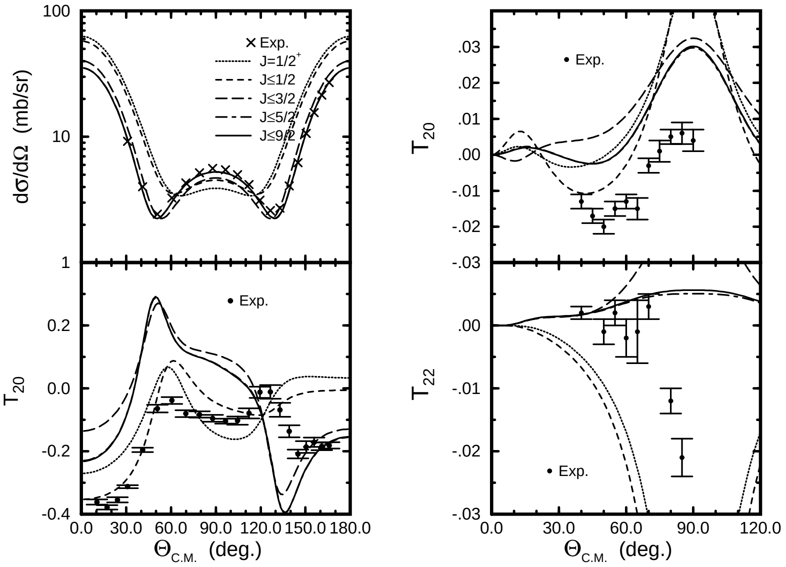

Fig. 1. dd − p^{3}H *(left) and* dd − dd *(right) observables for Bonn-B potential in channels $j \leq 1^+$ as we increase the number of* 3N *subsystem amplitudes from one ($J = 1/2^+$) to ten ($J \leq 9/2$). The deuteron laboratory energy is E_d* = 6.1 MeV *and the experimental points are from Ref. 13.*

From Fig. 1 one learns that 3N quantum numbers other than the triton ($J \neq 1/2^+$) are extremely important to the build up of 4N observables, even at this low energy. This effect is well known [14] and results from the importance of p-wave N − (2N) partial waves to four-particle scattering. Due to the presence of the NN tensor-force, relative N-(2N) d-waves are also strengthened. For this reason, in order to obtain converged 4N results, we include all J up to $7/2^+$.

Since n^{3}H − n^{3}H is an isospin $\mathcal{I} = 1$ reaction, one also needs to include $I = 3/2$ 3N subamplitudes in addition to $I = 1/2$. A similar study indicates that all J up to $5/2^+$ with $I = 3/2$ are needed for convergence. The net effect of adding 3N subamplitudes with higher J or I is shown in Fig. 2 for n^{3}H − n^{3}H observables, using Argonne V14 potential in channels $j \leq 1^+$.

Having fixed all upper limits on 3N and 4N quantum numbers needed for convergence, we now explore the underlying physics by changing the N – N interaction and increasing the number of 2N partial waves included in order to account for the

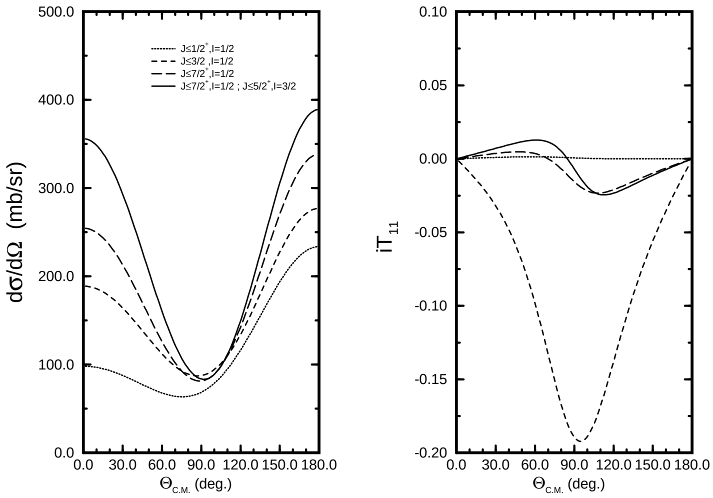

Fig. 2. $n^3H - n^3H$ *observables for Argonne V14 potential in channels* $j \leq 1^+$ *as we increase the number of* 3N *subamplitudes from one* ($J = 1/2^+, I = 1/2$) *to twelve* ($J \leq 7/2^+, I = 1/2; J \leq 5/2^+, I = 3/2$). *The neutron laboratory energy is* $E_n = 3.2$ MeV.

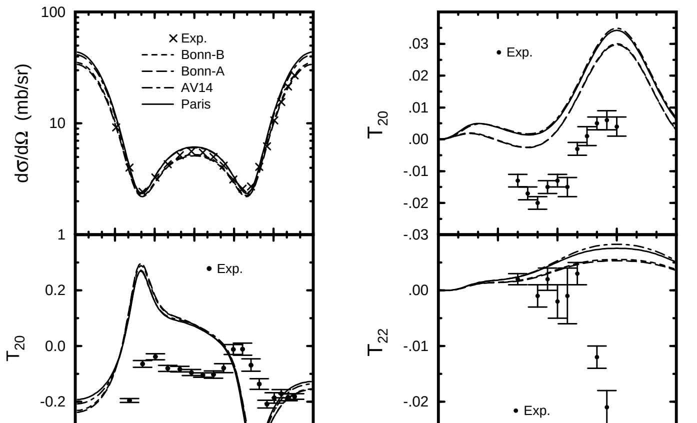

Fig. 3. $dd - p^3H$ *(left) and* $dd - dd$ *(right) observables at* $E_d = 6.1$ MeV *for different realistic interactions in partial waves* $j \leq 1^+$ ($^1S_0, ^3S_1 - ^3D_1$). *The experimental points are from Ref. 13.*

full wealth of the interaction in terms of its spin structure.

The first step is shown in Fig. 3 where dd – p^{3}H and dd – dd observables are calculated for different NN potentials taken in 2N partial waves with $j \leq 1^+$.

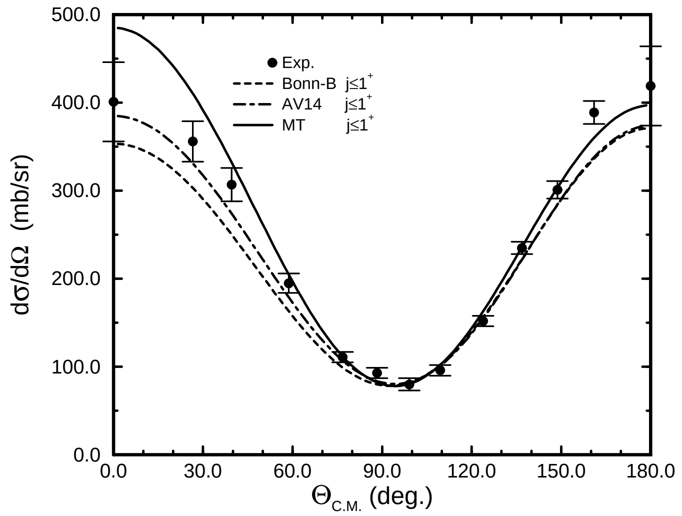

Fig. 4. n^{3}H – n^{3}H *differential cross section at* $E_n = 3.5$ MeV *for different realistic interactions in partial waves* $j \leq 1^+$. *The experimental points are from Ref. 15.*

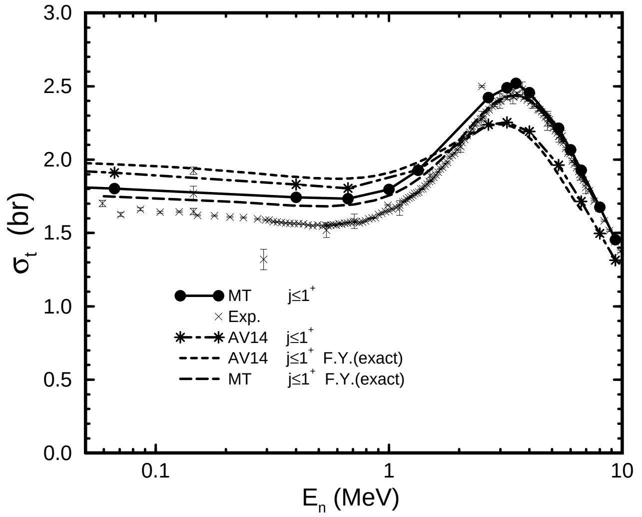

Fig. 5. Total neutron cross section versus energy for both AV14 (stars) and Malfliet-Tjon (circles) potential. The experimental points are from Ref. 16. The results of an exact Faddeev-Yakubovsky calculation are shown for comparison [4,11].

It is interesting to note that inspite of the well-known differences between these interactions, ranging from on-shell fitting to different 2N data (p – p versus n – p), to off-shell missmatch due to choice of underlying physics input (virtual bosons, form factors and nucleonic resonances), and triton binding, the results are very similar but in disagreement with the data. The same disagreement shows up in n^{3}H – n^{3}H elastic scattering as shown in Fig. 4 for the differential cross section at E_n = 3.5 MeV and in Fig. 5 for the total neutron cross section. Both Bonn-B and AV14 miss the data points at the peak of the resonance region, while a pure model potential such as Malfliet-Tjon follows closely the experimental total cross section. Although in Figs. 3 – 5 most of the disagreement may be attributed to the lack of higher N – N partial waves or use of rank-one EST representation of the N-N t-matrix, the purpose of these simplified calculations is threefold: a) to allow benchmark comparisons with other groups; b) to show that most realistic interactions behave in a similar way vis-a-vis the data; c) to set a framework through which one may be able to identify the contribution of higher N – N partial waves to 4N observables in order to learn about the spin structure of the NN interaction and its influence on the spectrum of the 4N system. The results of the first benchmark calculation [11] show that, at these energies, the rank one EST representation of the N – N t-matrix is a very good approximation. In Fig. 5, the exact work of Ciesielski et al. [4,11] is represented by the dashed line for AV14 and the long dashed line for Malfliet-Tjon potentials. Although no exact solution exists at present for the dd – dd and dd – ^{3}H observables, we also expect our method to be adequate.

The second step involves adding higher NN partial waves. This is shown in Fig. 6 for dd – dd and dd – p^{3}H and in Fig. 7 for n^{3}H – n^{3}H, leading to a remarkable but, in some cases, not sufficient improvement vis-a-vis the data. In all cases, we

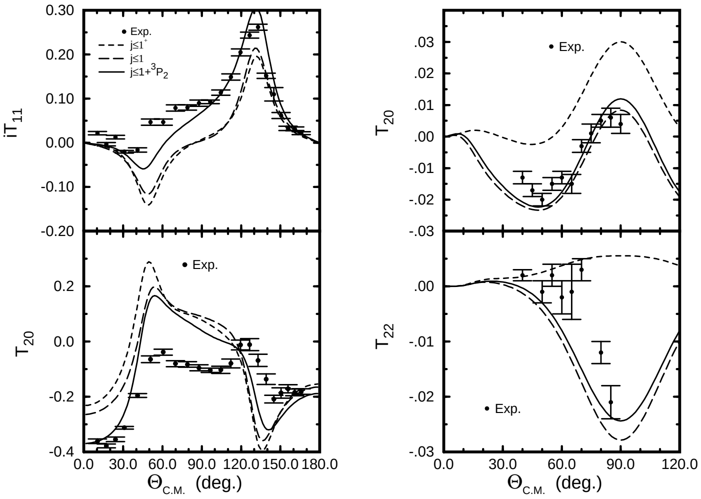

Fig. 6. dd – p^{3}H *(left) and* dd – dd *(right) observables at* $E_d = 6.1$ MeV *for Bonn-B potential in* NN *partial waves ranging from* $j \leq 1^+$ *to* $j \leq 1{+}^3\mathrm{P}_2$. *The data points are from Ref. 13.*

use the Bonn-B potential with all partial waves $j \leq 1$ plus an uncoupled ^{3}P$_2$. Unlike d – p tensor observables, all vector and tensor 4N observables are sensitive to NN p-waves, as noted in Fig. 6. Both iT_{11} in dd – p^{3}H and T_{20} in dd – dd show remarkable improvements when NN p-waves are added, particularly iT_{11} in dd – p^{3}H. Nevertheless, other observables, such as T_{20} in dd – p^{3}H, remain largely unexplained. As for T_{22} in dd – dd, recent measurements [17] indicate that the data

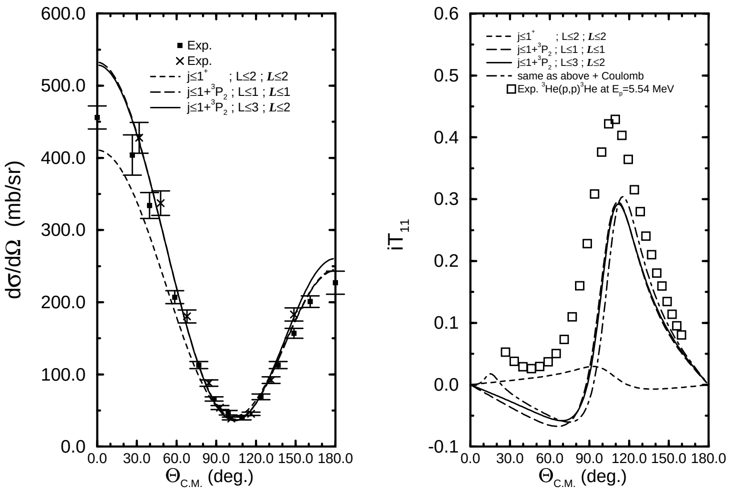

Fig. 7. n^{3}H – n^{3}H *differential cross section (left) and analysing power* iT_{11} *(right) at* $E_n = 6$ MeV *for Bonn-B potential in* NN *partial waves* $j \leq 1^+$ *and* $j \leq 1{+}^3$P$_2$. *The effect of changing the upper limit on* N – (2N) *and* N-(3N) *or* (2N)-(2N) *orbital angular momentum is also shown. The experimental points are from Refs. 18 and 19.*

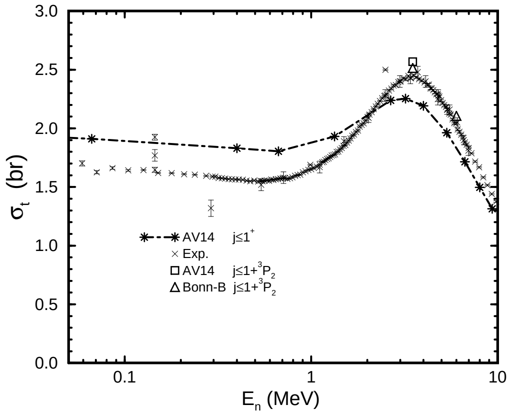

Fig. 8. Total neutron cross section versus energy for both AV14 and Bonn-B potentials taken with different number of NN *partial waves. The data are from Ref. 16.*

do not go as far down as plotted here, which, if confirmed, creates a new challenge for the theory. It is also worth noting that the same calculation produces dd – p^{3}H tensor observables that are one order of magnitude bigger than dd – dd results. Nevertheless, the largest shift due to NN p-waves is observed in Fig. 7 where the calculated experimental differential cross section and analysing power iT_{11} at $E_n = 6$ MeV move towards the data points. This is again confirmed in Fig. 8 for the total cross section as depicted by the squares (ArgonneV14) and the triangles (Bonn-B). To our knowledge, this is the first time NN p-waves can be directly associated to a major change ($> 10\%$) in a few-nucleon cross section. Although NN p-waves are small, their effect in N – (3N) $\mathcal{L} = 1$ phases gets amplified through the N-(2N) $L = 1$ subamplitudes which are known to be responsible for the rise of the total neutron cross section in this energy range.

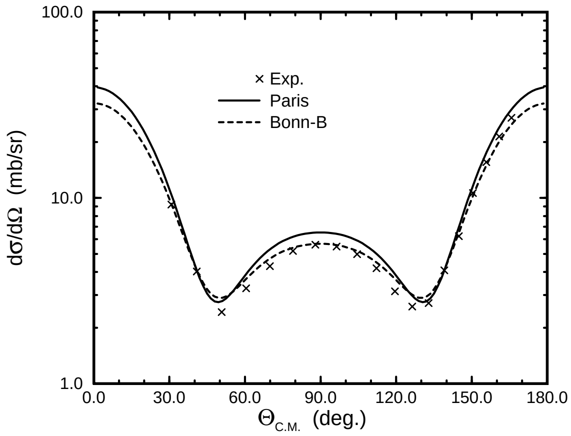

Fig. 9. Differential cross section for dd – p^{3}H *at* $E_d = 6.1$ MeV *for both Bonn-B and Paris potentials taken in* NN *partial waves* $j \leq 1+{}^3\mathrm{P}_2$. *The crosses are experimental points from Ref. 13.*

Finally, in Figs. 9 – 11, we show all dd – p^{3}H and dd – dd observables for Bonn-B and Paris potentials with all p-waves included. Although one finds that the 4N scattering observables are more sensitive to the 2N input than 3N observables, there are no dramatic differences that may dictate a preference between potentials. Everywhere one finds discrepancies that may be attributed to the lack of higher partial waves ($^3\mathrm{F}_2$,$^1\mathrm{D}_2$ or $^3\mathrm{D}_2$), higher rank in the NN t-matrix representation, possible failures of the method used to expand 3N subamplitudes and absence of 3N forces. Although the effect of adding higher partial waves is currently under investigation, we expect their contribution to be small, at least when compared with the p-waves already included. Higher rank in the NN t-matrix respresentation is bound to introduce changes in the 4N observables, but from the benchmark work already performed, it may not be responsible for large effects; on the contrary, we believe the dominant physics at this energy to be well represented by the rank one representation of the NN t-matrix in each partial wave j^p. Given the strong relation between the triton binding energy and n – ^{3}H scattering length, the 3N force is going to have a very strong effect on the n $-$ ^{3}H total cross section at threshold, as already shown [11] for a very simple 3N force model. Nevertheless, in the resonance

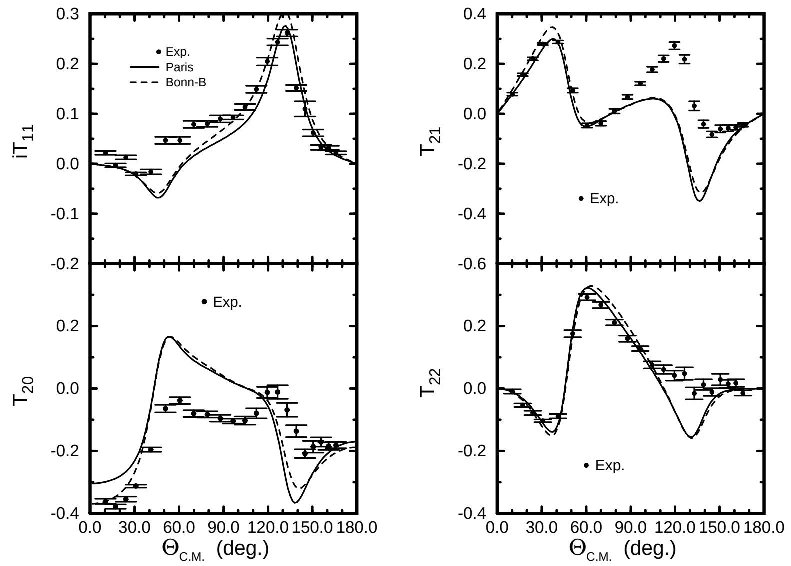

Fig. 10. dd $-$ p^{3}H *tensor analysing powers at* $E_d = 6.1$ MeV *for Bonn-B and Paris potentials taken in* NN *partial waves* $j \leq 1+^3$P$_2$.

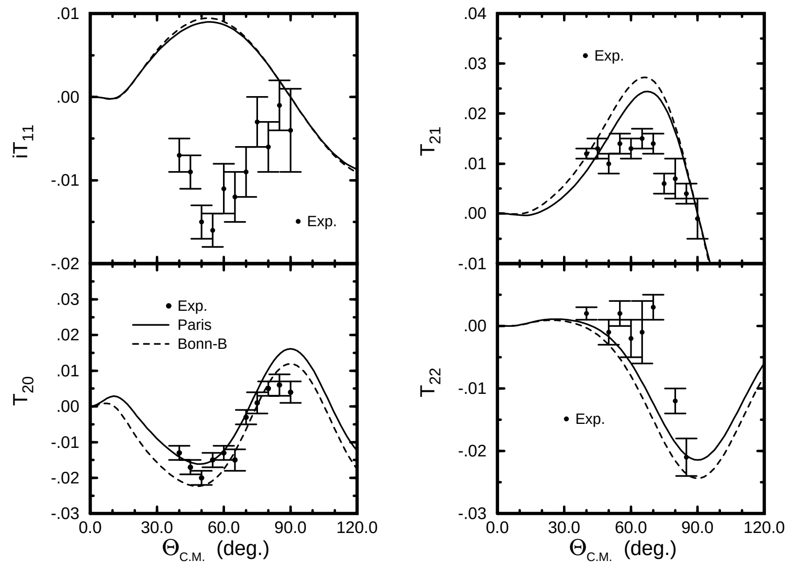

Fig. 11. dd $-$ dd *tensor analysing powers at* $E_d = 6.1$ MeV *for Bonn-B and Paris potentials taken in* NN *partial waves* $j \leq 1+^3$P$_2$

region that dominates the n − ^{3}H elastic scattering around $E_n = 3.5$ MeV, the 3N force has no effect. At present, there are no 4N scattering calculations that make use of realistic 2N + 3N force models, but if one takes into consideration that the 3N force plays a marginal role in low-energy 3N physics, it is reasonable to admit that 4N observables may show little sensitivity to its presence, at least not more than we already observe when we change the 2N forces.

3. Conclusions

We have solved AGS equations for all $1 + 3$ and $2 + 2$ four-nucleon amplitudes and calculated low-energy observables for nt → nt, dd → dd and dd → p^{3}H. For the NN interaction, we use the Paris, Bonn-A, Bonn-B and Argonne V14 potentials. The corresponding NN t-matrix is represented as a rank one operator through the EST expansion method. The results show the shortcomings of 2N force models in describing 4N observables. If present findings are confirmed by future calculations, one may be confronted with very interesting new physics such as the need for new 3N force models and/or a consistent description of 2N and 3N interactions. For now two very important conclusions may be drawn from the present work: 1) the small p-wave channel components of the NN interaction play a crucial role, not only in determining the size of vector and tensor observables in 4N scattering, but also the magnitude of the total cross section for n^{3}H → n^{3}H reaction in the resonance domain; 2) all tested realistic NN potentials are equally good or bad, depending on the observable one focus once attention.

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RAČUN OPSERVABLI nt – nt, dd – pt i dd – dd RASPRŠENJA PRIMJENOM REALISTIČNIH MEĐUDJELOVANJA

Rješavaju se jednadžbe Alta, Grassbergera i Sandhasa za četiri tijela za sustav četiri nukleona uz primjenu realističnih pretpostavki o NN-međudjelovanju u kanalima $^1S_0$, $^3S_1$–$^3D_1$, $^1P_1$, $^3P_0$, $^3P_1$ i $^3P_2$. Ishodi računa uspoređuju se s podacima za reakcije dd $\to$ dd, dd $\to$ p^{3}H i n^{3}H $\to$ n^{3}H. Računi pokazuju da nukleon – nukleon p-valovi snažno utječu na 4N opservable, ali neka neslaganja s podacima ukazuju na mogućnost pogreške u poznavanju stvarnih međudjelovanja.