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B-MESON – QUARK COUPLING CONSTANT AND DECAY WIDTH OF B*-MESON

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The pion – quark coupling constant ($g_{\pi qq}$) and the B-meson – quark coupling constant ($g_{\mathrm{B}^- qq}$) have been found in the processes $\mathrm{B}^{*-}\to\mathrm{B}^-\pi^0$ and $\mathrm{B}^{*-}\to\mathrm{B}^-\gamma$. Their decay widths have been calculated through the direct coupling of π^0 and B^- with quarks which are static inside B^{*-} meson.

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1. *Introduction*

We estimate the values of coupling constants from the decay widths of B-mesons. We consider the specific matter of decays such as $\mathrm{B}^{*-}\to\mathrm{B}^-\pi^0$ and $\mathrm{B}^{*}\to\mathrm{B}^-\gamma$. In the radiative decay of a vector meson (here B^*), the initial $q\overline{q}$ pair is converted into final $q\overline{q}$ pair(π^0) by the spin flip from the $s = 1$ state to the $s = 0$ state with the emission of a photon. The process follows according to the conventional quark model of the radiative decays of vector mesons.

The fact that $q\overline{q}$ pair contributes only 25% to the pion is acceptable [1] and the chiral invariance of the QCD Lagrangian under the condition $m_u, m_d \to 0$ gives rise to a phenomenological model of hadrons in which the pion field, treated as an elementary field Goldstone boson, couples directly to the quark fields [2]. We impose a similar status to the B$^-$ particle.

Suzuki and Bhadhuri [3] applied this model to vector-meson decay and used a free propagator in the static approximation to find pion – quark coupling constant from ρ meson decay. We shall study B^{*-}-meson decay using a free propagator and an effective quark mass to account for the effect of confinement.

2. Theory

In the phenomenological model of hadrons, if the quarks are treated as zero mass Dirac particles confined in a scalar potential $V(r)$, the axial vector current carried by the quarks is

$$A_\mu^i = \frac{1}{2}\overline{q}\gamma_5\gamma_\mu\tau_i q \tag{1}$$

This axial current is not conserved even if the quark masses are taken to be zero, as indicated by its four divergence

$$\partial_\mu A_\mu^i = iV(r)\overline{q}\gamma_5\tau^i q \tag{2}$$

Since the QCD Lagrangian, in the limit of zero quark mass, is globally invariant under chiral transformation in the (u,d) sector, the axial vector current is to be conserved in this model. Hence a pseudoscalar isovector field $\phi^i(x)$ [4] is introduced to conserve axial vector current in this phenomenology.

In the MIT Bag model, quarks are constituent particles of hadrons. Spectra may nicely be computed by perturbation method through gluonic exchange. Inside the bag pion cannot exist because of small region. Axial vector current constructed from massless quarks is given by

$$A_\mu^i = \frac{1}{2}\overline{q}\gamma_5\gamma_\mu\tau^i q. \tag{3}$$

Outside the bag, there is a pion field which carries an axial current

$$A_\mu^i = -f_\pi\partial_\mu\phi_\pi^i. \tag{4}$$

Applying bag boundary conditions which prevents the tunnelling of colour current from the bag, the total axial vector current is

$$A_\mu^i = \frac{1}{2}\overline{q}\gamma_\mu\gamma_5\tau^i q - f_\pi\partial_\mu\phi^i \tag{5}$$

The pion decay constant is $f_\pi = 93$ MeV. Inserting $\partial_\mu A_\mu^i = 0$ yields the equation

$$\partial_\mu^2\phi^i = (\frac{i}{f_\pi})V(r)\overline{q}\gamma_5\tau^i q. \tag{6}$$

Thus, for a pion field with mass m_π, we have

$$(\partial_\mu^2 + m_\pi^2)\phi^i = \frac{i}{f_\pi} V(r)\overline{q}\gamma_5\tau^i q, \tag{7}$$

yielding the usual PCAC relation $\partial_\mu A_\mu^i = m_\pi^2 f_\pi \phi^i$ at the quark level. The above Eq. (7) can be deduced from a Lagrangian given by:

$$\begin{aligned} L &= \overline{q}\gamma_\mu\partial_\mu q - \frac{1}{2}(\partial_\mu\phi^i)^2 + V(r)q\overline{q} + \frac{1}{2}m_\pi^2\phi_i^2 \\ &\quad - \frac{\mathrm{i}}{}f_\pi V(r)\overline{q}\gamma_5\tau^i q\phi^i - \frac{1}{4}f_\pi^2 V(r)q\overline{q}\phi_i^2 + O(\frac{1}{f_\pi^3}). \end{aligned} \tag{8}$$

The quark-pion interaction is of the form

$$L_{int} = \frac{iV(r)}{f_\pi}\overline{q}\gamma_5\tau^i q\phi^i. \tag{9}$$

To work with the above Lagrangian, it is very diffcult to conserve linear momentum and to remove divergencies for sharp boundaries [2] when quark propagator includes the summation of all possible intermediate states. Calculation becomes also much simpler if one assumes

$$L_{int} = ig_{qqM}\overline{q}\gamma_5\tau_i q\phi^i, \tag{10}$$

where g_{qqM} denotes the effective quark meson coupling strength that is no larger surface peaked. Comparing the interaction term in equation (9) with equation (10), we have

$$g_{qq\pi} = f_\pi^{-1}\langle V(r)\rangle \tag{11}$$

In the MIT bag model, the quark can be considered to be moving in the scalar potential $V(r)$ obeying the Dirac equation

$$-\mathrm{i}\,\gamma_\mu\partial_\mu\Psi + V(r)\Psi = 0, \tag{12}$$

where $V(r) = \frac{1}{2}\frac{G(r)}{F(r)}$, $\Psi = F^2 q$ and

$$L_{bag} = [\frac{1}{2}\mathrm{i}\,\overline{q}\gamma_\mu\partial_\mu q - B]F(r) - \frac{1}{2}q\overline{q}G(r). \tag{13}$$

B is the universal bag pressure, $F = \Theta(R - r)$ and $G(r) = -dF/dR$. Here we use static approximation and since we finally take the ratio of the decay widths of $\mathrm{B}^{*-}\to\mathrm{B}^-\gamma$ and $\mathrm{B}^{*-}\to\mathrm{B}^-\pi^0$, we hope that the errors due to the static approximation may cancel out.

3. Transition amplitude

3.1. $B^{*-} \to B^- \pi^0$ transition

For the process $B^{*-} \to B^- \pi^0$, the transition amplitude is calculated from the Feynman diagram of Fig. 1. The amplitude for the process using free quark propagator is given by:

$$M_{fi} = \sqrt{2} g_{B^- qq} g_{\pi^0 qq} \overline{V}^{s'}(p') \left[\gamma_5 \frac{(\not p - \not k) + m_q}{(p-k)^2 + m_q^2} \gamma_5 - \gamma_5 \frac{(\not p - \not k') + m_q}{(p-k')^2 + m_q^2} \gamma_5 \right] U^s(p). \tag{14}$$

If the quarks are taken on the mass shell, then $p^2 = m_q^2$. After calculation of the trace, one obtains

$$\Sigma |M_{fi}|^2 = \frac{16 g_{\pi^0 qq}^2 g_{B^- qq}^2 ({m_{\pi^0}}^2 - 2 p_0 w_k)^2}{m_q^2 ({m_{\pi^0}}^2 - 2\vec{p}\cdot\vec{k})(m_{B^-} - 2\vec{p}\cdot\vec{k}')^2} [|p_0|^2 |k|^2 - (\vec{p}\cdot\vec{k})^2], \tag{15}$$

where $p_0^2 = \vec{p}^2 + m_q^2$ and $w_k^2 = \vec{k}^2 + m_{\pi^0}^2$. Since the momentum distribution of the quarks in the vector meson is not known, this expression is difficult to be calculated unless some specific model is chosen. Alternatively, in the static approximation [3], we take the ratio of the decays $B^{*-} \to B^- \gamma$ and $B^* \to B^- \pi^0$ to cancel out the errors of static approximation. With $\vec{p} \to 0$, the above equation reduces to

$$\Sigma |M_{fi}|^2 = \frac{16 g_{\pi qq}^2 g_{B^- qq}^2 |\vec{k}|^2}{(m_{B^-}^2 - 2 m_q w_k')^2}. \tag{16}$$

3.2. $B^{*-} \to B^- \gamma$ transition

A similar calculations can be done for the process $B^{*-} \to B^- \gamma$ with the Feynman diagram of Fig. 2. The matrix element for the $B^{*-} \to B^- \gamma$ decay can be written as:

$$\Sigma_{s,s'} |M_{fi}|^2 = \frac{32}{9} e^2 g_{qqB^-}^2 \frac{w_k^2 - m_{B^-}^2}{(2 m_q m_{B^-} - m_{B^*}^2)^2}. \tag{17}$$

With the kinematical relations, the above equation reduces to

$$\Sigma |M_{fi}|^2 = \frac{2 e^2 g_{B^- qq}^2}{9 m_q^2}. \tag{18}$$

Now, using the fine structure constant $\alpha = \frac{1}{137}$, the constituent quark masses $m_b = 4500$ MeV, $m_u = 350$ MeV and the pion and B meson masses as: $m_{\pi^0} = 135$ MeV, $M_{B^-} = 5279$ MeV and $m_{B^{*-}} = 5324$ MeV [5], we have obtained

$$|\vec{k}|_{B^{*-} \to B^- \pi^0} = \sqrt{E_B^2 - m_{B^-}^2} = \sqrt{\frac{m_{B^{*-}}^2 + m_{B^-}^2 - m_{\pi^0}^2}{2 m_{B^{*-}}} - m_{B^-}^2} = 126.73 \text{MeV}. \tag{19}$$

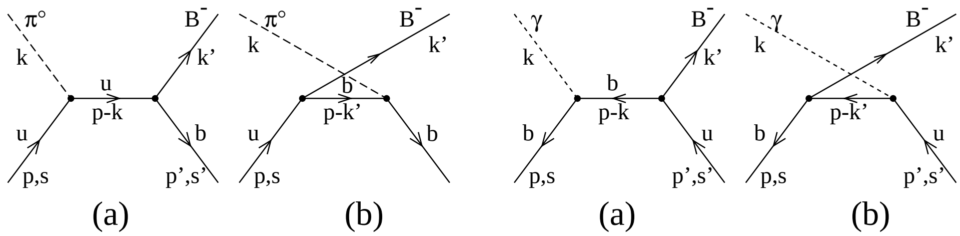

Fig. 1. Direct and exchange diagrams for $B^{*-}\rightarrow B^{-}\pi^{0}$*; where s and* $s^{'}$ *denote the spins of the quark and antiquark, respectively, and k and* $k^{'}$ *are the four momenta of the* π^0 *and* B^- *mesons, respectively.*

Fig. 2 (right). Direct and exchange diagrams for $B^{*-}\rightarrow B^{-}\gamma$ *decay.*

Also

$$|\vec{k}|_{B^{*-}\rightarrow B^{-}\gamma} = \frac{m^2_{B^{*-}} - m^2_{B^-}}{2m_{B^-}} = 45.19 \text{ MeV}. \tag{20}$$

We have also determined r from hydrogen like wave function for the B^{*-} meson. The wave functions is

$$\psi(r) = (\frac{1}{\pi a^3})^{1/2} \exp(-\frac{r}{a}), \tag{21}$$

where $a = 2a_0 = \frac{2}{m\alpha}$. At $r = 0$, $\psi(0) = (\frac{1}{\pi a^3})^{1/2} \approx 2.02 \times 10^3 \text{MeV}^{3/2}$. Here we have assumed the reduced mass (m) of the constituent quarks to be 324 MeV.

TABLE 1. Wave functions for B^{*-}*-meson.*

Separation of quarks (r in fm)	0.25	0.30	0.35	0.40	0.45	0.50
$\|\psi(0)\|^2$ (in 10^6 MeV^3)	20.287	11.74	7.39	4.95	3.47	2.54

where $|\psi(0)|^2 = (\frac{\lambda}{\pi})^{3/2} = [\frac{3}{8}\frac{(197.3)^2}{\pi r^2}]^{3/2}$ MeV^3 [6].

Using the normalisation for the wave function ψ and comparing our above results for $\psi(0)$, we have obtained $r = 0.4$ fm [3,6,7]. Now, the g_{B^-qq} coupling constant can be determined from the Goldberger – Treiman (GT) relation as:

$$g^2_{B^-qq}/4\pi = \frac{m_b^2}{m_{B^-} f_{B^-}/4\pi} = 1.633, \tag{22}$$

where $f_{B^-} = 187$ MeV [8] is the decay constant of the B^- meson.

4. *Determination of the coupling constant* $g_{\pi qq}$

The coupling constant $g_{\pi qq}$ can be obtained from

$$g^2_{\pi qq}/4\pi = \frac{\alpha \mathrm{B}^2 |\vec{k}|_{\mathrm{B}^{*-}\to \mathrm{B}^-\gamma}}{36 m_b^2 |\vec{k}|^2_{\mathrm{B}^{*-}\to \mathrm{B}^-\pi^0}}, \tag{23}$$

where $\mathrm{B}^2 = (m^2_{\mathrm{B}^-} - 2m_q m_{\mathrm{B}^-} + m_q^2)^2$ MeV^2. Therefore, the coupling constant $g_{\pi qq}$ is:

$$g^2_{\pi qq}/4\pi = 0.011. \tag{24}$$

5. $g_{\mathrm{B}^- qq}$ *coupling constant*

In this section, we first determine the decay widths of $\mathrm{B}^{*-}\to \mathrm{B}^-\gamma$ and $\mathrm{B}^{*-}\to \mathrm{B}^-\pi^0$ decays by using the coupling constant $g_{\mathrm{B}^- qq} = 1.633$, which is obtained by GT relation and then using these decay widths, we calculate the coupling constant $g_{\mathrm{B}^- qq}$ and compare these results with the earlier estimation for the coupling constant $g_{\mathrm{B}^- qq}$ from GT relation.

5.1. *Decay width of* B^{*-}*-meson*

5.1.1. *Decay width of* $B^{*-}\to B^-\pi^0$

The decay width of $\mathrm{B}^{*-}\to \mathrm{B}^-\pi^0$ decay can be obtained from:

$$\Gamma_{\mathrm{B}^{*-}\to \mathrm{B}^-\pi^0} = \Sigma_{\ s,s'} |M_{fi}|^2 \vec{k} |\psi(0)|^2 / M_{B^{*-}}\ . \tag{25}$$

Using Eqs. (16) and (19), we have obtained the decay width as:

$$\Gamma_{\mathrm{B}^{*-}\to \mathrm{B}^-\pi^0} = \frac{16 g^2_{\mathrm{B}^- qq} g^2_{\pi qq} |\vec{k}|^3 |\psi(0)|^2}{\mathrm{B}^2 M^{*-}_{\mathrm{B}}}. \tag{26}$$

Using the average value of $|\psi(0)|^2$, we have obtained the decay width of $\mathrm{B}^{*-}\to \mathrm{B}^-\pi^0$ to be 0.606 MeV.

The values of the coupling constant $g_{\mathrm{B}^- qq}$ from the $\mathrm{B}^{*-}\to \mathrm{B}^-\pi^0$ decay process are shown in Table 2.

5.1.2. *Decay width of* $B^{*-}\to B^-\pi^0$

The decay width of $\mathrm{B}^{*-}\to \mathrm{B}^-\gamma$ decay can be obtained from Refs. 9 and 10

$$\Gamma(\mathrm{B}^{*-} \to \mathrm{B}^-\gamma) = \Sigma |M_{fi}|^2 \vec{k} |\psi(0)|^2 / M_{B^{*-}}\ . \tag{27}$$

TABLE 2. Coupling constant from the $B^{*-} \to B^- \pi^0$ *decay.*

$\lvert\psi(0)\rvert^2$ (in 10^6 MeV^3)	20.29	11.74	7.39	4.95	3.47	2.54
$g^2_{B^- qq}/4\pi$	1.036	1.79	2.84	4.09	6.04	7.34

TABLE 3. Coupling constant from the $B^{*-} \to B^- \gamma$ *decay.*

$\lvert\psi(0)\rvert^2$ (in 10^6 MeV^3)	20.29	11.74	7.39	4.95	3.47	2.54
$g^2_{B^- qq}/4\pi$	1.035	1.79	2.84	4.08	6.03	7.34

Therefore, using Eq. (17) and the value of coupling constant obtained from GT relation $g_{B^- qq} = 1.633$, we have obtained the decay width as:

$$\Gamma(B^{*-} \to B^- \gamma) = 2.256 \text{ keV}. \tag{28}$$

Remembering that K for the decay $B^{*-} \to B^- \pi^0 = 126.73$ MeV, we finally arrive at the following values for the coupling constant $g_{B^- qq}$ shown in the Table 3.

TABLE 4. Coupling constant from the $B^{*-} \to B^- \pi^0$ *and* $B^{*-} \to B^- \pi^0$ *decays.*

r (in fm)	0.25	0.30	0.35	0.40	0.45	0.50
$g^2_{B^- qq}/4\pi$ from $B^{*-} \to B^- \pi^0$	1.036	1.790	2.840	4.089	6.040	7.340
$g^2_{B^- qq}/4\pi$ from $B^{*-} \to B^- \gamma$	1.035	1.788	2.840	4.080	6.030	7.340

To compare the coupling constants obtained from the $B^{*-} \to B^- \pi^0$ and $B^{*-} \to B^- \gamma$ decays, we present the results in Table 4. From the above results, we have observed that the coupling constant $g_{B^- qq}$ obtained from the GT relation is in a quite good agreement with that obtained from the decay processes $B^{*-} \to B^- \pi^0$ and $B^{*-} \to B^- \pi^0$ when we take the bag of radius 0.3 fm.

6. *Results and discussion*

From our analysis, resonably consistent values of the coupling constants of quarks with π^0 and B^- meson have been obtained. Under the static approximation and confinement of quarks within the mesonic volume, an interesting feature of our study is that for a bag radius of about 0.3 fm, the results are in confirmity with the calculations from Goldberger-Triemann relation.

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KONSTANTA VEZANJA B-MESON – KVARK I ŠIRINA RASPADA B*-MESONA

Određuju se konstante vezanja pion – kvark ($g_{\pi qq}$) i B-mezon – kvark ($g_{\mathrm{B}^- qq}$) u procesima $\mathrm{B}^{*-} \to \mathrm{B}^- \pi^0$ i $\mathrm{B}^{*-} \to \mathrm{B}^- \gamma$. Njihove se širine raspada računaju preko izravnog vezanja π^0 i B^- s kvarkovima koji miruju u B^{*-} mezonu.