ISSN 1330–0016
CODEN FIZBE7

# MASS AND FLAVOUR SPINORS FROM THE DIRAC EQUATION WITH TWO MASS PARAMETERS

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Received 27 January 1999; Accepted 6 April 1999

The choices of the standard equations for massive, massless and tachyonic fermions are examined, on the grounds of desirable features related to the symmetry operations. Linear superpositions of different mass states are revisited, and some possible probability interpretations are suggested.

PACS numbers: 03.65.Pm, 11.30.-j UDC 530.125, 539.124

Keywords: Dirac equation with two mass states, massive, massless and tachyonic fermions, symmetry operations

## 1. Introduction

In previous papers [1–6], the Dirac equation with two mass parameters and related topics were discussed. The approach was used to derive standard equations for massive, massless and tachyonic fermions. In particular, a massless equation was obtained, which differs from the usual one and does not produce a superfluous conserved current. In this paper, the aforementioned results are reformulated and justified on the grounds of desirable features related to the active symmetry operations (time reversal, spatial parity, etc.). It is seen that these operations (together with physical evidence and/or assumptions) are instrumental in determining which standard equations should be adopted for the description of fermionic matter. The outlined treatment is done before second quantization. Also, in Section 5, the flavoured neutrino model of Ref. 3 is revisited, with the aim of suggesting some probability interpretations.

Notation is rather conventional throughout the paper. Specifically, and unless otherwise noted, Greek (Latin) indices run through the values $0,1,2,3$ $(1,2,3)$ and the summation convention is applied to repeated up and down labels. Units are such that $\hbar = c = 1$. An attempt is made at distinguishing powers from superscripts: for instance, $(m)^2$ and $|v|^2$ are powers, while $x^2$ indicates a specific variable with superscript 2.

## 2. Dirac equation with two mass parameters

In a frame of reference $X$ of real spacetime coordinates $x = \{x^\lambda\}$ and pseudoeuclidean metric $g^{\mu\nu} = \mathrm{diag}\{+1,-1,-1,-1\}$, the Dirac equation with two mass parameters [1] may be written as follows

$$\not{P}\,\Upsilon(x) = \mathrm{M}\Upsilon(x)\,, \tag{1}$$

with

$$\not{P} = \mathrm{i}\gamma^\alpha \partial_\alpha \tag{2}$$

and

$$\mathrm{M} = \frac{1}{2}\left[(a+b)I - (a-b)\varepsilon\gamma^5\right], \tag{3}$$

where $a,b$ are complex constants and $\Upsilon(x)$ is a complex four-spinor. The Dirac matrices $\gamma^\lambda$ (in a fixed chosen representation) obey the usual rules

$$\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}I\,, \qquad (\gamma^\mu)^\dagger = \gamma^0\gamma^\mu\gamma^0\,, \tag{4}$$

with $I$ being the $4\times 4$ identity matrix. The matrix $\gamma^5 = \mathrm{i}\gamma^0\gamma^1\gamma^2\gamma^3$ is hermitian and unitary, and anticommutes with all $\gamma^\lambda$. For general reference on the Dirac equation and related topics, see, for instance Refs. 7–17. The value of the sign $\varepsilon = (-1)^{T+S}$ depends on the frame of reference [1,18]. Namely, the time-index $T$ and the space-index $S$ of $X$ are so defined: $T = 0$ if $t = x^0$ runs forward ($T = 1$ otherwise) and $S = 0$ if $s = \{x^\ell\}$ is a right-handed triplet ($S = 1$ otherwise). It is also reminded:

$$(\gamma^\mu)^* = B^\dagger\gamma^\mu B\,, \tag{5}$$

where $B$ is the (fixed chosen) unitary matrix associated with the charge conjugation operation [2], and the asterisk denotes complex conjugation.

The solutions $\Upsilon(x)$ of Eq. (1) are eigenstates of the squared four-momentum (S.F.M.) operator [1]

$$-\partial_\alpha g^{\alpha\beta}\partial_\beta \tag{6}$$

for the eigenvalue $ab$. Six cases can be identified, labeled with Roman numerals: (I) $a = 0 = b$; (II) $a\neq 0, b = 0$; (III) $a = 0, b\neq 0$; (IV) $ab > 0$; (V) $ab < 0$; (VI) $ab \notin \Re$.

Case (VI) is of unclear interpretation, and will not be considered here. For each of the other cases, one of the objectives of a previous paper (Ref. 1) was that of obtaining an equivalent "standard" equation which would simplify Eq. (1). This was accomplished by rewriting Eq. (1) as

$$\not{P}\,\Psi(x) = \mathrm{N}\Psi(x)\,, \tag{7}$$

by means of some appropriate linear transformation

$$\Upsilon(x) = H\Psi(x)\,. \tag{8}$$

Here, $H$ is an invertible $4\times 4$ matrix, chosen as to produce Eq. (7) with a simple enough (and "convenient" enough) mass matrix N. The mass matrix N should be simpler and more convenient than the original mass matrix M: for a discussion of the relevant criteria, see Sections 3 and 4. Due to Eqs. (1)–(3), (7) and (8), both $H$ and N are linear combinations of $I$ and $\varepsilon\gamma^5$. Specifically, $H$ may be expressed in terms of two complex parameters $e$ and $d$ as follows:

$$H = eI + d\varepsilon\gamma^5\,, \qquad e \neq \pm\, d\,, \tag{9}$$

and N results as

$$\mathrm{N} = (\gamma^0 H\gamma^0)^{-1}\mathrm{M}H\,. \tag{10}$$

In cases (II)–(VI), the covariance of Eqs. (1), (7) is shown by the usual methods employed for the massive Dirac equation [13]. Thus, the parameters $a, b, e, d$ are treated as scalars, and the spinorial transformations under the Poincaré group (passive transformations) may be realized as outlined in Ref. 2.

The standard equations for the various cases [1] are described in the remainder of this section. All quantities referring to a specific case are labeled with an ordinary number in place of the corresponding Roman numeral; the use of Roman numerals is retained in text, as to avoid confusion with references to numbered equations and to the bibliography. For each case, the values of the parameters $a$ and $b$ will be considered to be fixed.

**Case (I)** $[a_{(1)} = 0 = b_{(1)}]$

The only equation of type (7) obtainable by means of transformations like (8) is as follows:

$$\not{P}\,\Psi_{(1)}(x) = 0\,. \tag{11}$$

The form of Eq. (11) is identical to that of Eq. (1) in this case. Equation (11), in spite of its history, does not appear to be directly usable, since it has a redundant current: it will be here disregarded. However, see Refs. 1 and 2 for alternative applications.

**Case (II)** $[a_{(2)} \neq 0, b_{(2)} = 0]$

In this case, the original equation reads

$$\not{P}\,\Upsilon_{(2)}(x) = \frac{a_{(2)}}{2}(I - \varepsilon\gamma^5)\Upsilon_{(2)}(x)\,. \tag{12}$$

By means of transformations like (8), all forms of type (7) which can be obtained are given by

$$\not{P}\Psi_{(2)}(x) = \frac{m_{(2)}}{2}(I - \varepsilon\gamma^5)\Psi_{(2)}(x)\,, \tag{13}$$

where $m_{(2)}$ is an arbitrary nonvanishing complex constant. Equation (13) is in standard form if the constant $m_{(2)}$ is chosen to have some fixed real value [2], positive or negative.

For all possible (complex) values of $m_{(2)}$, the conserved current (aside from a multiplicative constant) may be expressed as follows:

$$j^{\mu}_{(2)}(x) = \overline{\Lambda}_{(2)}(x)\gamma^{\mu}\Lambda_{(2)}(x)\,, \tag{14}$$

where a bar over a spinor indicates its Dirac adjoint, and where $\Lambda_{(2)}(x)$ is the left-handed component of $\Psi_{(2)}(x)$ defined by [1]

$$\Lambda_{(2)}(x) = \frac{1}{2}(I - \varepsilon\gamma^5)\Psi_{(2)}(x)\,. \tag{15}$$

The normalization

$$\int j^{0}_{(2)}(t,s)\,\mathrm{d}s = 1 \tag{16}$$

infers a one-particle probability significance for the spinor $\Lambda_{(2)}(x)$: this is a consistent interpretation if $\Lambda_{(2)}(x)$ is regarded as the physically relevant part [1,2] of $\Psi_{(2)}(x)$. The integration extends to the entire three-dimensional space, taken with positive orientation (equivalently, only the volume where $\Lambda_{(2)}(x)$ is nonvanishing at time $t$ needs to be considered); d$s$ denotes the volume element.

**Case (III)** $[a_{(3)} = 0, b_{(3)} \neq 0]$

This case is similar to the previous one. With considerations like those of case (II), the standard form is

$$\not{P}\Psi_{(3)}(x) = \frac{m_{(3)}}{2}(I + \varepsilon\gamma^5)\Psi_{(3)}(x)\,, \tag{17}$$

with $m_{(3)}$ chosen to have some fixed real value, positive or negative.

**Case (IV)** $[a_{(4)}b_{(4)} > 0]$

In this case the standard form is of the Dirac type [1]. That is, either

$$\not{P}\Psi_{(4)}(x) = m_{(4)}\Psi_{(4)}(x)\,, \qquad m_{(4)} = [a_{(4)}b_{(4)}]^{1/2} > 0\,, \tag{18}$$

or

$$\not{P}\Psi_{(4)}(x) = -m_{(4)}\Psi_{(4)}(x)\,. \tag{19}$$

However, this case allows much freedom; for example:

$$\not{P}\Psi_{(4)}(x) = \mathrm{i}m_{(4)}\varepsilon\gamma^5\Psi_{(4)}(x) \tag{20}$$

is another simple equation of type (7) which can be obtained by means of a transformation (8). The next section will discuss why this choice and other possible ones are less convenient than (18) or (19). For Eqs. (18)–(20), and for similar choices such that

$$\mathrm{N}^{\dagger}_{(4)} = \gamma^0\mathrm{N}_{(4)}\gamma^0\,, \tag{21}$$

the conserved current (aside from a multiplicative constant) has the formal expression

$$j^{\mu}_{(4)}(x) = \overline{\Psi}_{(4)}(x)\gamma^{\mu}\Psi_{(4)}(x)\,. \tag{22}$$

The normalization

$$\int j^0_{(4)}(t,s)\,\mathrm{d}s = 1 \tag{23}$$

induces a one-particle probability interpretation for the spinor $\Psi_{(4)}(x)$.

**Case (V)** $[a_{(5)}b_{(5)} < 0]$
In this case, the standard equation [1] is taken to be

$$\not{P}\Psi_{(5)}(x) = -m_{(5)}\varepsilon\gamma^5\Psi_{(5)}(x)\,, \qquad m_{(5)} = [-a_{(5)}b_{(5)}]^{1/2} > 0\,, \tag{24}$$

or

$$\not{P}\Psi_{(5)}(x) = m_{(5)}\varepsilon\gamma^5\Psi_{(5)}(x)\,. \tag{25}$$

As in the previous case, other simple choices could be proposed; for example:

$$\not{P}\Psi_{(5)}(x) = \mathrm{i}m_{(5)}\Psi_{(5)}(x)\,. \tag{26}$$

The discussion is postponed to Section 4. For Eqs. (24)–(26), and for similar choices such that

$$\mathrm{N}^{\dagger}_{(5)} = -\gamma^0\mathrm{N}_{(5)}\gamma^0\,, \tag{27}$$

the conserved current (aside from a multiplicative constant) has the formal expression

$$j^{\mu}_{(5)}(x) = -\varepsilon\overline{\Psi}_{(5)}(x)\gamma^{\mu}\gamma^5\Psi_{(5)}(x)\,. \tag{28}$$

For general reference on tachyons, see, for instance Refs. 19 and 20. Also, for a variety of opinions: [21–27].

## 3. Massive and massless cases

This section will discuss cases (II) through (IV); case (V) is referred to the next section.

**Case (IV)** $[a_{(4)}b_{(4)} > 0]$

The massive Dirac equation (18) or (19) is one of the best known equations in physics, and this makes it an ideal choice for case (IV). More precisely, the Dirac equation is equipped with a number of active symmetry operations (charge conjugation, time reversal, etc.) which have standard mathematical expressions and physical meanings, and leave the equation invariant. As a matter of practice, it is not convenient to change the expressions of these operations. Thus, for example, consider the standard spatial parity [2,13] transformation (here displayed for some generic four-spinor $\Psi$):

$$\widetilde{\Psi}(x) = (\text{global phase factor})\, \mathrm{i}\gamma^0 \Psi(t, -s)\,. \tag{29}$$

Notice that each of Eqs. (18), (19) is invariant under (29), while Eq. (20) is not: in fact, it can be verified that (18), (19) are the only possible forms in case (IV) to manifest invariance under (29). In a rather tautological sense, this consideration makes either Eq. (18) or Eq. (19) the most convenient choice. The underlying assumption is that, for each of the cases (I)–(VI), all possible forms (7) represent the same physical reality, because of the simple linear relation implied by Eq. (8) in each case. Hence, if Eq. (20) or another form were here chosen in place of (18) or (19), the expression of the parity transformation would have to be modified (either directly, by means of a definition different from the one above, or, perhaps, in a more indirect fashion).

It is reminded that each of Eqs. (18), (19) is invariant under the standard charge conjugation [2,13] operation

$$\widetilde{\Psi}(x) = (\text{global phase factor})\, \gamma^5 B \Psi^*(x)\,. \tag{30}$$

Thus, $\Psi_{(4)}(x)$ may be taken to represent matter (e.g., electrons) as well as antimatter (e.g., positrons). In either situation, Eq. (18) is usually preferred to Eq. (19): see Ref. 10. See Section 68 of Ref. 28 concerning the issue of charge conjugation and intrinsic parities; also, Chapter XX of Ref. 13. Note, however, that a proper treatment of charge conjugation would require second quantization [2,14].

**Remark.** In the absence of interactions and second quantization, the distinction between matter and antimatter is formally uneasy; a possible definition is given in the following. An equation and its spinor are said to represent (or refer to) matter if the equation's positive energy solutions describe matter. A positive energy solution is here defined as an equation's solution which is eigenstate of the operator $E = \mathrm{i}\,(-1)^T \partial_0$ for some positive eigenvalue.

**Case (II)** $[a_{(2)} \neq 0, b_{(2)} = 0]$

This case, including transformation properties, was studied in Ref. 2. At any rate, the only choice here concerns the selection of the parameter $m_{(2)} \neq 0$. If this parameter is not chosen to be real, Eq. (13) is not manifestly invariant under the standard time reversal [2,13] operation

$$\widetilde{\Psi}(x) = (\text{global phase factor})\,\gamma^0 B \Psi^*(-t,s)\,. \tag{31}$$

Since a physical time reversal invariance has to be present (according to the experimental evidence), the choice of a non-real $m_{(2)}$ would end up suggesting a change of the expression of the time reversal operation for case (II). Thus, Eq. (13) becomes the standard form for case (II) provided a real nonvanishing $m_{(2)}$ is chosen. The actual value of $m_{(2)}$ is unimportant here [2]. However, it cannot be excluded that $[m_{(2)}]^2$ might become observable in some larger context, such as that of Sections 6 of Ref. 1 or that of Section 5 of this paper. For this reason (or, if anything else, for reasons of convenience), $m_{(2)}$ is considered to be fixed, in either of the sign options $\pm\,|\,m_{(2)}\,|$.

Even with real $m_{(2)}$, Eq. (13) is not invariant under the standard charge conjugation operation (30), as this transformation sends (13) into a form of type (17): see Ref. 2 for clarity. This appears to be in agreement with the experimental evidence, and does not make a case for modifying the expression of the charge conjugation operation for case (II) (a similar situation arises with parity). As a consequence, the spinor $\boldsymbol{\Psi}_{(2)}(x)$ represents either matter or antimatter exclusively. Specifically, given that massless fermionic matter appears to be left-handed, $\boldsymbol{\Psi}_{(2)}(x)$ refers to matter.

**Case (III)** $[\,a_{(3)} = 0, b_{(3)} \neq 0\,]$
This case can be approached similarly to case (II). However, due to the charge conjugation properties described above, case (III) is essentially redundant, in that $\boldsymbol{\Psi}_{(3)}(x)$ has the same role as the charge conjugate of $\boldsymbol{\Psi}_{(2)}(x)$.

## *4. Tachyons*

This case presents more ambiguity than the previous ones, due to the lack of experimental evidence. Still, case (V) may be examined on the ground of "reasonable assumptions" regarding the relevant properties possessed by tachyonic fermions [19]. It is noted that the existence of tachyons would not necessarily imply the existence of frames of reference moving at speeds faster than light (just like the existence of photons does not imply the possibility of frames moving at light speed). As a matter of fact, superluminal transformations of coordinates have not been considered in this paper.

The presence of the factor $(I - \varepsilon\gamma^5)$ in the equation for massless matter generates an appealing correlation, which, in lack of solid alternatives, may be exploited as a tool for the selection of the standard equation for case (V). Namely:

**(a)** with no formal changes in the active symmetry operations for case (V), if Eq. (18) is chosen as the standard matter equation for case (IV), then Eq. (24) refers to tachyonic matter;

**(b)** with no formal changes in the active symmetry operations for case (V), if Eq. (19) is chosen as the standard matter equation for case (IV), then Eq. (25) refers to tachyonic matter.

A desirable feature for tachyonic fermions is the presence of a physical time reversal invariance: indeed, each of Eq. (24), (25) is invariant under (31), while, for instance, Eq. (26) is not. It is also interesting to examine the charge conjugation operation. For example, take Eq. (24) and apply (30), to obtain:

$$\not{P}\,\widetilde{\Psi}_{(5)}(x) = m_{(5)}\varepsilon\gamma^5\widetilde{\Psi}_{(5)}(x)\,, \tag{32}$$

which is of the form of Eq. (25), not Eq. (24). According to the author's reading of the relevant literature, this too is appropriate, as is the analogous behaviour under spatial parity.

The various choices outlined above and in the previous sections can be put together in a compact formalism. For that, relabel cases (IV), (II), (V) (in this order) with the index $\omega = -1, 0, 1$. Then, with a different spinor symbol, write the standard equations as

$$\not{P}\,\Theta_\omega(x) = \mathrm{N}(\omega)\Theta_\omega(x)\,, \tag{33}$$

with

$$(\mathbf{a}) \qquad \mathrm{N}(\omega) = \frac{m_\omega}{2}\left[(I-\varepsilon\gamma^5) - \omega(I+\varepsilon\gamma^5)\right], \tag{34}$$

or (making use of the convenient replacement $m_0 \to -m_0$):

$$(\mathbf{b}) \qquad \mathrm{N}(\omega) = -\frac{m_\omega}{2}\left[(I-\varepsilon\gamma^5) - \omega(I+\varepsilon\gamma^5)\right], \tag{35}$$

where each $\Theta_\omega(x)$ refers to matter (massive, massless, tachyonic).

Equations (33), (34) were introduced in a previous paper (Ref. 1), but with the values of $m_\omega$ all being the same. The interpretation was that of a Dirac particle with three possible mass states: these states were linearly combined in Ref. 3. In the next section, some probability interpretations are examined, which are associated with the aforementioned superpositions.

## 5. *Neutrino flavours*

Here, Eqs. (33), (34) are taken with $m_\omega = m > 0$. Each state $\Theta_\omega(x)$ is a mass eigenstate corresponding to the eigenvalue $-(m)^2\,\omega$ of the S.F.M. operator. The linear superpositions

$$\Phi_f(x) = v_f^{\omega}\Theta_\omega(x)\,, \qquad f = -1, 0, 1 \tag{36}$$

are introduced [3], where $(v_f^\omega)$ is a $3\times 3$ unitary matrix of mixing coefficients. The spinor $\Phi_f(x)$ is regarded as a particle flavour: specifically, a neutrino flavour [3]. The transformation properties of flavour spinors are not discussed in this paper, but some relevant probability interpretations will be suggested.

Equation (36) is patterned after Pontecorvo's formulation [29]. However, each component $\Theta_\omega(x)$ of the flavour spinor $\Phi_f(x)$ has its own expression of the conserved current, and, in particular, the tachyonic current does not induce a clear probability interpretation

[19]. Thus, without second quantization, the meaning of $\Phi_f(x)$ is much less manifest than in Pontecorvo's model (and, even with second quantization, tachyons are notoriously difficult to treat). At any rate, it is still possible to obtain probabilistic information from Eq. (36). For example, if the non-tachyonic components of $\Phi_f(x)$ are normalized as indicated at Eqs. (16), (23), the values of the squared magnitudes of the mixing coefficients may be understood as follows:

$$|\, v_f^{\omega}\,|^2 = p\,(\omega \mid f)\,, \qquad (\omega = -1, 0)\,, \tag{37}$$

where $p\,(\omega \mid f)$ denotes the conditional probability that a flavour $f$ will be measured to have a value $-(m)^2\,\omega$ of the S.F.M. operator. Then, by normalization of probability

$$p\,(1 \mid f) = 1 - [\,p\,(-1 \mid f) + p\,(0 \mid f)\,] = |\, v_f^1\,|^2, \tag{38}$$

regardless of the actual normalization of $\Theta_1(x)$. The procedure entailed by Eqs. (37), (38) has to be specified more carefully: if a neutrino is detected (equivalently, produced or absorbed) with flavour $f$ at a (macroscopically small) spacetime neighborhood $N\,(\mathtt{Q})$ "surrounding" a point $\mathtt{Q}$ of fixed coordinates $q = \{q^{\lambda}\}$, a measurement of this particle's S.F.M. performed at $N\,(\mathtt{Q})$ returns the value $-(m)^2\,\omega$ with a probability $p\,(\omega \mid f)$. The detection of the neutrino with flavour $f$ at $N\,(\mathtt{Q})$ implies that the spinor representing this particle for $x \in N\,(\mathtt{Q})$ is a superposition as indicated at the right-hand side of Eq. (36), with non-tachyonic components normalized within the spatial extent of $N\,(\mathtt{Q})$. For consistency, it is assumed that the usual equations of motion are not applicable within $N\,(\mathtt{Q})$, as measurements are taking place: see Ref. 30.

## 6. Conclusions

For each flavour, the quantity

$$\sum_{\omega} \left[-(m)^2 \omega\right] p\,(\omega \mid f) \tag{39}$$

represents the mean value of the S.F.M. of the flavour, in the probabilistic interpretation outlined in the previous section. In particular, a choice of the mixing coefficients such that

$$p\,(-\omega \mid f) = p\,(\omega \mid f) \tag{40}$$

reduces to zero the S.F.M. mean values for all flavours: thus, the possibility arises of treating the "average" behaviour of each flavour as purely massless. It is hoped that this concept (admittedly vague) and the issue of flavour oscillations can be addressed in future work.

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## MASA I OKUS SPINORA IZ DIRACOVE JEDNADŽDE S DVA MASENA PARAMETRA

Ispituje se odabir standardnih jednadžbi za masene, bezmasene i tahionske fermione na osnovi poželjnih odlika koje su vezane s operacijama simetrije. Ponovno se razmatraju linearna dodavanja masenih stanja i predlažu moguća tumačenja vjerojatnosti.