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A STUDY OF THE FOURTH-GENERATION QUARKS AND FCNC

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The decay of the fourth-generation down-type quark $b' \to b\gamma$ and $b' \to bg$ has been studied as an extension of the standard model using evolution of the mass-based fourth generation CKM matrix with CP violation phase equal to zero. Range of the masses of fourth generation down-type quark b' and up-type quark t' have been taken with due observance of the constraint imposed by the present experimental value of the ρ parameter, keeping in view the mass difference of the fourth generation quark doublet. The decay width of $b' \to b\gamma$ also has been studied, however, with correction up to the leading QCD logarithms for six active flavours after the W boson, t and t' quarks have been integrated out using operator product expansion. Strong interaction coupling constant is taken on matching scale of Z boson mass.

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1. Introduction

Recent experimental and theoretical studies have stimulated the attemps to demonstrate the existence of the fourth generation in the quark sector. With the assumed b' down-type and t' up-type fourth-generation quarks, the down-type quarks would be d,s,b and b' and the up-type quarks u,c,t and t'. At the large colliders, attempts are made to find signature for the decay modes like $b' \to b\gamma$ and $b' \to bg$ [2,3].

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One may study the b' quark decay separately or as an extension of the formulation for the three generations to the four generations.

For the third generation, it is well known that the radiative b decay is extremely sensitive to the structure of fundamental interactions at the electroweak scale. This has a chequered career; this process has departure from other flavour changing neutral current (FCNC) processes which normally arise at the tree level in the standard model (SM). Actually, one-loop W-exchange diagrams generate this decay at the lowest order in the SM. The model is based on the gauge group $SU(3)_C\times SU(2)_L\times U(1)$, where the quark couples to the W boson yielding the weak current [4].

The radiative b' quark decay may be treated in the same way as that of b quark decay. The latter has a rate of the order of $G_F^2\alpha_{QED}$, and all other FCNC processes involving leptons and photons are of the order $G_F^2\alpha_{QED}^2$. At the same time, the long-range strong interactions are expected to play a minor role in the exclusive process of the radiative B-meson decay (by requiring $E_\gamma > (m_B^2 - m_D^2)/2m_B$), so its effect cannot be ignored while considering the inclusive process of radiative b quark decay. However, it is known that the short-distance effect of QCD [5–7] due to gluon exchange between the quark lines of the leading one-loop electroweak diagrams enhance the radiative b decay rate in the SM by 2 to 5 times, depending on the top quark mass. Further important contributions in the related domain have been made by Deshpande, Trampetić and others [8–11], where long-distance effects have been duly considered. Other $q \to q'\gamma$ transitions are very hard to observe. Recently, the $\Gamma(b \to s\gamma)$ and the top quark mass has been measured [1,12].

In view of these new developments, it is the purpose of the present paper to analyse the theoretical uncertainties in the calculation of the decay width $\Gamma(b' \to b\gamma)$. Firstly, this requires the choice of the fourth generation quark masses which are not free parameters, rather they are constrained by the experimental value of the ρ parameter. The ρ parameter, in terms of the transverse part of the W- and Z-boson self energies at zero momentum transfer, is given in Ref. 13,

$$\rho = \frac{1}{1-\Delta\rho};\ \Delta\rho = \frac{\Pi_{ZZ}(0)}{M_Z^2} - \frac{\Pi_{WW}(0)}{M_W^2}. \tag{1}$$

The choice of the pair of masses $(m_{b'},\ m_{t'})$, keeping in view the upper bound of 550 GeV for the mass of the t' quark coming from partial-wave unitarity [14], the model-independent lower bound of the b' quark mass which has been set [15] as 45 GeV, and constraint on the mass difference of the fourth generation quark doublet enforced by the experimental value of the ρ parameter, has been recently done in Ref. 16, and we are free to utilize it.

Secondly, the dominant uncertainty in the existing leading logarithmic QCD calculations is due to the choice of the renormalization scale μ [17]. In any finite-order of perturbation theory, such an uncertainty is inherent. Several papers [18–20] have recently analysed this problem. However, the μ-dependence of the relevant amplitudes is reduced considerably by the inclusion of next-to-leading-order correction. It may be noted that the scale uncertainty in the leading-order calculation of this decay rate is particularly large, it amounts to around $\pm 25\%$. Therefore we have the restrictions on the SM or its extensions, which can be obtained with the help of the experimental findings and the leading order approximations, which are substantially weaker than found without taking the theoretical uncertainties into account.

The $\mu-$dependence, present in the branching ratios, can be reduced in the same manner as it was done for other decays [18–20]. The full next-to-leading log calculation of radiative decays would require consideration of three-loop mixing between certain effective operators. Before one undertakes such an effort, it is much better to make a formal analysis of the considered decay at the leading logarithmic level, and to check to what extent the $\mu-$dependence can be reduced once all the necessary calculations have been performed. A review by Greub et al. on FCNC process of radiative b decays and on the next-to-leading logarithmic results has appeared in the proceedings of a recent Symposium [21]. Chetyrkin et al. [22] have obtained the results for three-loop anomalous dimensions while analyzing $B \to X_s\gamma$ decay and report the branching ratio $\mathcal{B}(B \to X_s\gamma)$ to be $(3.28 \pm 0.33) \times 10^{-4}$. The predictions of the SM are in conformity with the CLEO data at 2σ level. So, we may proceed following the same way. However, for the present, the analysis of $b' \to b\gamma$ decay is restricted to the leading logarithmic calculations, to be given in the next section.

The paper is organized as follows: Section 2 summarises the results of the formulation of $b' \to b\gamma$ in four generations for any $m_{b'}$, i.e., $m_{b'} < M_W$ and even beyond the W boson mass. In Sect. 3 calculations of the decay width for $b' \to bg$ for four generations has been considered along with the ratio of $\Gamma(b' \to b\gamma)$ to $\Gamma(b' \to bg)$; Section 4 summarises the results of the leading logarithmic (LL) calculations, adopting operator product expansion which is used to evolve anomalous dimension matrix for four generations and to perform leading logarithmic calculation for the decay width taking the value of $m_{b'} < M_W$. In Sect. 5 the results are discussed.

2. Calculation of $\Gamma(b' \to b\gamma)$ for $m_{b'}$ below and above M_W

The transition $b' \to b\gamma$ occurs through the process of one-loop penguin-type diagram which admits W boson and u, c, t, t' quark exchange. It may be treated as a direct extension of the SM. The matrix element for this process is given by [23]

$$\begin{aligned}\mathcal{M}(b' \to b\gamma) &= \frac{eG_F}{4\sqrt{2}\pi^2}\{\mathrm{i}\varepsilon_\mu \bar{b}\sigma^{\mu\nu} q_\nu (b'_R m_{b'} + b'_L m_b)\} \\ &\times \sum_{i=u,c,t,t'} V^*_{ib} V_{ib'} (I(x_i) - I(x_u)). \end{aligned} \tag{2}$$

V_{pq}'s are elements of the four generations CKM matrix, calculated as prescribed in Ref. 16, $x_i{=}m_i^2\,/\,M_W^2$, the function $I(x)$ is the Inami and Lim function [24], ε_μ is photon polarization vector, q_μ is photon momentum, and $\sigma^{\mu\nu} = (i/2)[\gamma^\mu, \gamma^\nu]$, $b'_R = (1/2)(1+\gamma_5)b'$, $b'_L = (1/2)(1-\gamma_5)b'$.

In this weak process of loop-induced coupling, the Glashow-Iliopoulos-Maiani (GIM) [25] cancellation is important. Taking this into account, and with the LL approximation to the first order in α_s [26], the QCD corrections read:

$$\begin{aligned} Corrections &= \sum_{i=u,c,t,t'} V_{ib}^* V_{ib'} \frac{e\alpha_s G_F\sqrt{2}}{6\pi^3} \ln\left(m_i^2 / M_W^2\right), \quad \text{for} \quad m_i < M_W \\ &= 0, \quad \text{for} \quad m_i > M_W. \end{aligned} \tag{3}$$

Combining Eqs. (2) and (3), and neglecting the terms of the order $m_b^2 / m_{b'}^2$, one has

$$\Gamma(b' \to b\gamma) = \frac{\alpha G_F^2 m_{b'}^5}{128\pi^4} \times \tag{4}$$

$$\left| V_{ub}^* V_{ub'} I(x_u) + V_{cb}^* V_{cb'} \left(I(x_c) + \frac{4\alpha_s(m_{b'})}{3\pi} \left[\ln\frac{m_c^2}{M_W^2} - \ln\frac{m_u^2}{M_W^2} \right] \right) + \right.$$

$$\left. V_{tb}^* V_{tb'} \left(I(x_t) - \frac{4\alpha_s(m_{b'})}{3\pi} \ln\frac{m_u^2}{M_W^2} \right) + V_{t'b}^* V_{t'b'} \left(I(x_{t'}) - \frac{4\alpha_s(m_{b'})}{3\pi} \ln\frac{m_u^2}{M_W^2} \right) \right|^2$$

where

$$I(x) = \frac{2x}{3(x-1)} + \frac{7x}{4(x-1)^2} + \frac{x}{2(x-1)^3} - \frac{(3x^3 - 2x^2)}{2(x-1)^4} \ln x. \tag{5}$$

The above expression is valid for $m_{b'} < M_W$ as well as for $m_{b'} > M_W$.

3. *Decay width* $\Gamma(b' \to bg)$ *and the ratio to* $\Gamma(b' \to b\gamma)$

For the decay channel $b' \to bg$ the one-loop calculation is straightforward from Ref. 23, as an extension of SM. It has been ascertained in Ref. 26 that for this decay mode, even large QCD corrections cannot allow this process to reach the few percent level. Thus, even for the extension to fourth generation, we neglect the QCD corrections and following Refs. 23 and 27, the decay width may be written as

$$\Gamma(b' \to bg) = \frac{G_F^2 \alpha_s(m_{b'}) m_{b'}^5}{128\pi^4} C_2(F) \left| \sum_{i=u,c,t,t'} V_{ib}^* V_{ib'} F(x_i) \right|^2, \tag{6}$$

where $C_2(F) = 4/3$, $x_i = m_i^2 / M_W^2$ and $F(x)$ is the Inami and Lim function for this decay. Here the terms of the order $m_b^2 / m_{b'}^2$ have been neglected. The ratio is given by

$$R = \frac{\Gamma(b' \to b\gamma)}{\Gamma(b' \to bg)} = \frac{3\alpha}{4\alpha_s(m_{b'})} \left| \frac{N_1}{N_2} \right|^2 \tag{7}$$

where,

$$N_1 = V_{ub}^* V_{ub'} I(x_u) + \tag{8}$$

$$V_{cb}^* V_{cb'} \left(I(x_c) + \frac{4\alpha_s(m_{b'})}{3\pi} \left[\ln \frac{m_c^2}{M_W^2} - \ln \frac{m_u^2}{M_W^2} \right] \right) +$$

$$V_{tb}^* V_{tb'} \left(I(x_t) - \frac{4\alpha_s(m_{b'})}{3\pi} \ln \frac{m_u^2}{M_W^2} \right) + V_{t'b}^* V_{t'b'} \left(I(x_{t'}) - \frac{4\alpha_s(m_{b'})}{3\pi} \ln \frac{m_u^2}{M_W^2} \right)$$

and $N_2 = \sum_{i=u,c,t,t'} V_{ib}^* V_{ib'} F(x_i)$. Here R is a function of $m_{t'}$ for a given $m_{b'}$.

4. The leading logarithmic calculations of $b' \to b\gamma$ decay

This section briefly summarises the leading logarithmic calculation of $b' \to b\gamma$ decay in general terms. The theoretical uncertainties due to the dependence on the mass scale μ can be reduced after performing such calculations.

The QCD corrections to the $b' \to b\gamma$ decay contain large logarithms $\ln(M_W^2/m_{b'}^2)$ which have to be resummed with the renormalization group equation (RGE). In order to do this, one has to introduce an effective Hamiltonian built of operators of dimension higher than four,

$$H^{eff} \;=\; -V_{tb'} V_{tb}^* \frac{G_F}{\sqrt{2}} \sum_{i=1}^{8} Q_i(\mu) C_i(\mu) \equiv -V_{tb'} V_{tb}^* \frac{G_F}{\sqrt{2}} Q^T(\mu) C(\mu) \tag{9}$$

where V_{ij} are the CKM matrix elements, $Q_i(\mu)$ are the relevant operators and $C_i(\mu)$ are the corresponding Wilson coefficients in the mass scale μ. The complete set of operators necessary in the $b' \to b\gamma$ after t quark, t' quark and W boson have been integrated out is the following [28] :

$$\begin{aligned}
Q_1 &= \left(\bar{b}_\alpha c^\beta\right)_{V-A} \left(\bar{c}_\beta b'^\alpha\right)_{V-A} . \\
Q_2 &= \left(\bar{b}_\alpha c^\alpha\right)_{V-A} \left(\bar{c}_\beta b'^\beta\right)_{V-A} . \\
Q_3 &= \left(\bar{b}_\alpha b'^\alpha\right)_{V-A} \sum_{q=u,d,s,c,b,b'} \left(\bar{q}_\beta q^\beta\right)_{V-A} . \\
Q_4 &= \left(\bar{b}_\alpha b'^\beta\right)_{V-A} \sum_{q=u,d,s,c,b,b'} \left(\bar{q}_\beta q^\alpha\right)_{V-A} . \\
Q_5 &= \left(\bar{b}_\alpha b'^\alpha\right)_{V-A} \sum_{q=u,d,s,c,b,b'} \left(\bar{q}_\beta q^\beta\right)_{V+A} .
\end{aligned}$$

$$
\begin{aligned}
Q_6 &= \left(\bar{b}_\alpha b'^\beta\right)_{V-A} \sum_{q=u,d,s,c,b,b'} \left(\bar{q}_\beta q^\alpha\right)_{V+A} . \\
Q_7 &= (e/8\pi^2)\, m_{b'}\, \bar{b}_\alpha \sigma^{\mu\nu} (1+\gamma_5)\, b'^\alpha F_{\mu\nu} \\
Q_8 &= (g/8\pi^2)\, m_{b'}\, \bar{b}_\alpha \sigma^{\mu\nu} (1+\gamma_5)\, (T^a)^\alpha_\beta b'^\beta G^a_{\mu\nu} \qquad (10)
\end{aligned}
$$

where T^a are the SU(3)$_c$ Gell-Mann colour matrices normalized by $Tr(T^a T^b) = 1/2\delta^{ab}$, $(\bar{q}q')_{V\pm A} = \bar{q}\gamma_\mu (1\pm\gamma_5)q'$, Q_1, Q_2 are the current-current operators, Q_3, Q_4, Q_5, Q_6 are the QCD penguin operators and Q_7, Q_8 are the "magnetic penguin" operators.

In the latter operators, the terms proportional to b-quark mass have been neglected in comparison to b' quark mass, since they give only a correction of the order $m_b^2/m_{b'}^2$ to the decay rate ($m_{b'} \geq 45$ GeV and $m_b = 4.6$ GeV). In the present case $N = 3$, number of up-type quarks is $u = 2$, of down-type quarks $d = 4$ and the number of active flavours $f = u + d = 6$.

The inclusion of the couplings in the definition of Q_7 and Q_8 allows us to discuss the evolution of the renormalization group in full analogy with the case of the usual $\Delta S = 1$ or $\Delta B = 1$ Hamiltonians for the nonleptonic decays of the b quark. Following Ref. 29, one may write

$$C(\mu) = \bar{U}(\mu, M_W)\, C(M_W). \qquad (11)$$

Here the renormalization group evolution matrix may be written as

$$\bar{U}(m_1, m_2) = T_g \exp\left(\int_{g(m_2)}^{g(m_1)} \mathrm{d}g'\, \bar{\gamma}^T(g')/\beta(g')\right) \qquad (12)$$

with $m_1 < m_2$. Here T_g denotes such ordering in the coupling constants that they increase from right to left.

Next, $\bar{\gamma}(g)$ is the 8×8 anomalous dimension matrix of the operator Q_i and $\beta(g)$ is the usual renormalization group function which governs evolution of α_s.

Keeping the first two terms in the expansion for $\bar{\gamma}(g)$ and $\beta(g)$, one has

$$\bar{\gamma}(g) = \gamma^{(0)} \frac{g^2}{16\pi^2} + \bar{\gamma}^{(1)} \frac{g^4}{(16\pi^2)^2} + \ldots \qquad (13)$$

$$\beta(g) = -\beta_0 \frac{g^3}{16\pi^2} - \beta_1 \frac{g^5}{(16\pi^2)^2} + \ldots \qquad (14)$$

where $\beta_0 = (11\,N - 2n_f)/3$ and $\beta_1 = (34/3)\,N^2 - (10/3)\,Nn_f - 2c_f n_f$, N is the number of colours, n_f the number of active flavours and $c_f = (N^2-1)/(2N)$. With this, one finds

$$\bar{U}(m_1, m_2) = \left(\bar{1} + \frac{\alpha_s(m_1)}{4\pi}\bar{J}\right) \bar{U}^{(0)}(m_1, m_2) \left(\bar{1} - \frac{\alpha_s(m_2)}{4\pi}\bar{J}\right) \qquad (15)$$

where $\bar{U}^{(0)}(m_1,m_2)$ denotes evolution matrix in the leading logarithmic approximation (LL), $\bar{1}$ is unit matrix and $\bar{J}$ summarizes the next-to-leading corrections to this evolution. If

$$\bar{\gamma}_D^{(0)} \equiv \bar{V}^{-1}\bar{\gamma}^{(0)T}\bar{V}, \quad \text{and} \quad \bar{G} \equiv \bar{V}^{-1}\bar{\gamma}^{(1)T}\bar{V}, \tag{16}$$

where $\bar{\gamma}_D^{(0)}$ denotes a diagonal matrix whose diagonal elements are the components of the vector $\gamma^{(0)}$, then

$$\bar{U}^{(0)}(m_1,m_2) = \bar{V}\left[\left(\frac{\alpha_s(m_2)}{\alpha_s(m_1}\right)^P\right]_D \bar{V}^{-1} \quad \text{with} \quad P = \frac{\gamma^{(0)}}{2\beta_0}. \tag{17}$$

For the matrix $\bar{J}$ one has

$$\bar{J} \;=\; \bar{V}\bar{S}\bar{V}^{-1} \tag{18}$$

where the elements of $\bar{S}$ are given by

$$S_{ij} \;=\; \delta_{ij}\gamma_i^{(0)}\frac{\beta_1}{2\beta_0^2} - \frac{G_{ij}}{2\beta_0 + \gamma_i^{(0)} - \gamma_j^{(0)}}, \tag{19}$$

with $\gamma_i^{(0)}$ denoting the element of $\gamma^{(0)}$, and G_{ij} the element of $\overline{G}$.

The function $\beta(g)$ governs the evolution of the effective coupling constant $\bar{g}$,

$$\frac{d\bar{g}^2}{dt} = \bar{g}\beta(\bar{g}), \qquad t = \ln\frac{Q^2}{\mu^2}, \qquad \bar{g}(0) = g, \tag{20}$$

Q being some scale with the solution

$$\begin{aligned}\frac{\alpha_s(Q^2)}{4\pi} &= \frac{\bar{g}^2(Q^2)}{16\pi^2} \\ &= \frac{1}{\beta_0\ln(Q^2/\Lambda^2_{\overline{MS}})} - \frac{\beta_1}{\beta_0^3}\frac{\ln\ln(Q^2/\Lambda^2_{\overline{MS}})}{\ln^2(Q^2/\Lambda^2_{\overline{MS}})} + \mathrm{O}\left(\frac{1}{\ln^3(Q^2/\Lambda^2_{\overline{MS}})}\right)\end{aligned} \tag{21}$$

where Λ is the QCD scale parameter which has been taken to be in the MS scheme. Truncating at the second term, one has for the flavour f, the following expression for $\alpha_f(Q)$

$$\alpha_f(Q) \;=\; \frac{4\pi}{\beta_0\ln(Q^2/\Lambda_f^2)}\left[1 - \frac{\beta_1}{\beta_0^2}\frac{\ln\ln(Q^2/\Lambda_f^2)}{\ln(Q^2/\Lambda_f^2)}\right] \tag{22}$$

with Λ_f in the MS scheme. The continuity of $\alpha_f(Q)$ has been understood.

The ratio $\alpha_s(m_2)/\alpha_s(m_1)$ in Eq. (17) above has now to be calculated with the help of the two-loop renormalization group equation for $\alpha_s(\mu)$. Nowadays it is customary to calculate with $\alpha_s(M_Z)$ as an initial condition for this equation. A straightforward calculation gives the solution of the form

$$\alpha_s(\mu) \quad = \quad \frac{\alpha_s(M_Z)}{\alpha(\mu)}\left(1-\frac{\beta_1}{\beta_0}\frac{\alpha_s(M_Z)}{4\pi}\frac{\ln\alpha(\mu)}{\alpha(\mu)}\right), \tag{23}$$

where

$$\alpha(\mu) \quad = \quad 1-\beta_0\frac{\alpha_s(M_Z)}{2\pi}\ln(\frac{M_Z}{\mu}). \tag{24}$$

Thus $\eta=\alpha_s(M_W)/\alpha_s(\mu)$ can now be calculated.

To find the anomalous dimension matrix, the calculation of the one-loop matrix elements of the operators Q_1, Q_2, Q_3, Q_4, Q_5 and Q_6 is encountered with logarithmic divergences in the effective theory, which correspond to the logarithm of M_W in the full theory. To calculate the anomalous dimension matrix from this, we just pick up a term of the order α_s^0. Since these logarithms are not induced by QCD effects, they are present even without any QCD.

Calculations to leading logarithmic (LL) level were done following Ref. 30, which gives the scheme-independent matrix $\bar{\gamma}^{(0)\,eff}$ defined in (13) that governs the LL QCD corrections to $b'\to b\gamma$.

Taking $\bar{J}$=0, Eq. (11) reads

$$\begin{aligned} C(\mu) \quad &= \quad \bar{U}(\mu,M_W)C(M_W) \;=\; \bar{U}^{(0)}(\mu,M_W)C(M_W) \\ &= \quad \bar{V}\left[\eta^{\gamma^{(0)}}/2\beta_0\right]_D \bar{V}^{-1}C(M_W) \end{aligned} \tag{25}$$

where $\bar{V}$ diagonalizes $\bar{\gamma}^{(0)}$, i.e.,

$$\bar{\gamma}_D^{(0)} \equiv \bar{V}^{-1}\bar{\gamma}^{(0)T}\bar{V} \tag{26}$$

where $\gamma^{(0)}$ are the diagonal elements of $\bar{\gamma}_D^{(0)}$. $C(M_W)$ may also be expanded as

$$C(M_W) \quad = \quad C^{(0)}(M_W)+\frac{\alpha_s(M_W)}{4\pi}C^{(1)}(M_W) \tag{27}$$

and to get only LL approximation, one may take $C^{(1)}(M_W)$=0 and this will give $C^{(0)\,eff}(\mu)$.

In the 't Hooft and Veltman (HV) scheme, this matrix is equal to $\bar{\gamma}^{(0)}$ of the Eq. (13). For the sake of simplicity, in this paper we do not take into account the virtual effects of the fourth generation up-type quark t'. The figures are taken from Ref. 31 for (i) the leading-order matching conditions at the top-quark scale for the 1PI Green functions in the full SM and in the intermediate effective field theory, and (ii) for the matching conditions at μ=

M_W for four quarks and two quarks 1PI Green functions in the intermediate effective-field theory and effective-field theory below W scale, however, with replacement of b by b' and of s by b.

For the evolution from $\mu = M_W$ to $\mu = m_{b'}$, with the number of active flavours $f= 6$, $Q_1= Q_u= 2/3$, $Q_2= Q_d= -1/3$, $\bar{Q}= 2Q_u+4Q_d= 0$, one gets the anomalous dimension matrix [32] for the operators given in (10) as:

$$\bar{\gamma}^{(0)} = \begin{pmatrix} -2 & 6 & 0 & 0 & 0 & 0 & 0 & 3 \\ 6 & -2 & -2/9 & 2/3 & -2/9 & 2/3 & 416/81 & 70/27 \\ 0 & 0 & -22/9 & 22/3 & -4/9 & 4/3 & -464/81 & 626/27 \\ 0 & 0 & 14/3 & 2 & -4/3 & 4 & -32/27 & 194/9 \\ 0 & 0 & 0 & 0 & 2 & -6 & 32/9 & -68/3 \\ 0 & 0 & -4/3 & 4 & -4/3 & -12 & -32/27 & -274/9 \\ 0 & 0 & 0 & 0 & 0 & 0 & 32/3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -32/9 & 28/3 \end{pmatrix} . \qquad (28)$$

The eight eigenvalues are $\{4., -8., -13.5484, 6.81377, -5.97777,\ 2.26796, 9.33333, 10.6667\}$ and the corresponding eigenvectors are:

1st	2nd	3rd	4th
0.707107	0.707107	–0.0228936	–0.069976
0.707107	–0.707107	0.044064	–0.102792
0.0	0.0	–0.0492333	–0.62544
0.0	0.0	0.218597	–0.756705
0.0	0.0	–0.350862	0.159358
0.0	0.0	–0.909224	–0.127852
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0

5th	6th	7th	8th
–0.19046	–0.0448565	0.802291	–1.2376
0.126268	–0.0319076	1.01544	–1.27937
–0.890227	–0.332954	6.84462	–13.1557
0.376687	–0.281159	7.69721	–14.18
0.288386	–0.977828	–2.90253	6.39838
0.383446	0.0436695	–0.230236	1.42456
0.0	0.0	0.0	1.0
0.0	0.0	1.0	–2.66667

Neglecting the effects of the fourth generation up quark t', one has the SM values of the Wilson coefficients collected from Ref. 24 at the M_W mass scale as a function of $x = m_t^2 / M_W^2$, written as $C_i(W)$, which are functions of the mass of t quark and the mass of W boson, and are given by:

$$
\begin{aligned}
C_2(W) &= -1 \\
C_7(W) &= \frac{8x^3+5x^2-7x}{24(x-1)^3} - \frac{3x^3-2x^2}{4(x-1)^4}\ln x \\
C_8(W) &= +\frac{x^3-5x^2-2x}{8\,(x-1)^3} + \frac{3x^2}{4(x-1)^4}\ln x.
\end{aligned} \tag{29}
$$

Thus we have from Eq. (17) the diagonal matrix $\left[(\alpha_s(M_W)\,/\,\alpha_s(m_{b'}))^P\right]_D$

$$
\begin{aligned}
&\mathit{DiagonalMatrix}[\eta^{2/7}, \eta^{-4/7}, \eta^{-0.967742857}, \eta^{0.486697857}, \eta^{-0.426983571}, \\
&\qquad \eta^{0.161997142}, \eta^{0.666666428}, \eta^{0.761907142}] \equiv \mathit{DiagonalMatrix}[\{a,b,c,d,e,f,g,h\}],
\end{aligned} \tag{30}
$$

where $\eta = \alpha_s(M_W)\,/\,\alpha_s(m_{b'})$, taken at $\mu{=}m_{b'}$ mass scale; The coefficients C_3, C_4, C_5, C_6 are now assumed to vanish at the matching scale $\mu{=}M_W$.

Now, setting $\bar{J}=0$ and $C^{(1)}=0$, Eqs. (15) and (27) give $C^{(0)\,eff}(\mu)$ in the leading logarithmic approximation at the $\mu = m_{b'}$ scale, just by straightforward calculations.

The results are given below:-

$$
C_1(\mu) = (0.5\,a - 0.5\,b)\,C_2(W), \tag{31}
$$

$$
C_2(\mu) = (0.5\,a + 0.5\,b)\,C_2(W), \tag{32}
$$

$$
\begin{aligned}
C_3(\mu) = {} & (-0.0625\,a + 0.125\,b - 0.0121761\,c + 0.0400364\,d \\
& -0.0964069\,e + 0.00604658\,f)\,C_2(W),
\end{aligned} \tag{33}
$$

$$
\begin{aligned}
C_4(\mu) = {} & (-0.0625\,a - 0.125\,b + 0.0167488\,c + 0.0890896\,d \\
& +0.0786656\,e + 0.00299592\,f)\,C_2(W),
\end{aligned} \tag{34}
$$

$$
\begin{aligned}
C_5(\mu) = {} & (-0.00258045\,c - 0.0377387\,d \\
& +0.0110945\,e + 0.0292247\,f)\,C_2(W),
\end{aligned} \tag{35}
$$

$$
\begin{aligned}
C_6(\mu) = {} & (-0.0427815\,c + 0.0338142\,d \\
& +0.019852\,e - 0.0108847\,f)\,C_2(W),
\end{aligned} \tag{36}
$$

$$
\begin{aligned}
C_7(\mu) = {} & 1.0\,h\,C_7(W) + 2.66667\,(g-h)\,C_8(W) + (-0.45\,a \\
& -0.107143\,b - 0.0130141\,c - 0.84745\,d - 0.00906636\,e \\
& -0.00179489\,f + 2.70784\,g - 1.27937\,h)\,C_2(W),
\end{aligned} \tag{37}
$$

$$
\begin{aligned}
C_8(\mu) = {} & 1.0\,g\,C_8(W) + (-0.062918\,c - 1.06153\,d \\
& +0.0911357\,e + 0.0178728\,f + 1.01544\,g)\,C_2(W).
\end{aligned} \tag{38}
$$

Above formulae are the values of effective Wilson coefficients at the μ=$m_{b'}$ mass scale. The resulting leading logarithmic expression for $\Gamma(b' \to b\gamma)$ may be read as

$$\Gamma(b' \to b\gamma) \quad = \quad \frac{\alpha G_F^2 m_{b'}^5}{128\pi^4} |V_{tb}^* V_{tb'}|^2 |C_7^{(0)\,eff}(m_{b'})|^2, \tag{39}$$

where V_{pq} is a 4×4 CKM matrix element and $C_7^{(0)\,eff}(m_{b'})$ is given by (37), (29) and (30).

5. Results and conclusion

For the calculation, in the present case, β_0=$\frac{1}{3}(11\,N - 2\,n_f)$= 7, and β_1= 18. The values of the constants α^{-1}=137.036, G_F=1.166372 $\times$ 10^{-5} GeV^2, and M_Z =91.187 were taken from Ref. 33, and M_W=80.33 [34] has been used. The value of $\alpha_s(M_Z^2)$ has been taken as 0.1157 [16]. The values of $\alpha_s(\mu)$ at the M_Z mass scale for different values of $m_{b'}$ were calculated with the help of Eqs. (23) and (24).

$\Gamma(b' \to b\gamma)$, $\Gamma(b' \to bg)$, and R are calculated by Eqs (4), (6) and (7), respectively, and are given in Table 1.

TABLE 1. Calculated widths of $b' \to b\gamma$ and $b' \to bg$ decays and the ratio $R = \Gamma(b' \to b\gamma)/\Gamma(b' \to bg)$, for m_t= 180 GeV.

$m_{b'}$ (GeV)	$m_{t'}$ (GeV)	$\Gamma(b' \to b\gamma)$ (eV)	$\Gamma(b' \to bg)$ (eV)	R
45	110	0.066	0.146	0.453
	115	0.062	0.121	0.515
50	110	0.119	0.249	0.477
	115	0.106	0.206	0.514
	120	0.096	0.171	0.559
60	110	0.296	0.621	0.478
	115	0.266	0.517	0.515
	120	0.238	0.427	0.558
	130	0.211	0.299	0.674
85	110	0.169	3.506	0.483
	115	1.504	2.909	0.517
	120	1.347	2.415	0.557
	130	1.125	1.682	0.669
	140	1.026	1.231	0.833
	150	1.072	1.021	1.049
90	110	2.252	4.650	0.484
	120	1.788	3.202	0.558
	130	1.490	2.230	0.668
	140	1.356	1.356	0.831
	150	1.413	1.349	1.047

$\Gamma(b' \to b\gamma)$ is monotonically increasing for increasing $m_{b'}$ and fixed $m_{t'}$, but monotonically decreasing for the increasing $m_{t'}$ and $m_{b'}$ fixed. $\Gamma(b' \to bg)$ is monotonically increasing for increasing $m_{b'}$ and fixed $m_{t'}$, but monotonically decreasing for increasing $m_{t'}$ and fixed $m_{b'}$ mass.

Scenario for the b' quark mass below the W boson mass : For the range of mass 110 GeV$\leq m_{t'} \leq$120 GeV, $\Gamma(b' \to bg) > \Gamma(b' \to b\gamma)$, however, $\Gamma(b' \to b\gamma)$ increases steadily.

Scenario for the b' mass above the W boson mass : Calculations were done only for $m_{b'}$ =85 GeV and 90 GeV, and for the values of $m_{t'}$ ranging from 110 GeV to 140 GeV, $\Gamma(b' \to bg) > \Gamma(b' \to b\gamma)$. But at $m_{t'}$=150 GeV, the position is reversed: $\Gamma(b' \to b\gamma)$ becomes greater than $\Gamma(b' \to bg)$.

So, the formulation is very sensitive to the t' quark mass in the range 130 GeV to 150 GeV. The mass of the t' quark is constrained by the experimental value of the ρ parameter. For $m_{b'}$ =60 GeV, t' mass is not allowed above 130 GeV, also for the b' quark mass of 85 GeV and 90 GeV, the mass of the t' quark is not permitted above 150 GeV; the calculations were truncated at this stage.

From the sharply increasing monotonicity of $\Gamma(b' \to b\gamma)$, it is expected that more enhancement may be achieved if QCD corrections up to the leading logarithmic or the next-to-leading order calculations could be done.

The QCD corrected values of $b' \to b\gamma$ were calculated using Eq. (39) for $m_t = 175$ GeV and 180 GeV and $m_{b'} = 45$, 50 and 60 GeV. The mass of the fourth generation up-type quark has been taken in the permissible range of 110–130 GeV for the calculation of the CKM matrix elements, but it has been observed that the values are not that sensitive to the t' quark mass, and these calculations are reported only for $m_{t'}$= 110 GeV. They are given in Table 2.

TABLE 2. LL QCD corrected $b' \to b\gamma$ decay.

$m_{b'}$ (GeV)	m_t (GeV)	$\Gamma(b' \to b\gamma)$ (eV)
45	175	0.1420
	180	0.1461
50	175	0.2375
	180	0.2446
60	175	0.5745
	180	0.5923

The decay width is much more sensitive to the $m_{b'}$ quark mass than to the top quark mass. For an increase of mass of $m_{b'}$ from 45 GeV to 50 GeV, the enhancement is by the factor of 1.6, from 45 GeV to 60 GeV, the enhancement is by a factor of 4.05, and from 50 GeV to 60 GeV it is by a factor of 2.4; the effect of m_t may account for approximately 1%. Further, it is observed that there is an enhancement of the decay width from that given in Table 1 (without the QCD corrections), and this rate is approximately doubled in the present case when the QCD corrections are taken into account.

The introduction of the effect of the virtual exchange of the fourth generation up quark t' has been discussed in Refs. 35 and 36. Here, we utilize this approach. For this, one has to rewrite $C_7(W)$ as $C_7(m_t, M_W)$ and $C_8(W)$ as $C_8(m_t, M_W)$, and with these, the Wilson coefficients of the dipole operators at the W mass scale in the limit of vanishing up and charm quark masses can be written simply by replacing the term $V_{tb}^* \, V_{tb'} \, C_i(m_t, M_W)$ by $V_{tb}^* \, V_{tb'} \, C_i(m_t, M_W) \; + \; V_{t'b}^* \, V_{t'b'} \, C_i(m_{t'}, M_W)$ where V_{pq} represent the 4×4 *CKM* matrix elements and $i = 7, 8$.

So, the fourth-generation Wilson coefficient at the W-mass scale can be written as

$$C_{i,4th\ gener.}(m_t, M_W) = C_i(m_t, M_W) + \frac{V_{t'b}^* \, V_{t'b'}}{V_{tb}^* \, V_{tb'}} C_i(m_{t'}, M_W), \quad i = 7, \ 8.$$

Equation (39) yields $\Gamma(b' \to b\gamma)$ by replacing $C_7^{(0)\,eff}(m_{b'})$ by $C_{7,4th\ gener.}^{(0)\,eff}(m_{b'})$ which is obtained from Eq. (37) with $C_7(W)$ and $C_8(W)$ replaced by $C_{7,4th\ gener.}(m_t, M_W)$ and $C_{8,4th\ gener.}(m_t, M_W)$, respectively. The results are shown in Table 3.

TABLE 3. LL QCD corrected $b' \to b\gamma$ decay considering the fourth generation.

$m_{b'}$ (GeV)	m_t (GeV)	$m_{t'}$ (GeV)	$\Gamma(b' \to b\gamma)$ (eV)
45	175	110	0.365988
	180	110	0.372515
50	175	110	0.624785
		120	0.639156
	180	110	0.636124
		120	0.650437
60	175	110	1.566482
		120	1.611991
		130	1.644182
	180	110	1.595772
		120	1.641211
		130	1.673314

The introduction of the t' quark in the arena shifts the observations of the previous paragraph slightly. For an increase of mass of $m_{b'}$ from 45 GeV to 50 GeV, the enhancement is by the factor of 1.7, from 45 GeV to 60 GeV, the enhancement is by a factor of 4.28, and from 50 GeV to 60 GeV it is by a factor of 2.5. The ratio of the results for $\Gamma(b' \to b\gamma)$ with and without the consideration of the t' quark mass in the internal loop lies in the range 2.58 to 2.8. So, the fourth-generation up-quark mass has quite large effects. It is expected that the next-to-leading order calculations may enhance the rate further.

If we take the mass of t' quark as the free parameter, and taking the values of $m_{t'}$ of 400, 500, 600, 700, 800, and 900 GeV, the calculation of $b' \to b\gamma$ for $m_{b'} = 45$ GeV without LL QCD corrections yields the following results, namely; 0.0901 eV, 0.0699 eV, 0.0531 eV, 0.0398 eV, 0.0295 eV and 0.0216 eV, respectively. The results are comparable to the values arrived at for $m_{t'}$=110 GeV and 115 GeV, i.e., 0.064 eV (average).

Testing ground of this formulation are the future experimental measurements, as at present time there is no evidence for the b' quark, except for some implicit signature as reported in Ref. 3. The search for new physics is conducted through a three-prong attack: (i) direct production of new particles at high energy colliders, (ii) deviations from SM predictions in precision measurements, and (iii) indirect observation of new physics in rare or forbidden processes. The present formulation was done keeping in mind that a fourth generation is consistent with the LEP/SLC data as long as the fourth neutrino is heavy, i.e., $m_{\nu_4} \geq M_Z/2$, and that such a heavy fourth neutrino could mediate a see-saw type mechanism thus generating a small mass for $\nu_{e,\mu,\tau}$. And the possibility of a fourth family of fermions may be taken as a popular potential extension.

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PROUČAVANJE ČETVRTE GENERACIJE KVARKOVA I NEUTRALNIH STRUJA KOJE MIJENJAJU OKUS

Razmatraju se raspadi četvrte generacije kvarkova tipa "dolje" $b' \to b\gamma$ i $b' \to bg$ kao proširenje standardnog modela, primjenom evolucije CKM matrice četvrte generacije zasnovane na masi, s fazom kršenja CP simetrije jednakom nula. Izabrali smo područje masa kvarkova četvrte generacije tipa "dolje" b' i tipa "gore" t' gledajući ograničenje koje postavlja sadašnja eksperimentalna vrijednost parametra ρ, držeći u vidu razliku masa kvarkovskog dubleta četvrte generacije. Također se proučava širina raspada $b' \to b\gamma$, ali s popravkom do vodećih QCD logaritama za šest aktivnih okusa, nakon što su W bozon, te t i t' kvarkovi izintegrirani primjenom razvoja operatorskog umnoška. Konstanta jake interakcije uzela se je u skladu s masom Z bozona.