ISSN 1330–0016
CODEN FIZBE7

COLLECTIVE-FIELD FLUCTUATIONS AROUND THE WALL SOLUTION OF THE CHERN-SIMONS THEORY

IVAN ANDRIĆ, VELIMIR BARDEK and LARISA JONKE [1]

Theoretical Physics Division, Rudjer Bošković Institute, P.O. Box 1016, 10001 Zagreb, Croatia

Received 8 June 1998; Accepted 7 September 1998

We consider a large-N Chern-Simons theory for the attractive bosonic matter (Jackiw-Pi model) in the Hamiltonian collective-field approach based on the $1/N$ expansion. We show that the dynamics of low-lying density fluctuations around the semiclassical wall solution is governed by the Calogero Hamiltonian. The relationship between the Chern-Simons coupling constant κ and the Calogero statistical parameter λ signalizes some sort of statistical transmutation accompanying the dimensional reduction of the initial problem.

PACS numbers: 11.10.Lm, 74.20.Kk, 03.65.Sq, 05.30.-d UDC 539.12

Keywords: collective field, Jackiw-Pi model, large-N Chern-Simons theory, Calogero model

1. Introduction

Gauge models of a scalar field with a Chern-Simons [1] term admit static soliton solutions [2,3]. A lot is known about their structure, stability and connection with the Bogomol'nyi limit. These vortices were shown to exist only in the presence of a suitably tuned quartic contact interaction. What we want to address here is how to analyze the quantum dynamics of low-lying density fluctuations around such a solution. Using the collective-field approach based on the $1/N$ expansion, we show that the Chern-Simons theory for the attractive bosonic matter (Jackiw-Pi model) possesses the so-called wall solution, of which

[1]E-mail addresses: andric@thphys.irb.hr, bardek@thphys.irb.hr, larisa@thphys.irb.hr

the thickness goes to zero as N goes to infinity. This observation substantially simplifies the problem of quantum fluctuations around such a configuration and allows us to identify their dynamics with that of the Calogero model.

2. Chern-Simons theory

The Lagrangian for non-relativistic matter coupled to the Abelian Chern-Simons gauge field is [1,2]

$$\mathcal{L} = \mathrm{i}\psi^\dagger D_0\psi - \frac{1}{2}(\mathbf{D}\psi)^\dagger(\mathbf{D}\psi) + \frac{1}{2}\kappa\varepsilon^{\alpha\beta\gamma}A_\alpha\partial_\beta A_\gamma, \tag{1}$$

where $D_\mu = \partial_\mu + \mathrm{i}A_\mu$ is the covariant derivative of the gauge field and $\psi(\mathbf{r})$ is the non-relativistic matter field. The corresponding Hamiltonian is given by

$$H = \frac{1}{2}\int \mathrm{d}^2\mathbf{r}(\mathbf{D}\psi(\mathbf{r}))^\dagger(\mathbf{D}\psi(\mathbf{r})), \tag{2}$$

while the equations for the gauge field are

$$B = \varepsilon_{ij}\partial_i A_j = -\frac{1}{\kappa}\psi^\dagger\psi, \tag{3a}$$

$$\partial_0 A_i + \partial_i A_0 = -\frac{1}{\kappa}\varepsilon_{ij}J^j, \tag{3b}$$

where J^i is the gauge-invariant matter current:

$$\mathbf{J} = \frac{1}{2i}(\psi^\dagger\mathbf{D}\psi - (\mathbf{D}\psi)^\dagger\psi). \tag{4}$$

The Chern-Simons field has no physical degrees of freedom and is solved in terms of the matter field. The solutions of the gauge-field equations (3) in the Coulomb gauge $\nabla\mathbf{A} = 0$ are

$$\mathbf{A}(\mathbf{r}) = -\frac{1}{2\pi\kappa}\hat{\mathbf{n}}\times\int \mathrm{d}^2\mathbf{r}'\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^2}\psi^\dagger(\mathbf{r}')\psi(\mathbf{r}'), \tag{5a}$$

$$A^0(\mathbf{r}) = -\frac{1}{2\pi\kappa}\hat{\mathbf{n}}\times\int \mathrm{d}^2\mathbf{r}'\ln|\mathbf{r}-\mathbf{r}'|\left(\nabla'\times\mathbf{J}(\mathbf{r}')\right), \tag{5b}$$

where $\hat{\mathbf{n}}$ is the unit vector perpendicular to the plane in which the fields move. Using (5) we can rewrite the Hamiltonian (2) only in terms of matter fields, and quantize it by requiring the following bosonic commutation relations at equal times:

$$[\psi(\mathbf{r},t),\psi^\dagger(\mathbf{r}',t)] = \delta(\mathbf{r}-\mathbf{r}'), \tag{6}$$

with all other commutators vanishing. Now, we use the identity [2]

$$(\mathbf{D}\psi)^\dagger(\mathbf{D}\psi) = (D_+\psi)^\dagger(D_+\psi) + \varepsilon^{ij}\partial_i J_j + B\psi^\dagger\psi, \tag{7}$$

where $D_+ = D_1 + \mathrm{i}D_2$. With sufficiently well-behaved fields, so that the integral over all the space of $\nabla \times \mathbf{J}$ vanishes, we have

$$H = \frac{1}{2}\int \mathrm{d}^2\mathbf{r}(D_+\psi(\mathbf{r}))^\dagger(D_+\psi(\mathbf{r})) - \frac{1}{2\kappa}\int \mathrm{d}^2\mathbf{r}(\psi^\dagger(\mathbf{r})\psi(\mathbf{r}))^2. \tag{8}$$

If we add the following contact term to our system:

$$H_{\mathrm{int}} = g\int \mathrm{d}^2\mathbf{r}(\psi^\dagger(\mathbf{r})\psi(\mathbf{r}))^2, \tag{9}$$

the possibility of the Bogomol'nyi saturation arises. For a special choice of the coupling constant, $g = 1/2\kappa$, the Hamiltonian has the minimum value when $\psi(\mathbf{r})$ satisfies the static self-dual equation

$$D_+\psi(\mathbf{r}) = 0. \tag{10}$$

The addition of the contact interaction term H_{int} to our Hamiltonian (8) has been motivated by the possibility of reaching the self-dual limit, but it has already been shown [4] that once a renormalization is taken into account, the term (9) appears naturally.

3. Collective-field formulation of the model

From now on, we investigate the corresponding ground-state configuration in the large-N limit. Therefore, we reformulate the Jackiw-Pi Hamiltonian in terms of collective fields [5]. By decomposing the field $\psi(\mathbf{r})$ into the phase and the amplitude:

$$\psi(\mathbf{r}) = \mathrm{e}^{\mathrm{i}\pi(\mathbf{r})}\sqrt{\rho(\mathbf{r})}\,, \; \psi^\dagger(\mathbf{r}) = \sqrt{\rho(\mathbf{r})}\,\mathrm{e}^{-\mathrm{i}\pi(\mathbf{r})}, \tag{11}$$

we obtain the Hamiltonian in the form

$$H = \frac{1}{2}\int \mathrm{d}^2\mathbf{r}\rho(\mathbf{r})\left\{\nabla\pi(\mathbf{r}) + \hat{\mathbf{n}}\times\left[\frac{1}{2}\frac{\nabla\rho(\mathbf{r})}{\rho(\mathbf{r})} + \frac{1}{2\pi\kappa}\int \mathrm{d}^2\mathbf{r}'\rho(\mathbf{r}')\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^2}\right]\right\}^2, \tag{12}$$

while the self-dual equation yields

$$\nabla\pi(\mathbf{r}) + \hat{\mathbf{n}}\times\left[\frac{1}{2}\frac{\nabla\rho(\mathbf{r})}{\rho(\mathbf{r})} + \frac{1}{2\pi\kappa}\int \mathrm{d}^2\mathbf{r}'\rho(\mathbf{r}')\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^2}\right] = 0. \tag{13}$$

Here the new fundamental variables are the collective field (number density) $\rho(\mathbf{r}) = \psi^\dagger(\mathbf{r})\psi(\mathbf{r})$ and the operator $\pi(\mathbf{r})$, i.e. the canonical conjugate of the field $\rho(\mathbf{r})$. By using the Baker-Campbell-Hausdorf identity, it can be shown that $\rho(\mathbf{r})$ and $\pi(\mathbf{r})$ satisfy the correct commutation relation

$$[\rho(\mathbf{r}'), \nabla\pi(\mathbf{r})] = \mathrm{i}\nabla\delta(\mathbf{r}-\mathbf{r}'). \tag{14}$$

Since

$$\int d^2\mathbf{r}\rho(\mathbf{r}) = \int d^2\mathbf{r}\psi^\dagger(\mathbf{r})\psi(\mathbf{r}) = N, \tag{15}$$

where N is the total number of bosonic particles in the system, the conjugate momentum $\pi(\mathbf{r})$ goes as $1/N$ in order to satisfy the commutation relation (14). This fact allows to perform the $1/N$ expansion in the collective-field Hamiltonian and the self-dual equation (10) and get some insight into the structure of the semiclassical leading approximation and the next-to-leading quantum fluctuations. Performing the $1/N$ expansion and disregarding the kinetic term, we obtain the semiclassical equation for the lowest-energy collective-field configuration $\rho_0(\mathbf{r})$ [6]:

$$\frac{1}{2}\frac{\nabla\rho_0(\mathbf{r})}{\rho_0(\mathbf{r})} + \frac{1}{2\pi\kappa}\int d^2\mathbf{r}'\rho_0(\mathbf{r}')\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^2} = 0. \tag{16}$$

There exists an interesting solution, depending only on one variable, let us say x, for definiteness. Equation (16) reduces to

$$\frac{1}{2}\frac{\partial}{\partial x}\ln\rho_0(x) + \frac{1}{2\kappa}\int dx'\rho_0(x')\mathrm{sign}(x-x') = 0, \tag{17}$$

where we have used the result:

$$\int_{-L/2}^{L/2}\frac{dy'}{(x-x')^2+(y-y')^2} = \frac{\pi}{|x-x'|} + O(1/L)\ , \ x\neq x'. \tag{18}$$

Here, we have restricted the domain of the collective field $\rho(\mathbf{r})$ to an arbitrarily large but finite interval of length L along the y axis. It is always assumed that the limits $L\to\infty$ and $N\to\infty$ are taken simultaneously at the end of the calculations, keeping N/L fixed and large (thermodynamic limit). Applying the derivative with respect to x, we can transform Eq. (16) into the differential one:

$$\frac{1}{2}\frac{\partial^2}{\partial x^2}\ln\rho_0(x) + \frac{1}{\kappa}\rho_0(x) = 0. \tag{19}$$

This equation has a positive and normalizable solution given by

$$\rho_0(x) = \frac{A}{\cosh^2 Bx}, \tag{20}$$

where

$$A = \frac{N^2}{4\kappa L^2}\ , \ B = \frac{N}{2\kappa L}. \tag{21}$$

With increasing number of particles N, our wall solution $\rho_0(x)$ becomes thinner, finally taking the form of the δ-function distribution:

$$\rho_0(x) = \frac{N}{L}\delta(x). \tag{22}$$

Here, we have used the well-known representation of the δ-function:

$$\delta(x) = \lim_{\varepsilon\to 0} \frac{\exp(x/\varepsilon)}{\varepsilon[1+\exp(x/\varepsilon)]^2}\ ,\ \varepsilon = \frac{\kappa L}{N}. \tag{23}$$

Actually, the particles are restricted by their statistical interaction (effectively, the attractive δ-function interaction, see Ref. 7) to move along the y axis in the ground state.

4. *Quantum fluctuations*

At this point, we analyze the dynamics of the collective-field excitations around the wall-like solution of the Jackiw-Pi model. To this end, we perform the $1/N$ expansion of the collective field $\rho(\mathbf{r})$ in the form

$$\rho(\mathbf{r}) = \rho_0(\mathbf{r}) + \eta(\mathbf{r}), \tag{24}$$

where $\rho_0(\mathbf{r})$ is the ground-state semiclassical configuration and $\eta(\mathbf{r})$ a small density quantum fluctuation around $\rho_0(\mathbf{r})$. To proceed, we make the conjecture that the quantum fluctuation $\eta(\mathbf{r})$ contains $\rho_0(\mathbf{r})$ as a factor. This conjecture has been proved in different models in one and two dimensions, up to the quadratic terms in the field $\eta(\mathbf{r})$ and the momentum $\pi(\mathbf{r})$ in the collective Hamiltonian[5,8]. The expression (24) turns into

$$\rho(\mathbf{r}) = \rho_0(x)\tilde{\rho}(y), \tag{25}$$

suggesting that the residual one-dimensional fluctuations $\tilde{\rho}(y)$ exist in the y space. To find the dynamics of these fluctuations, we introduce the factorization form (25) into the collective Hamiltonian (12). It can be readily shown that the Hamiltonian reduces to

$$\begin{aligned} H = {} & \frac{N}{2L}\int \mathrm{d}x\mathrm{d}y\delta(x)\tilde{\rho}(y)\times \\ & \left[\frac{\partial\pi}{\partial x} - \frac{1}{2}\frac{\partial}{\partial y}\ln\tilde{\rho}(y) - \frac{N}{2L\pi\kappa}\int \mathrm{d}x'\mathrm{d}y'\delta(x')\tilde{\rho}(y')\frac{y-y'}{(x-x')^2+(y-y')^2}\right]^2 + \\ & \frac{N}{2L}\int \mathrm{d}x\mathrm{d}y\delta(x)\tilde{\rho}(y)\times \\ & \left[\frac{\partial\pi}{\partial y} + \frac{1}{2}\frac{\partial}{\partial x}\ln\rho_0(x) + \frac{N}{2L\pi\kappa}\int \mathrm{d}x'\mathrm{d}y'\delta(x')\tilde{\rho}(y')\frac{x-x'}{(x-x')^2+(y-y')^2}\right]^2 . \end{aligned} \tag{26}$$

According to Eq. (17) and the limiting form of the solution $\rho_0(x)$, Eq. (22), we interpret $\partial_x \ln\rho_0(x)$ as $-N/(\kappa L)$ sign$x$, and obtain

$$\begin{aligned} H = {} & \frac{N}{2L}\int \mathrm{d}y\tilde{\rho}(y)\left[\left.\frac{\partial\pi}{\partial x}\right|_{x=0} - \frac{1}{2}\frac{\partial}{\partial y}\ln\tilde{\rho}(y) - \frac{N}{2L\pi\kappa}⨍\mathrm{d}y'\frac{\tilde{\rho}(y')}{(y-y')}\right]^2 \\ & + \frac{N}{2L}\int \mathrm{d}y\tilde{\rho}(y)\left.\left(\frac{\partial\pi}{\partial y}\right)^2\right|_{x=0} . \end{aligned} \tag{27}$$

The integral over y' in the above expression must be evaluated by the principal-value prescription. Namely, because of the presence of the δ function, the integral kernel in the Hamiltonian (26)

$$\frac{y-y'}{(x-x')^2+(y-y')^2} \quad \text{changes to} \quad \frac{y-y'}{\varepsilon^2+(y-y')^2},$$

and as ε diminishes to zero, by definition, we have

$$\lim_{\varepsilon\to 0}\frac{y-y'}{\varepsilon^2+(y-y')^2} \equiv \mathcal{P}\frac{1}{y-y'}.$$

Having in mind the commutator (14) and the factorization structure of $\rho(\mathbf{r})$, Eq. (25), it can be shown by the chain rule of functional differentiation that the canonical momentum $\pi(\mathbf{r})$ transforms into

$$\pi(\mathbf{r}) \equiv -\mathrm{i}\frac{\delta}{\delta\rho(\mathbf{r})} = -\mathrm{i}\int \mathrm{d}z\frac{\delta\tilde{\rho}(z)}{\delta\rho(\mathbf{r})}\frac{\delta}{\delta\tilde{\rho}(z)} = -\mathrm{i}\frac{\delta(0)}{\rho_0(x)}\frac{\delta}{\delta\tilde{\rho}(y)} \equiv \frac{\delta(0)}{\rho_0(x)}\tilde{\pi}(y). \tag{28}$$

Accordingly, its derivation with respect to x vanishes:

$$\partial_x\pi(\mathbf{r})|_{x=0} = -\frac{L}{N}\frac{\partial_x\rho_0(x)}{\rho_0(x)}\bigg|_{x=0}\tilde{\pi}(y) = 0. \tag{29}$$

The Hamiltonian (27) is finally written in terms of the one-dimensional collective field $\tilde{\rho}(y)$ and its conjugate momentum $\tilde{\pi}(y)$ as

$$H = \frac{L}{2N}\int \mathrm{d}y\tilde{\rho}(y)\left(\frac{\partial\tilde{\pi}}{\partial y}\right)^2 + \frac{N}{2L}\int \mathrm{d}y\tilde{\rho}(y)\left(\frac{1}{2}\frac{\partial}{\partial y}\ln\tilde{\rho}(y) + \frac{N}{2L\pi\kappa}⨍\mathrm{d}y'\frac{\tilde{\rho}(y')}{(y-y')}\right)^2. \tag{30}$$

This is nothing but the collective-field Calogero Hamiltonian [9,10]. In order to establish a full correspondence, we should rescale the field $\tilde{\rho}(y) \to c\rho(y)$ and the momentum $\tilde{\pi}(y) \to \pi(y)/c$, so that the Hamiltonian becomes

$$H = \frac{L}{cN}\left\{\frac{1}{2}\int \mathrm{d}y\rho(y)\left(\frac{\partial\pi}{\partial y}\right)^2 + \frac{1}{2}\int \mathrm{d}y\rho(y)\left[\frac{\lambda-1}{2}\frac{\partial}{\partial y}\ln\rho(y) + \lambda⨍\mathrm{d}y'\frac{\rho(y')}{y-y'}\right]^2\right\}. \tag{31}$$

Here the constant c, the Chern-Simons coupling constant κ and the Calogero statistical parameter λ are interrelated by

$$\frac{N}{L}c = \lambda - 1 \text{ and } \frac{1}{2\pi\kappa}\frac{N^2c^2}{L^2} = \lambda, \tag{32}$$

finally leading to the relation

$$2\pi\kappa = \frac{(\lambda-1)^2}{\lambda}. \tag{33}$$

One can see that the relation (33) is invariant against the duality transformation $\lambda \to 1/\lambda$, reflecting the well-known symmetry of the Calogero model [11]. Simultaneously with the dimensional reduction, our system exhibits some sort of statistical transmutation.

Acknowledgements

This work was supported by the Ministry of Science and Technology of the Republic of Croatia under contract No. 00980103.

References

1) R. Jackiw and S. Templeton, Phys. Rev. **D23** (1981) 2291; J. Schonfeld, Nucl. Phys. **B185** (1981) 157; S. Deser, R. Jackiw and S. Templeton, Phys. Rev. Lett. **48** (1982) 975;

2) R. Jackiw and S. Y. Pi, Phys. Rev. Lett. 64 (1990) 2969; Phys. Rev. **D42** (1990) 3500; Prog. Theor. Phys. (Suppl.) **107** (1992) 1;

3) J. Hong, Y. Kim and P. Y. Pack, Phys. Rev. Lett. **64** (1990) 2230; R. Jackiw and E. Weinberg, Phys. Rev. Lett. **64** (1990) 2234; Z. F. Ezawa, M. Hotta and A. Iwazaki, Phys. Rev. **D44** (1991) 452; P. K. Ghosh, Phys. Rev. **D49** (1994) 5458; I. V. Barashenkov and A. O. Harin, Phys. Rev. **D52** (1995) 2471;

4) S.-J. Kim, Phys. Lett. **B343** (1995) 244; S.-J. Kim and C. Lee, Phys. Rev. D55 (1997) 2227;

5) A. Jevicki and B. Sakita, Nucl. Phys **B165** (1980) 511; Phys. Rev. **D22** (1980) 467;

6) I. Andrić and V. Bardek, Phys. Rev. **D47** (1993) 5546; Mod. Phys. Lett. **A7** (1992) 3267; Phys. Lett. **B318** (1993) 99;

7) I. Andrić, V. Bardek and L. Jonke, *Calogero-Sutherland model from the excitations of Chern-Simons vortices*, hep-th/9507110;

8) J. Shapiro, Nucl. Phys. **B184** (1981) 218; I. Andrić, V. Bardek and L. Jonke, Fizika **B4** (1995) 93;

9) F. Calogero, J. Math. Phys. **10** (1969) 2191, 2197; J. Math. Phys. **12** (1971) 419;

10) I. Andrić, A. Jevicki and H. Levine, Nucl. Phys. **B215** (1983) 307; I. Andrić and V. Bardek, J. Phys. **A21** (1988) 2847;

11) J. A. Minahan and A. P. Polychronacos, Phys. Rev. **B50** (1994) 4236.

FLUKTUACIJE KOLEKTIVNOG POLJA OKO ZIDNOG RJEŠENJA CHERN-SIMONSOVE TEORIJE

Razmatra se Chern-Simonsova teorija za privlačnu bozonsku tvar (Jackiw-Pi model) u pristupu hamiltonijana kolektivnog polja zasnovanog na $1/N$ razvoju. Pokazuje se da dinamikom niskoležećih fluktuacija gustoće oko poluklasičnog zidnog rješenja upravlja Calogerov hamiltonijan. Odnos izmedu Chern-Simonsove konstante vezanja κ i Calogerovog statističkog parametra λ ukazuje na neku vrstu statističke transmutacije koja prati smanjenje dimenzija početnog problema.

æ