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POLE-DOMINANCE MODEL AND THE TWO-GAMMA DECAY OF B AND D MESONS

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We give a detailed study of the two-gamma decay of some heavy pseudoscalar mesons using the pole-dominance model. The transition matrix elements of P_L to bare glueball (G^0) are also computed, and it is found that the P_L to glueball contribution is comparable with that of P_L to π^0. From our results, we have obtained the two-gamma decay width of D_L and an upper limit of the two-gamma decay widths of B_L and B_{sL} mesons.

1. Introduction

In our previous paper [1], we have evaluated two-gamma decay widths of some heavy preudoscalar mesons in the Standard Model, and it has been found that the decay widths are very small. The reason for this is that in our earlier paper we have taken into consideration only the short distance contribution, neglecting the long distance contribution for these decays. The long distance contribution for $P_L \to 2\gamma$ decay can be taken into account by the meson pole model. Hence, we have started with a phenomenological meson pole model and for this purpose we have taken the contributions from $\pi^0, \eta, \eta', \iota, \eta_c$ and η_b pseudoscalar poles. Here we want to mention that η, η' and ι are SU(3) mixed states

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of quarkonium and gluonium. Now, in order to obtain the two-gamma decay width of D^0 meson, we should consider the $20''$ dimensional representation SU(4) symmetry, because in the standard theory, H_W can be decomposed into the form $H_w \approx c_- O_- + c_+ O_+ (+$ penguin). Then H_w transforms like $H_w \approx 20''$ under SU(4) symmetry unless the penguin contribution is magically enhanced. For simplicity, we may assume that in the case of B meson decay, SU(4) symmetry is formed with the (u, d, s and b) quarks instead of (u, d, s and c) quarks. Here, for simplicity, we also assume η_b as a pure bottonium state and η_c as a pure charmonium state. For calculating two-γ decay width of P_L meson, we have taken into account the glueball and charm (or bottom) contribution along with η and η' poles. Therefore, the mixing of these pseudoscalars with the glueball is to be taken into consideration. The existence of pure glueball is a clear prediction of QCD, but even now the experimental detection of such particles is not possible. The meson pole approach has been described by other authors [2-6] to consider the long distance effects for $K_L \to 2\gamma$ decay. Hence, in our paper, we consider the effects of all possible poles on the $P_L \to 2\gamma$ decay by assuming SU(4) symmetry.

2. Theory

The pole contributions to the amplitude of $P_L \to 2\gamma$ decay are given by

$$A(P_L \to 2\gamma) = \sum_{P_i} \langle \pi^0 | H_W | P_L^0 \rangle \frac{A(P_i \to 2\gamma)}{m^2_{P_L^0} - m^2_{P_i}}, \tag{1}$$

where the summation over P_i is over all possible poles, that is $\pi^0, \eta, \eta', \iota, \eta_c$ and η_b; P_L refers to D_L, B_L and B_{sL}. In general, π^0, η, η' and ι will mix, but it has been shown that the mixing of π^0 with others is very small and can be neglected. Now, η is strongly dominated by η_8 component and small part of η_1 and G^0, while η' contains a small η_8 component and large share of other states with η_1 and G^0. Also, ι should be primarily a superposition of G^0 and η_1 and a very small component η_8. The physical states $|\eta\rangle, |\eta'\rangle$ and $|\iota\rangle$ can be expressed in terms of the standard SU(3) states belonging to the 1 + 8 representation, and their mixing matrix is given below as

$$\begin{aligned} |\eta\rangle &= a_1 |\eta_8\rangle + a_2 |\eta_1^{SU(3)}\rangle + a_3 |G^0\rangle, \\ |\eta'\rangle &= b_1 |\eta_8\rangle + b_2 |\eta_1^{SU(3)}\rangle + b_3 |G^0\rangle, \\ |\iota\rangle &= d_1 |\eta_8\rangle + d_2 |\eta_1^{SU(3)}\rangle + d_3 |G^0\rangle, \end{aligned} \tag{2}$$

where

$$\begin{array}{lll} a_1 = \;\; 0.97 & a_2 = 0.22 & a_3 = -0.004 \\ b_1 = -0.24 & b_2 = 0.89 & b_3 = -0.016 \\ d_1 = \;\; 0 & d_2 = 0.02 & d_3 = \;\; 0.092\,. \end{array}$$

Let us define the ratio of the various decay amplitudes of P^0 as

$$R_1 = \frac{\langle \eta_8 | H_W | P^0 \rangle}{\langle \pi^0 | H_W | P^0 \rangle}, \tag{3}$$

$$R_2 = \frac{\langle \eta_1^{SU(3)} | H_W | P^0 \rangle}{\langle \pi^0 | H_W | P^0 \rangle}, \tag{4}$$

$$R_3 = \frac{\langle G^0 | H_W | P^0 \rangle}{\langle \pi^0 | H_W | P^0 \rangle}, \tag{5}$$

$$R_4 = \frac{\langle \eta_{20} | H_W | P^0 \rangle}{\langle \pi^0 | H_W | P^0 \rangle}, \tag{6}$$

where P^0 refers to D^0, B^0 and B_s^0. Here the weak Hamiltonian H_W for the decay of P^0 mesons actually belongs to the $20''$ dimensional representation of SU(4). The SU(4) CG coefficient is the product of [SU(2) singlet factor]×[SU(3) isoscalar factor]× [CG coefficients for SU(2)]. In the SU(4) consideration, using [7], we have obtained R_1, R_2 and R_4 as $\sqrt{1/3}$, $\sqrt{2/3}$ and $\sqrt{1/3}$, respectively. For simplicity, we define $|\eta_q\rangle$ as

$$|\eta_q\rangle = |\bar{q}q\rangle, \tag{7}$$

where q indicates to b or c quark and $|\eta\rangle, |\eta'\rangle$ and $|\iota\rangle$ are SU(3) mesons, so they are free from the $|\eta_{20}\rangle$ state. We can express $|\eta_q\rangle$ in terms of three states of $20''$ dimensional representation of SU(4) in addition to gluonium state $|G^0\rangle$

$$|\eta_q\rangle = p_1|\eta_8\rangle + p_2|\eta_1^{SU(3)}\rangle + p_3|G^0\rangle + p_4|\eta_{20}\rangle, \tag{8}$$

with $p_1 = 0, p_2 = \sqrt{1/3}, p_3 = 0$ and $p_4 = -\sqrt{1/3}$. Taking all the meson poles into account, we can write the two-gamma decay amplitude of P_L as

$$A(P_L \to 2\gamma) = \frac{A(\pi^0 \to 2\gamma)\langle \pi^0 | H_W | P^0 \rangle}{m_{P^0}^2 - m_\pi^2} \times \tag{9}$$

$$\left\{ 1 + \frac{m_{P^0}^2 - m_\pi^2}{m_{P^0}^2 - m_\eta^2} \frac{A(\eta \to 2\gamma)}{A(\pi^0 \to 2\gamma)} (R_1 a_1 + R_2 a_2 + R_3 a_3) + \right.$$

$$\frac{m_{P^0}^2 - m_\pi^2}{m_{P^0}^2 - m_{\eta'}^2} \frac{A(\eta' \to 2\gamma)}{A(\pi^0 \to 2\gamma)} (R_1 b_1 + R_2 b_2 + R_3 b_3) +$$

$$\frac{m_{P^0}^2 - m_\pi^2}{m_{P^0}^2 - m_\iota^2} \frac{A(\iota \to 2\gamma)}{A(\pi^0 \to 2\gamma)} (R_1 d_1 + R_2 d_2 + R_3 d_3) +$$

$$\frac{m_{P^0}^2 - m_\pi^2}{m_{P^0}^2 - m_{\eta_q}^2} \frac{A(\eta_q \to 2\gamma)}{A(\pi^0 \to 2\gamma)} (R_2 p_2 + R_4 p_4) \Big\} .$$

3. Calculations

3.1. Determination of R_3

The lowest order contributions to $P_L \to G^0(2g)$ is shown in Figs. 1a and b. The evaluation of these diagrams is straightforward and can be carried out in a way similar to the decay $P_L \to 2\gamma$ [1]. Here we need to change the coupling of the quark photon vertex $Qe\overline{q}\gamma_\mu q A^\mu$ to the quark gluon vertex $g_s \overline{q}\gamma_\mu (\lambda^c/2) q G^c_\mu$, where $(\lambda_c/2)$ is the SU(3)$_c$ generator. The amplitude of the $P_L \to 2g$ transition is given by

$$A(P_L \to 2g) = \frac{9 G_F \alpha_s}{8\pi} f_P \overline{G}^c_{\mu\nu} G^c_{\mu\nu} G \sum Re(V_{im} V^*_{in})[A^I_i + A^R_i], \tag{10}$$

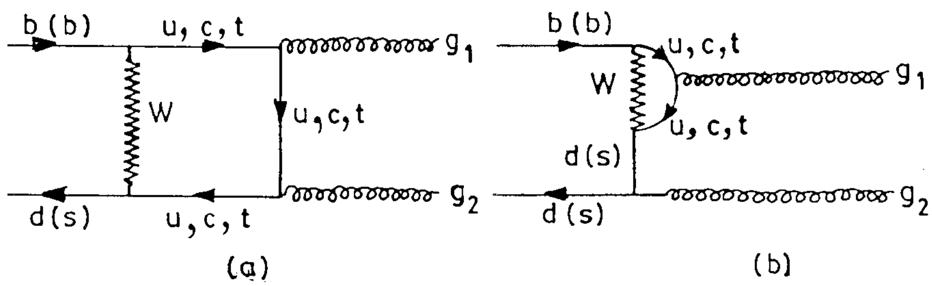

Fig. 1. The lowest-order contributions of P_L (i.e. B_L and B_{sL}) to glueball transition amplitude: a) the reducible contribution, and b) the reducible contribution from the penguin diagram. Note: In the case of D meson, b and d will be replaced by c and u, and u, c and t will be replaced by d, s and b, respectively.

where V_{ij} is the Kobayashi-Maskawa matrix element, $i = u, c, t$ for B and B_{sL} and $m = d, n = b$ for B_L and $m = s$ and $n = d$ for B_{sL} meson, whereas for D meson $i = d, s, b$ and $m = c$ and $n = u$. A^I_i is the irreducible contribution and is obtained from Fig. 1a as

$$A^I_i = \frac{4x_i}{x_B} \int_0^1 \frac{\mathrm{d}y}{y} \ln \left[1 - y(1-y) \frac{x_P}{x_i} \right] , \tag{11}$$

where $x_i = (m_i^2/m_W^2), x_P = (m_P^2/m_W^2)$ and A^R_i is the reducible contribution and is obtained from Fig. 1b as:

$$A^R_i = \xi_{P_L} \left[\frac{1 - 5x_i - 2x_i^2}{(1-x_i)^3} - \frac{6x_i^2 \ln x_i}{(1-x_i)^4} \right] . \tag{12}$$

ξ_{P_L} is a kinematical factor and is given by

$$\xi_{P_L} = \frac{m^2_{p^0}}{16} \langle P_L | \frac{1}{q_1 p_1} + \frac{1}{q_2 p_2} + \frac{1}{q_1 p_2} + \frac{1}{q_2 p_1} | P_L \rangle, \tag{13}$$

where $p_{1,2}$ are the momenta of the valence quarks inside the P^0 mesons.

Using the value $\xi_\pi = 0.055$ [5], we have obtained $\xi_B = 41.60, \xi_{Bs} = 35.19$ and $\xi_D = 10.94$. To convert $\overline{G}^c_{\mu\nu} G^c_{\mu\nu}$ into a glueball, we parametrise as

$$\langle 0 | \alpha_s \overline{G}^c_{\mu\nu} G^c_{\mu\nu} | G^0 \rangle = f_G m^2_G. \tag{14}$$

Using the QCD sum rule, we have $f_G = 0.242$ GeV for $m_G = 1.4$ GeV. Taking the strong coupling constants $\alpha_s = 0.28$ [8], $f_B = 187$ MeV, $f_{Bs} = 207$ MeV and $f_{Ds} = 208$ MeV [9], and substituting the values of all parameters in equation (10), we have obtained the amplitudes of P_L to G^0 decay as

$$\begin{aligned} \langle G^0 | H_W | B_L \rangle &= 2.12 \times 10^{-8} \text{ GeV}^2, \\ \langle G^0 | H_W | B_{sL} \rangle &= 4.15 \times 10^{-7} \text{ GeV}^2, \\ \langle G^0 | H_W | D_L \rangle &= 1.77 \times 10^{-8} \text{ GeV}^2. \end{aligned} \tag{15}$$

Now, using the results of Ref. 10 and 11, we have obtained the Cabibbo favoured mixing amplitudes for $P_L \to \pi^0 \pi^0$ decay as

$$\begin{aligned} \langle \pi^0 | H_W | B_L \rangle &< 4.47 \times 10^{-8} \text{ GeV}^2, \\ \langle \pi^0 | H_W | B_{sL} \rangle &< 7.16 \times 10^{-7} \text{ GeV}^2, \\ \langle \pi^0 | H_W | D_L \rangle &= 8.45 \times 10^{-8} \text{ GeV}^2. \end{aligned} \tag{16}$$

Therefore, the values of R_3 for B_L, B_{sL} and D_L, are, respectively, the following

$$\begin{aligned} R_3 &= \frac{\langle G^0 | H_W | B_L \rangle}{\langle \pi^0 | H_W | B_L \rangle} > 0.58, \\ R_3 &= \frac{\langle G^0 | H_W | B_{sL} \rangle}{\langle \pi^0 | H_W | B_{sL} \rangle} > 0.47, \\ R_3 &= \frac{\langle G^0 | H_W | D_L \rangle}{\langle \pi^0 | H_W | D_L \rangle} = 0.21. \end{aligned} \tag{17}$$

The above results show that the glueball to P_L transition amplitude is not negligible in comparison with the π^0 to P_L transition amplitude. In our numerical calculation, we may take any value greater than 0.58 and 0.47 of R_3 for B_L and B_{sL}, respectively. For simplicity, we use $R_3 = 0.58$ for B_L and $R_3 = 0.47$ for B_{sL}.

3.2. Estimation of the decay amplitudes of the pseudoscalar mesons π^0, η and η'

The two-gamma decay amplitudes of the pseudoscalar mesons can be obtained from the experimental values of the decay widths of π^0, η and η' [11]

$$\begin{aligned} \Gamma(\pi^0 \to 2\gamma) &= 7.73 \text{ eV}, \\ \Gamma(\eta \to 2\gamma) &= 0.46 \text{ keV}, \\ \Gamma(\eta' \to 2\gamma) &= 4.26 \text{ keV}. \end{aligned} \tag{18}$$

From these data, we can easily estimate the magnitude of decay amplitudes of the above pseudoscalar mesons as

$$\begin{aligned} A(\pi^0 \to 2\gamma) &= 2.52 \times 10^{-5} \text{ MeV}^{-1}, \\ A(\eta \to 2\gamma) &= 2.39 \times 10^{-5} \text{ MeV}^{-1}, \\ A(\eta' \to 2\gamma) &= 3.12 \times 10^{-5} \text{ MeV}^{-1}. \end{aligned} \tag{19}$$

3.3. Estimation of $\iota \to 2\gamma$ decay amplitude

To estimate the $\iota \to 2\gamma$ decay amplitude, we first need to compute the amplitude of $G^0 \to 2\gamma$ decay. The latter amplitude is estimated in [12] in the chiral Lagrangian as

$$A(G^0 \to 2\gamma) = 0.23 A(\pi^0 \to 2\gamma). \tag{20}$$

But the short distance contribution of the above decay, estimated from the Euler-Heisenberg diagram, is found to be very small [13]

$$A(G^0 \to 2\gamma) = 0.03 A(\pi^0 \to 2\gamma). \tag{21}$$

It follows that [14]

$$A(\iota \to 2\gamma) = A(\pi^0 \to 2\gamma) \left[1.63 s_2 \left(\frac{f_\pi}{f_{\eta_1}} \right) + 0.23 c_2 \right]. \tag{22}$$

TABLE 1. Dependence of $A(\iota \to 2\gamma)$ on s_2.

s_2	$\lvert c_2 \rvert$	$A(\iota \to 2\gamma)$ (MeV^{-1})
– 0.40	0.84	$(1.16 – 2.13)\times 10^{-5}$
– 0.20	0.98	$(0.25 – 1.39)\times 10^{-5}$
0.00	1.00	0.58×10^{-5}
0.10	0.99	$(0.16 – 1.23)\times 10^{-5}$

From Table 1, we observe that the magnitude of the amplitude of $\iota \to 2\gamma$ varies in the range $(0.16 - 2.13) \times 10^{-5}$ MeV^{-1} with s_2. In the following calculations, we use $A(\iota \to$

$2\gamma) = 1.15 \times 10^{-5}$ MeV^{-1}. With the above data, we have obtained the decay amplitudes of $P_L \to 2\gamma$ decay as

$$\begin{aligned} A(B_L \to 2\gamma) &< 4.32 \times 10^{-11}\ \mathrm{MeV}^{-1}, \\ A(B_{sL} \to 2\gamma) &< 2.92 \times 10^{-10}\ \mathrm{MeV}^{-1}, \\ A(D_L \to 2\gamma) &= 1.62 \times 10^{-11}\ \mathrm{MeV}^{-1}. \end{aligned} \tag{23}$$

Therefore, the two-gamma decay widths of B_L, B_{sL} and D_L in the pole-dominance model are as follows

$$\begin{aligned} \Gamma(B_L \to 2\gamma) &< 1.37 \times 10^{-12}\ \mathrm{MeV}, \\ \Gamma(B_{sL} \to 2\gamma) &< 6.59 \times 10^{-11}\ \mathrm{MeV}, \\ \Gamma(D_L \to 2\gamma) &= 8.47 \times 10^{-15}\ \mathrm{MeV}. \end{aligned} \tag{24}$$

4. Results and discussion

From our results, we have obtained an upper limit of the two-gamma decay widths of the heavy pseudoscalar B and B_s mesons and the partial decay width of the D meson. In the pole-dominance model, the two-gamma decay amplitudes of B_L, B_{sL} and D_L mesons are larger than that of the quark model. Pole dominance model predictions are

$$\begin{aligned} A(B_{sL} \to 2\gamma) &< 2.92 \times 10^{-10}\ \mathrm{MeV}^{-1}, \\ A(B_L \to 2\gamma) &< 4.32 \times 10^{-11}\ \mathrm{MeV}^{-1}, \\ A(D_L \to 2\gamma) &= 1.62 \times 10^{-11}\ \mathrm{MeV}^{-1}, \end{aligned}$$

while the quark model predictions [1] are

$$\begin{aligned} A(B_{sL} \to 2\gamma) &< 0.90 \times 10^{-10}\ \mathrm{MeV}^{-1}, \\ A(B_L \to 2\gamma) &< 1.19 \times 10^{-11}\ \mathrm{MeV}^{-1}, \\ A(D_L \to 2\gamma) &= 0.88 \times 10^{-11}\ \mathrm{MeV}^{-1}. \end{aligned}$$

We conclude that the quark model results for the two-gamma decay widths of heavy P_L mesons should be enhanced when we take into account the long-distance effect by meson pole model. Actually, the total decay width of P_L mesons will be the sum of the short-distance as well as of long-distance contributions. Moreover, from our reults for the two-gamma decay widths of B_{sL}, B_L and D_L mesons, we observed that as the meson becomes heavy, the two-gamma decay width increases.

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DVOFOTONSKI RASPAD B I D MEZONA U MODELU PREVLADAVAJUĆIH POLOVA

Podrobno se proučavaju dvofotonski raspadi teških pseudoskalarnih mezona na osnovi modela prevladavajućih polova. Izračunavaju se matrični elementi prijelaza P_L u golu gluonsku loptu (G^0) i nalazi da je doprinos tog procesa usporediv s doprinosom prijelaza P_L u π^0. Izračunate su širina dvofotonskog raspada mezona D_L i gornje granice širina dvofotonskih raspada B_L i B_{sL} mezona.