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ARE THE LEPTONS A THREE-FERMION COMPOSITE OR A FERMION-BOSON COMPOSITE

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By considering the leptons to be either a three-fermion composite or a fermion-boson composite, we discuss the signatures for each model for spin polarization precession experiments, assuming that Markov jump processes are operative for the precessing particles in high magnetic fields.

1. *Introduction*

Certainly one of the major problems in all of modern day physics is centered around finding a more symmetric and more fundamental theory of weak, strong and electromagnetic interactions that reduces to the Standard Model in the GeV range [1]. Technicolor [2], grand–unification [3], supersymmetry [4], compositeness [5] and supergravity theories [6] all represent alternatives to the Standard Model that are intended to reveal a deeper level of understanding of the fundamental interactions. The chiral structure of the weak current, the origin of the quark masses, the origin of the generations, the origin of the quark mixing angles, the fundamental mechanism of symmetry breaking and the apparent absence of strong CP violations all represent challenging problems that a more fundamental theory should explain [7]. In this note we adopt the pathway of compositeness, mainly because all previous systems (atoms, nuclei, hadrons) have revealed a composite structure and it seems natural that the quarks, leptons, gauge bosons and Higgs particles should also possess a composite structure [8]. The usual manner through which the composite structure of particles is proved is through anomalous magnetic moments, form–factors and rare decays [9]. We however choose a different window through which to probe for compositeness, namely by using the corrections induced by discrete time quantum theory and Markov environmental processes, which give rise to distinct signatures for spin polarization precession of particles in a strong magnetic field [10–12]. The rudiments of discrete time quantum theory actually go back to the pioneering thoughts of Wheeler [13], Finkelstein [14] and Bombelli et al. [15] who pictured the substratum as a discrete structure with Minkowski space, and field theory emerging after an averaging process over a discrete combinatoric world. In field theory both Snyder [16] and t'Hooft [17] have studied discrete lattice versions of QED and quantum gravity, and T. D. Lee [18] used a discrete time structure to make path integrals more well defined. Actually, Caldirola [19,20] was the first investigator to study a discrete time difference quantum theory which was the result of a microscopic uncertainty principle in time, suggesting that due to the uncertainty between a particle's sense of time (microuniverse) and the surrounding averaged out frame of synchronous observes, a discrete time difference should replace a time derivative in the quantum equation of motion, expressing the uncertainty in the response time of the particle to an externally applied Hamiltonian [21]. We have applied these ideas to electron spin resonance [22], electron spin polarization precession [23], spectral shifts in hydrogen [24] and in the search for hidden internal quantum numbers of particles [25]. We have also applied the idea to the problem of looking for gauge-boson composite structure [26]. The above applications are really applications of a discrete time difference theory operating in a background of continuous space-time. Recently, we have discussed pure discrete time jump processes (Markov processes) and the influence they have on spin polarization precession of a composite gauge boson in a magnetic field (Refs. 11 and 12). Our analysis demonstrated that the spin polarization amplitude is sensitive to the internal structure of the particle being studied, with distinct signatures for a two–preon composite structure (with identical preons), as oposed to a two–preon composite structure with different preons. Both of these signatures differ from the signature produced for a point like (non-composite gauge boson). In the present note, we extend these models to the problem of lepton composite structure where the lepton can be three–fermion composite or a fermion–spin-one–boson composite. The signatures produced in spin polarization precession for these two models are distinctly different and provide us with a potential probe to the composite structure of leptons.

2. Lepton composite structure and spin polarization precession induced by Markov environmental processes

We begin by recalling the approach used in Refs. 11 and 12. We first consider a system of a three-fermion [27,28] (two identical, one different) composite lepton in a magnetic field ($B_z = B$). The Hamiltonian is

$$((q)_I = (q)_{II} = -e_1, \;\; m_I = m_{II} = m_1, \;\; q_{III} = -e_2, m_{III} = m_2)$$

$$H = \frac{e_1}{m_1} S_{z_1} B + \frac{e_1}{m_1} S_{z_2} B + \frac{e_2}{m_2} S_{z_3} B. \tag{2.1}$$

For the composite wave function we choose

$$\Psi_\uparrow = \frac{1}{\sqrt{2}} \left(\alpha(1)\beta(2) - \beta(1)\alpha(2)\right) \alpha(3) \quad \text{spin–up state,} \tag{2.2}$$

$$\Psi_\downarrow = \frac{1}{\sqrt{2}} \left(\alpha(1)\beta(2) - \beta(1)\alpha(2)\right) \beta(3) \quad \text{spin–down state.} \tag{2.3}$$

Here the two identical fermions are in an anti–symmetric state. Also, if the charge of the composite lepton is $-e$ and the composite mass m, we have

$$\frac{e_1}{m_1} = \frac{e_2}{m_2} = \frac{e}{m} \tag{2.4}$$

(to insure that the spin–up state has $E_+ = e\hbar B/2m$ and the spin down state has $E_- = -e\hbar B/2m$).

For the spin–up state (Eq. (2.2)) we have the total wave function including t

$$\Psi_+ = \left(\frac{\alpha\beta - \beta\alpha}{\sqrt{2}}\right) \alpha \exp(-\frac{\mathrm{i}E_+}{\hbar} t) \tag{2.5}$$

and for spin–down state

$$\Psi_- = \left(\frac{\alpha\beta - \beta\alpha}{\sqrt{2}}\right) \beta \exp(-\frac{\mathrm{i}E_-}{\hbar} t). \tag{2.6}$$

We now consider a state initially polarized in the x-direction so that

$$\Psi = \frac{1}{\sqrt{2}} \Psi_+ + \frac{1}{\sqrt{2}} \Psi_-. \tag{2.7}$$

Eq. (2.7) insures $< S_x >_{t=0} = \Psi^+ (S_{x_1} + S_{x_2} + S_{x_3}) \Psi = \hbar/2$.

We now modify Eq. (2.7) at time t to account for Markov environmental jump processes. For a two (up and down state) step process for each preon we have [29]

$$P_n(+) = \frac{p}{p+q} + (1-p-q)^n \left(\frac{1}{2} - \frac{p}{p+q}\right),$$

$$P_n(-) = \frac{q}{p+q} + (1-p-q)^n \left(\frac{1}{2} - \frac{q}{p+q}\right), \tag{2.8}$$

where the initial probabilities of $\pm$ are $1/2$ for both up and down state of the preons. We call $P_n(+)$, $P_n(-)$ the Markov probability for up and down states after

n steps of preons 1 and 2 (both identical preons 1 and 2 have same $P_n(+)$, $P_n(-)$), and $P_{0_n}(+)$, $P_{0_n}(-)$ the Markov probability after n steps for preon 3. Here p_1 and q_1 refer to preons 1 and 2; and p_2, q_2 refer to preon 3 in the two-state transition matrix,

$$M_{1,2} = \begin{array}{c} \\ + \\ - \end{array} \begin{array}{c} \begin{array}{cc} + & - \end{array} \\ \begin{pmatrix} 1-q_1 & q_1 \\ p_1 & 1-p_1 \end{pmatrix} \end{array}, \quad M_3 = \begin{array}{c} \\ + \\ - \end{array} \begin{array}{c} \begin{array}{cc} + & - \end{array} \\ \begin{pmatrix} 1-q_2 & q_2 \\ p_2 & 1-p_2 \end{pmatrix} \end{array}. \tag{2.9}$$

Adjoining the Markov probabilities to Eq. (2.7), we have assuming the initial probabilities of up and down equal 1/2

$$\Psi = \sqrt{2}\left(\sqrt{P_n(+)P_n(-)P_{0_n}(+)}\alpha\beta\alpha - \sqrt{P_n(-)P_n(+)P_{0_n}(+)}\beta\alpha\alpha\right)\exp(-\frac{\mathrm{i}E_+}{\hbar}t)$$
$$+\sqrt{2}\left(\sqrt{P_n(+)P_n(-)P_{0_n}(-)}\alpha\beta\beta - \sqrt{P_n(-)P_n(+)P_{0_n}(-)}\beta\alpha\beta\right)\exp(-\frac{\mathrm{i}E_-}{\hbar}t). \tag{2.10}$$

We now consider

$$< S_x >= \Psi^+(S_{x_1} + S_{x_2} + S_{x_3})\Psi \tag{2.11}$$

where

$$S_{x_1} = S_{x_2} = S_{x_3} = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \tag{2.12}$$

The result of calculating Eq. (2.11) for the x–spin polarization after n Markov steps at time t is after much labour

$$< S_x >= 4\hbar\sqrt{P_n(+)^2 P_n(-)^2 P_{0_n}(+)P_{0_n}(-)}\cos\frac{eB}{m}t. \tag{2.13}$$

Thus the distinctive feature of the three–preon (fermion) model of the lepton in Eq. (2.13) is an amplitude of $< S_x >$ that varies with n according to the coefficient in Eq. (2.13).

We now turn to a (fermion–spin-one–boson) composite model of a charged lepton of charge $-e$ and mass m. For the Hamiltonian in a z–component magnetic field we have

$$H = \frac{e}{m_1}S_{z_1}B + \frac{e_2}{m_2}S_{z_2}B \tag{2.14}$$

assuming again Eq. (2.4). Thus $H = eB(S_{z_1} + S_{z_2})/m$. For the spin one component we have

$$\begin{aligned} &U_{1,1} = \text{basic function of } S = 1, \;\; S_z = 1, \\ &U_{1,0} = \text{basic function of } S = 1, \;\; S_z = 0, \\ &U_{1,-1} = \text{basic function of } S = 1, \;\; S_z = -1, \\ &\alpha,\ \beta = \text{spin } 1/2 \text{ states of } \;S_z = \pm\tfrac{1}{2}. \end{aligned} \tag{2.15}$$

For the spin–up ($S_z = 1/2$) composite state we have

$$\Psi_+ = \left(\sqrt{\frac{2}{3}}U_{1,1}\beta - \sqrt{\frac{1}{3}}U_{1,0}\alpha\right)\exp(-\frac{\mathrm{i}E_+}{\hbar}t) \tag{2.16}$$

the spin–down state is

$$\Psi_- = \left(\sqrt{\frac{1}{3}}U_{1,0}\beta - \sqrt{\frac{2}{3}}U_{1,-1}\alpha\right)\exp(-\frac{\mathrm{i}E_-}{\hbar}t) \tag{2.17}$$

$$E_\pm = \pm\frac{e\hbar B}{2m}. \tag{2.18}$$

For a combination of Ψ_+ and Ψ_- that gives $< S_x >_{t=0} = \hbar/2$, we have

$$\Psi = \frac{1}{\sqrt{2}}\Psi_+ + \frac{1}{\sqrt{2}}\Psi_-$$

or

$$\Psi = \left(\sqrt{\frac{1}{3}}U_{1,1}\beta - \sqrt{\frac{1}{6}}U_{1,0}\alpha\right)\exp(-\frac{\mathrm{i}E_+}{\hbar}t)$$
$$+\left(\sqrt{\frac{1}{6}}U_{1,0}\beta - \sqrt{\frac{1}{3}}U_{1,-1}\alpha\right)\exp(-\frac{\mathrm{i}E_-}{\hbar}t). \tag{2.19}$$

We now replace the numerical coefficients in Eq. (2.19) by Markov probabilities so as to reduce to Eq. (2.19) at $t = 0$. This can be done by setting $P_{n=0} = 2/5$ for probability of $S_z = 1$ state at $t = 0$, $P_{n=0}(-1) = 2/5$, $P_{n=0}(0) = 1/5$ for the spin–1 boson. Also $\overline{P}_{n=0}(1/2) = 1/2$, $\overline{P}_{n=0}(-1/2) = 1/2$ for initial probabilities of spin 1/2 fermion. Thus Eq. (2.19) becomes ammended by the Markov processes for the $S = 1$ and $S = 1/2$ preons

$$\Psi = \left(\sqrt{\frac{5}{3}}\sqrt{P_n(+1)\overline{P}_n\left(-\frac{1}{2}\right)}U_{1,1}\beta - \sqrt{\frac{5}{3}}\sqrt{P_n(0)\overline{P}_n\left(\frac{1}{2}\right)}U_{1,0}\alpha\right)\exp(-\frac{\mathrm{i}E_+}{\hbar}t)$$
$$+\left(\sqrt{\frac{5}{3}}\sqrt{P_n(0)\overline{P}_n\left(-\frac{1}{2}\right)}U_{1,0}\beta - \sqrt{\frac{5}{3}}\sqrt{P_n(-1)\overline{P}_n\left(\frac{1}{2}\right)}U_{1,-1}\alpha\right)\exp(-\frac{\mathrm{i}E_-}{\hbar}t). \tag{2.20}$$

$P_n(+1)$, $P_n(0)$ and $P_n(-1)$ are calculated from the 3×3 transition matrix

$$M = \begin{array}{c c} & \begin{array}{ccc} + & 0 & - \end{array} \\ \begin{array}{c} + \\ 0 \\ - \end{array} & \begin{pmatrix} 1-\overline{q}-\overline{q}^2 & \overline{q} & \overline{q}^2 \\ \overline{p} & 1-\overline{p}-\overline{q} & \overline{q} \\ \overline{p}^2 & \overline{p} & 1-\overline{p}-\overline{p}^2 \end{pmatrix} \end{array}. \tag{2.21}$$

Here

$\overline{p}$ = probability of going from $-1 \to 0$ and $0 \to 1$,
$\overline{p}^2$ = probability of going from $-1 \to 1$,
$\overline{q}$ = probability of going from $1 \to 0$ and $0 \to -1$,
$\overline{q}^2$ = probability of going from $1 \to -1$.

The two state Markov processes $\overline{P}_n(\pm 1/2)$ are calculated from Eq. (2.8) with $\overline{p}$, $\overline{q}$ for the spin 1/2 preon.

Using Eq. (2.21), we have [29]

$$(P_n(+1), P_n(0), P_n(-1)) = \left(\frac{2}{5}, \frac{1}{5}, \frac{2}{5}\right) M^n. \tag{2.22}$$

Using Eq. (2.20), we have, after a long calculation, where $\overline{P}_n(+)$, $\overline{P}_n(-)$ are expressed in Eq. (2.8) for spin 1/2 preons

$$< S_x >= \Psi^+ (S_{x_1} + S_{x_2}) \Psi \tag{2.23}$$

$$S_{x_1} = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad S_{x_2} = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{2.24}$$

$$\begin{aligned} < S_x >= \hbar \Bigg[& \frac{5}{3}\sqrt{2}\sqrt{P_n(0)P_n(+1)\overline{P}_n^2\left(-\frac{1}{2}\right)} - \frac{5}{3}\sqrt{P_n^2(0)\overline{P}_n\left(-\frac{1}{2}\right)\overline{P}_n\left(+\frac{1}{2}\right)} \\ & + \frac{5}{3}\sqrt{2}\sqrt{P_n(0)P_n(-1)\overline{P}_n^2\left(+\frac{1}{2}\right)}\Bigg] \cos\frac{eB}{m}t. \end{aligned} \tag{2.25}$$

We see from Eq. (2.25) that the variation of the spin polarization amplitude (S_x) with n is fundamentally different than in Eq. (2.13) for the three–fermion composite model.

3. Conclusion

The above relations in Eq. (2.13) and Eq. (2.25) are unique signatures for a three–fermion composite lepton and a fermion–spin-one–boson model of a lepton, respectively. Numerous authors [30,31] have proposed fermion–scalar models, but I did not find any fermion–spin-one–boson models in the literature (here the spin 3/2 states are most likely so massive that they won't appear in the GeV range). A possibility here would be that the spin 1/2 fermion is the supersymmetric partner of the spin–one boson, the beautiful feature of such a model would be that the supersymmetry would be the required internal symmetry [32] that would keep the composite light compared to the composite scale. In the three–fermion model, two of the preons could form a core with the third fermion forming a hydrogen–like radial excitation with various excited states [33,34]. If high field spin polarization precession ever became a practical probe to particle properties, the above individual Markov jump processes for the individual constituent preons composing the leptons would produce clear signatures for composite structure through Eq. (2.13) and Eq. (2.25). The above analysis represents a first attempt at probing individual preon properties, most composite models use the structural features of the composite system to predict anomalous magnetic moments, form–factors and mixing angles. It is hoped that keen ingenuity on the part of the experimental community will make the above measurements possible in the near future. Lastly, the index n in the Markov process is most likely some function of time and it could very well be that $t = n\tau$, where τ is the fundamental time interval between Markov jumps. It could however be that n is an ordering process that refers to a completely independent time variable. If this were the case, the continuous time of classical physics contained in the function $\cos(eBt/m)$ of Eq. (2.13) and Eq. (2.25) would have to be reconciled with the independent discrete Markov jump time (n).

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JESU LI LEPTONI SASTAVLJENI OD TRI FERMIONA ILI FERMIONA I BOZONA

Predviđa se razlika u mjerenjima spinske polarizacije u magnetskom polju uz pretpostavku da su leptoni trofermionske odnosno bozonsko–fermionske složenice.