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THE ROLE OF THE JWKB TREATMENT IN THE SHARP CUT–OFF APPROXIMATION FOR HEAVY IONS COLLISIONS

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We show how an adiabatic approximation to the usual coupled–channels formalism gives a simple way of calculation and understanding the heavy–ion fusion inelastic cross–section. We did the calculation for the ^{58}Ni + ^{58}Ni system by considering only one transition, the ground–state to ground–state transition. A very simple method to calculate the above–barrier inelastic cross–section has been proposed, which can be handled entirely within the framework of the direct–reaction theory. The essence of the method is the role of the JWKB treatment in the determination of the partial waves which contribute to the fusion. We apply the schematical model to the calculation of inelastic cross–section for the system ^{58}Ni + ^{58}Ni. Results of the model are extensively discussed.

1. *Introduction*

Fusion of two heavy ions has been investigated for many years [1,2]. A lot of experimental data are available and many features concerning this process are now understood. However, there still remain open questions which deserve further studies. In particular, there exist some models which can reproduce well fusion data for restricted cases, but a global understanding of the fusion is still not known.

Investigations of deep inelastic reactions have shown that dissipation of collective energy into intrinsic excitation plays an important role in the fusion process [3]. The most direct approach to the fusion problem is to make a full quantal coupled–channels calculation. Broglia et al. [4] and Dasso et al. [5] use a very simplified coupled–channels model, using one–dimensional potentials, in which the intrinsic states are degenerate and the coupling can be removed by a unitary transformation. Lindsay and Rowley [6] also use this kind of model, but discuss angular–momentum–coupling effects explicitely. Jacobs and Smilansky [7] use a path-integral method based on influence functionals, where the essential approximation is that the tunnelling is not treated in a self–consistent way. The influence functional method gives a better approximation but is also more complicated. The path-integral method is equivalent to a version of the WKB method given by Brink, Nemes and Vautherin [8] for the coupling of a collective variable r to the intrinsic degrees of freedom ξ. In a previous paper, Bougouffa et al. [9] have shown that the coupled–channels model used by Dasso et al. [5] may be extended to three dimensions and to enlarge the investigation to states with angular momentum $l \neq 0$ in the adiabatic approximation. Also, we must ignore the difference in centrifugal potentials and use the one corresponding to $l_\alpha = 1$ in all channels, i.e. the entrance–channel relative angular momentum, where the system of coupled differential equations can be decoupled by using a theorem of coupled differential equations (Cao [10]). The qualitative features of the results are very similar.

In this paper, we use the kind of model where the barrier occurs at a large radius which is required for the scattering of two heavy ions in which the coupling can be removed by using a theorem of coupled differential equations. With this suggestion, we show the role of the JWKB treatment in the use of the sharp cut–off approximation in the coupling–channels formalism with schematical model for heavy ion problems. We also show that this role consists in determining the number of the partial waves which contribute significantly to the above–barrier inelastic cross–section.

The effects of coupling of a 0^+ ground state to a low–lying 2^+ excited state may be studied by using the JWKB approximation to the separated system with central potentials differing by an amount related to the coupling strength. The important feature of our method is that the full angular–momentum algebra is included rigorously, allowing all T matrix elements of physical interest to be generated from the two T matrices obtained from schematic model. In particular, the inelastic cross–section may be obtained in a manner which is numerically simpler than the distorted–wave approximation, which is also more accurate for strongly deformed systems.

2. *Exact two–channel model*

In order to make clear the method which must be employed in dealing with inelastic collisions, we will first consider the scattering of two spin–zero nuclei, one of which has a 0^+ excited state at an excitation energy Q with a coupled potential

$V_{\alpha\beta}$ to ground state. The two coupled equations for the radial wave function $\Psi(r)$ for the partial waves l_0 and l_1 are then [14]:

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}r^2} + \frac{2\mu}{\hbar^2}(E_0 - V_{00}) - \frac{l_0(l_0+1)}{r^2}\right]\Psi_0 = \frac{2\mu}{\hbar^2}V_{01}\Psi_1,$$

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}r^2} + \frac{2\mu}{\hbar^2}(E_1 - V_{11}) - \frac{l_1(l_1+1)}{r^2}\right]\Psi_1 = \frac{2\mu}{\hbar^2}V_{10}\Psi_0, \tag{1}$$

where $\Psi_0 = \Psi_{\mathrm{el}}^{l_0}(r)$, $\Psi_1 = \Psi_{\mathrm{inel}}^{l_1}(r)$, μ is the reduced mass, and $E_1 = E_0 - Q$. Q can be varied. Assume that couplings to the ground state are similar for each channel, i.e. $V_{01} = V_{10}$.

For the scattering of two heavy ions, the reduced mass is large and the barrier occurs at a large radius where the difference in centrifugal barriers is small for the various channels, if the total angular momentum l is not too large. For nuclei where Q is small, we can then define an extended adiabatic approximation with the following assumptions: we ignore the excitation energy Q, but in addition we must ignore the difference in centrifugal potentials in Eqs. (1) and use the one corresponding to $l_0 = l_1 = l$ in all channels, i.e. to the entrance–channel relative angular momentum.

Under these conditions, we can decouple Eqs. (1) using the theorem on the separation of a system of coupled differential equations [10], and the system may always be completely separated if and only if the quantity $\gamma = V_{01}/(V_{00} - V_{11})$ is independent of r.

The separated equations of the system (1) are now

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}r^2} + \frac{2\mu}{\hbar^2}(E - V_l^{\mp}(r))\right]\phi^{\pm} = 0, \tag{2}$$

where

$$\frac{2\mu}{\hbar^2}V_l^{\pm}(r) = \frac{l(l+1)}{r^2} + \frac{2\mu}{\hbar^2}V^{\pm}(r),$$

$$V^{\pm}(r) = \frac{1}{2}[V_{00}(r) + V_{11}(r)] \pm \frac{1}{2}\sqrt{[V_{00}(r) - V_{11}(r)]^2 + 4V_{01}^2(r)}.$$

The functions Ψ_0 and Ψ_1 in (1) may then be recovered by the inverse transformation

$$\Psi = X^{-1}\phi, \tag{3}$$

where the matrix X is defined in Ref. 9.

However, the boundary conditions also need to be considered, since Ψ_0, the elastic wave function, consists of an incoming wave as well as an outgoing wave, whereas Ψ_1 has only an outgoing part.

Both uncoupled Eqs. (2) correspond to simple elastic scattering problems, and by using the JWKB approximation of the uncoupled integral equation, we obtain the corresponding proper phase shift $\eta_l^{\pm}$ of the system (2)

$$\eta_l^{\pm} = (2l+1)\frac{\pi}{4} - kr_0^{\pm} + \int\limits_{r_0^{\pm}}^{\infty} \left[\sqrt{F_1^{\pm}} - k\right] \mathrm{d}r, \tag{4}$$

where

$$F_1^{\pm} = k^2 - \frac{\left(l+\frac{1}{2}\right)^2}{r^2} - \frac{2\mu}{\hbar^2} V^{\pm}(r)$$

and

$$k^2 = \frac{2\mu}{\hbar^2} E.$$

By using the inverse transformation (3) and with the $\overline{S}$–matrix defined by

$$\overline{S} = \exp(-2\mathrm{i}\eta_l^{\pm}), \tag{5}$$

we obtain directly the inelastic total cross-section

$$\sigma^{\mathrm{ine}} = \sum_{l=0}^{\infty} \sigma_l^{\mathrm{ine}}, \tag{6}$$

where

$$\sigma^{\mathrm{ine}} = \pi \lambda\!\!\!^{-}{}^2 (2l+1)\chi_l^2 \sin^2(\eta_l^+ - \eta_l^-), \tag{7}$$

$$\chi_l = \frac{1-a^2}{1+a^2}, \tag{8}$$

$$a = 2\gamma + \sqrt{1+4\gamma^2} \tag{9}$$

and $\lambda\!\!\!^{-} = 1/k$ is the asymptotic wavelength.

3. *Applications*

For energies safely above the barrier, many models use the sharp cut–off approximation [11], which defines the cut–off angular momenta for fusion $l_{\mathrm{cut}}(E)$ as

$$\sigma^{CF} = \pi \lambda\!\!\!^{-}{}^2 (l_{\mathrm{cut}} + 1)^2. \tag{10}$$

Values for $l_{\rm cut}$ obtained from the experimental cross–sections σ^{CF} using Eq. (10) are presented by Beckerman et al. [12]. The concept of an $l_{\rm cut}$ is not particularly useful at center–of–mass energies near and below that of the fusion barrier. For massive target–projectile combinations, the centrifugal barrier rises slowly with increasing angular momenta at low angular momenta and the corresponding transmission coefficients decrease slowly. Therefore, $l_{\rm cut}$ values have been given by Beckerman [12] for data at above–barrier energies only, and Eq. (10) is not very easy to use because it depends on $l_{\rm cut}$ which is not known explicitly.

In order to further support our conclusions presented in Ref. 9, we improved our model calculations. Our model is designed to meet two requirements:

(a) It should be simple enough to allow for a quantum-mechanical exact solution. This should guarantee that the results are free from such approximation as JWKB, Hill–Wheeler, averaging over probabilities, etc...

(b) The model should at least schematically reproduce some new features which are not predicted in our previous approach [9].

In the applications which follow, we use Gaussian forms for $V_{\alpha\alpha}$ and $V_{\alpha\beta}$:

$$V_{\alpha\alpha} = V_\alpha \exp\left[-\frac{(r-r_b)^2}{2\sigma^2}\right], \quad \alpha = 0, 1, \tag{11}$$

$$V_{\alpha\beta} = F \exp\left[-\frac{(r-r_b)^2}{2\sigma^2}\right], \quad \alpha \neq \beta, \tag{12}$$

with the same position and twice the center curvature of the s–wave barrier [13]. This assumption is good if the coupling potential does not alter the barrier shape to much. The coupling strength is determined by F. The choice of the form–factor reflects our picture of a neck degree of freedom which should be excited only for rather close nuclear contact.

In our applications, F does not carry angular momentum. This is assumed for the sake of simplicity and in order to minimize the assumptions about the coupling potential. It should not imply that the neck or any other relevant degree of freedom does not actually couple different partial waves. Note, however, that the multipole vibrations of separated nuclei are not the normal modes of the system in contact. One might, therefore, expect that the degrees of freedom relevant for fusion do not carry a specific angular momentum [13].

In the following, we shall use units of amu, MeV and fm for mass, energy and distance, respectively. The calculation procedure is to evaluate first the roots $r_0^\pm$ of $F_1^\pm$ by using the fixed–point iteration procedure, then to calculate the proper phase shifts, which are defined in Eq. (4) by using the Simpson integration method, and finally to obtain the inelastic cross–section from Eqs. (6) and (7).

As the firs step, we plot in Fig. 1 the interaction potential $V_l^\pm(r)$ defined in Eq. (2) for a head–on collision between two heavy ions, as a function of distance, r, separating their respective centres of mass, with the purpose to observe still more closely the influence of two–channel coupling. On the left, $V_l^0(r)$ stands for a single fusion barrier in the entrance channel, whereas the split barrier of the right are shifted by an amount $\pm(1/2)\sqrt{(V_0 - V_1)^2 + 4F^2}$ where F is the two channel coupling strength. This is done for the ^{58}Ni + ^{58}Ni system where the parameters are simulated to the s–wave potential that is located at $r = r_b$.

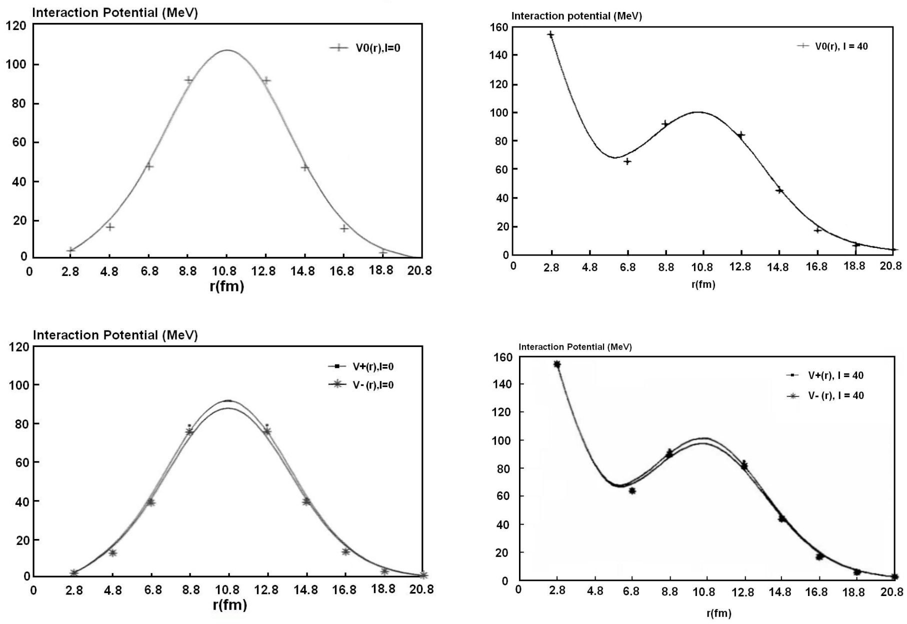

Fig. 1. Effective partial–wave interaction–potential near the barrier for ^{58}Ni + ^{58}Ni in our schematical model. $V_l(r)$ stands for single fusion barrier in the entrance channel, whereas the split barrier are shifted by an amount $\pm F$, the later being the two–channel coupling strength. The parameters are $\mu = 29.0$, $r_b = 10.8$, $\sigma = 3.0$, $F = 2$, $V_0 = V_1 = 95.6$, $Q = 0$.

On the other hand, it is important to note, that the cross–sections formulas (6) and (7), derived as described above, require that $\eta_l^{\pm}$, defined in Eq. (4), must be a real number, i.e. that the functions $F_1^{\pm}$, which appear in the integrand of the integral in Eq. (4), must be positive for $r > r_0^{\pm}$. In this way the cut–off angular momentum $l_{\rm cut}$ is defined. For the largest l value which still allows fusion to occur, namely $l_{\rm cut}$, we have the following condition:

$$k^2 = V_{l_{\rm cut}}^{\pm}(r_b), \tag{13}$$

where $V_{l_{\rm cut}}^{\pm}(r_b)$ is the height of the fusion barrier for a head–on collision and r_b its location corresponding to $l = l_{\rm cut}$. Equation (12) can be rewritten as:

$$\frac{\left(l_{\rm cut} + \frac{1}{2}\right)^2}{r^2} + \frac{1}{2}\frac{2\mu}{\hbar^2}\left[V_0 + V_1 \pm \sqrt{(V_0 - V_1)^2 + 4F^2}\right] = k^2, \tag{14}$$

which gives the cut–off angular momentum $l_{\rm cut}$.

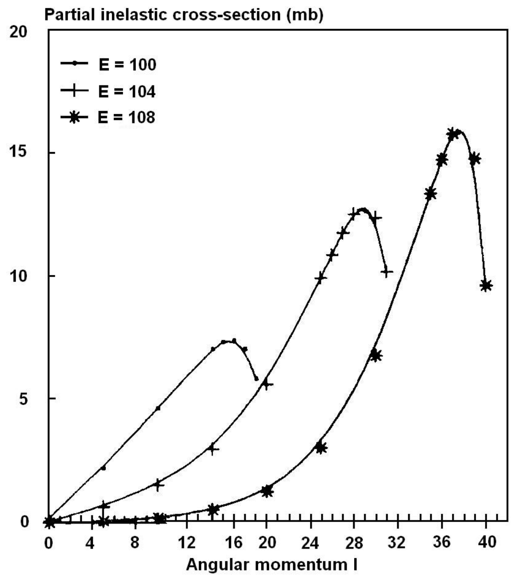

Fig. 2. Partial inelastic cross–section for ^{58}Ni + ^{58}Ni *at three different values of energy. The parameters are* $\mu = 29.0$, $r_b = 10.8$, $\sigma = 3.0$, $F = 2$, $V_0 = V_1 = 95.6$, $Q = 0$, $E = 100, 104, 108$.

The situation described in our way is quite different from that used in Eq. (9) [6], and it is interesting to see that the JWKB treatment is available only above barrier fusion of heavy ions. However, extension to a sub–barrier fusion case is possible at the cost of more complications.

In Fig. 2., we illustrate what has been said above. We plot the partial inelastic cross–sections, which are defined in Eq. (6), as a function of l. The calculation was done for the ^{58}Ni + ^{58}Ni system at three different values of the center–of–mass energy. We see that they have similar features, and the maximum of the partial inelastic cross–section occurs for l values just near the cut–off angular momentum $l_{\rm cut}$. When the center–of–mass energy increases, the maximum is shifted to larger values of l. In Table 1 we give the predictions for the total inelastic cross–section in the coupled–channel formalism with our schematical model, at energies well above the barrier, where the number $l_{\rm cut}$ of partial waves, which contribute significantly to the above barrier inelastic cross–section, has been estimated from Eq. (14).

The dependence on the coupling can be illustrated by considering the total inelastic cross–section as a function of coupling strength F at three different values of energy, $E = 100, 104, 108$ MeV. Figure 3 shows the results of calculations made assuming a perfectly matched excited channel ($V_0 = V_1$, $Q = 0$). However, the same oscillatory behaviour is more pronounced at higher energy.

TABLE 1.
Partial wave contribution to the total inelastic cross–section in the coupling–channel formalism with our schematic model, at energies well above the barrier. The parameters are $\mu = 29.0$, $r_b = 10.8$, $\sigma = 3.0$, $F = 2$, $V_0 = V_1 = 95.6$, $Q = 0$, $E = 100, 104, 108$.

E (MeV)	$\sigma^{\rm ine}$ (mb)	$l_{\rm cut}$ (h)
100	82.8	19
104	151.1	31
108	165.2	40

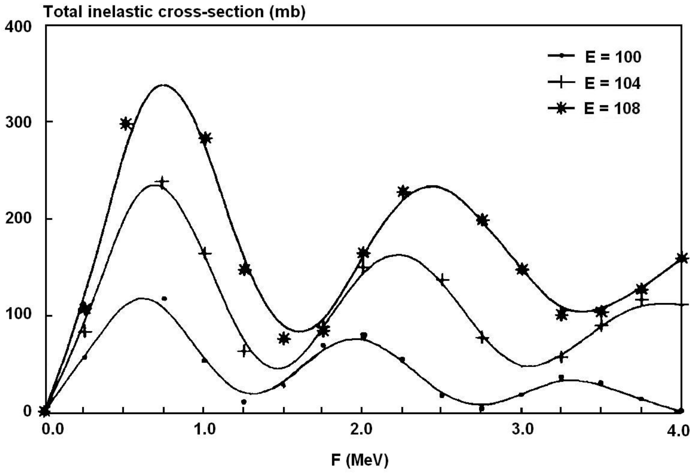

Fig. 3. Effect of magnitude of the coupling strenght F on the total inelastic cross–section in the adiabatic approximation. The parameters are simulated to the s–wave potential for ^{58}Ni + ^{58}Ni: $\mu = 29.0$, $r_b = 10.8$, $\sigma = 3.0$, $F = 2$, $V_0 = V_1 = 95.6$, $Q = 0$, $E = 100, 104, 108$.

To complete this subsection, we show some results where the entrance channel potential V_0 and the strength of coupling are fixed, but the potential V_1 in the excited channel (near–resonance case) is allowed to change. In this case the function

$$\gamma = \frac{V_{01}(r)}{V_{00}(r) - V_{11}(r)} \tag{15}$$

is independent of r and the quantity a in Eq. (9) is also independent of r. Therefore, Eqs. (1) may be separated. The total inelastic cross–sections are shown in Fig. 4 at energies above the barrier where the use of the JWKB treatment is allowed. It is assumed that only the partial waves up to an orbital angular momentum $l_{\rm cut}$ whose potential barrier height equals E, contribute to fusion. This result is similar to that obtained by Dasso et al. [5], T. Udagawa et al. [15] and Beckerman et al. [12] for the same system.

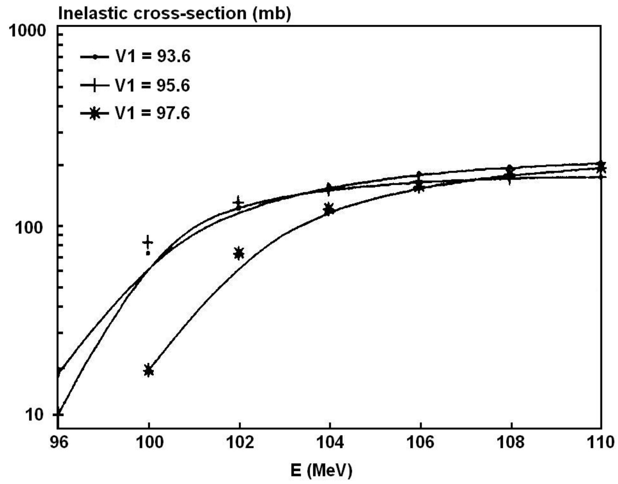

Fig. 4. Effect on the total inelastic cross–section of varying the barrier height in the excited channel in the two–channel model for ^{58}Ni + ^{58}Ni. *The parameters are* $\mu = 29.0$, $r_b = 10.8$, $\sigma = 3.0$, $F = 2$, $V_0 = 95.6$, $V_1 = 93.6, 95.6, 97.6$, $Q = 0$, $E = 100, 104, 108$.

4. *Summary and conclusion*

In this paper, we have derived a simple method of calculating the above–barrier inelastic cross–section for the scattering of two heavy ions, and we have shown how the coupled–channels equations for this scattering problem can be easily decoupled by using the theorem of separation of coupled differential equations in our adiabatic approximation (near resonance case $V_0 \neq V_1$) which is valid for many nuclei. We expect that our calculation is fairly realistic.

We have described the fusion of two heavy ions in terms of the simple model that uses the Gaussian forms of potentials. We have investigated the role of the JWKB treatment in this schematical model, in which the fusion is assumed to take place if and only if the two ions go over the potential barrier. With this mind, we have previously investigated the case when two channels are coupled with the same angular momentum and noted the effects of varying the interaction strength. In particular, the important role of the coupling in enhancing the above barrier inelastic cross–section for the system ^{58}Ni + ^{58}Ni has been pointed out.

The results obtained in this work show the important role of the JWKB treatment which assumes that only partial waves up to an orbital angular momentum $l_{\rm cut}$, whose potential barrier height equals E, contribute to fusion.

Our results can be extended to phenomenological problems of scattering, to include the angular–momentum coupling effects at the cost of more complication. This work is in progress. An analogous situation occurs in molecular chemistry when one brings two hydrogen atoms together in order to form a hydrogen molecule [14].

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ULOGA JWKB POSTUPKA U APROKSIMACIJI NAGLOG PREKIDA ZA SUDARE

Pokazuje se kako primjena adijabatske aproksimacije omogućuje proračun neelastičnog udarnog presjeka pri fuziji teških iona. Račun je proveden za sistem ^{58}Ni + ^{58}Ni i prijelaze iz osnovnih u osnovno stanje. Bit metode je JWKB postupak za određivanje parcijalnih valova koji doprinose fuziji.