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SCALAR CONSTANT OF MOTION OF THE ELECTROMAGNETIC FIELD AND ITS CONSEQUENCES

KRUNOSLAV LJOLJE

Academy of Science and Art of Bosna and Hercegovina, Sarajevo, Bosna and Hercegovina

Dedicated to Professor Mladen Paić on the occasion of his 90$^{\text{th}}$ birthday

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Existence of a scalar constant of motion of the electromagnetic field is considered and its consequences are analysed.

1. Introduction

The electromagnetic field is a real field. Due to this fact, there is no a scalar constant of motion of this field in the conventional theory as well as in the dual symmetrical theory [1]. The Lagrangian density in the dual symmetric theory is given by

$$\mathcal{L} = K\left(-\frac{1}{2}F_{\alpha\beta}F^{\alpha\beta} + F^2 - G^2\right) = K\left[\left(-\mathrm{i}\partial_\alpha\Phi\gamma^\alpha\right)\left(\mathrm{i}\partial_\beta\gamma^\beta\Phi\right)\right], \tag{1}$$

where

$$F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha - \frac{1}{2}\varepsilon_{\alpha\beta\xi\zeta}(\partial^\xi b^\zeta - \partial^\zeta b^\xi),$$

$$G = \partial_\alpha A^\alpha, \quad F = \partial_\alpha b^\alpha \tag{2}$$

$$\Phi = \begin{pmatrix} -A_y & \mathrm{i}A_x \\ -f & \mathrm{i}A \\ -b_x & \mathrm{i}b_y \\ b_z & -\mathrm{i}\varphi \end{pmatrix}. \tag{3}$$

γ^α are Dirac matrices, K is a constant ($1/8\pi$ for the EM field) and A^α, b^α are Lagrange's variables. The quantities A^α make a fourvector, b^α a pseudo fourvector in the case of EM field while in the case of massless Dirac field they compose a bispinor.

The canonical equations are given by

$$\partial_\alpha(F^{\alpha\beta} + g^{\alpha\beta}G) = 0,$$

$$\partial_\alpha(\tilde{F}^{\alpha\beta} + g^{\alpha\beta}F) = 0, \tag{4}$$

$$\tilde{F}^{\alpha\beta} = \frac{1}{2}\varepsilon^{\alpha\beta\xi\zeta}F_{\xi\zeta}, \;\; g^{00} = -g^{11} = -g^{22} = -g^{33} = 1, \;\; g^{\alpha\beta} = 0, \;\; \alpha{\neq}\beta.$$

For the bispinor (Dirac) field, they can be written in the form

$$\mathrm{i}\partial_\alpha\gamma^\alpha\Psi = 0,$$

$$\Psi = \mathrm{i}\partial_\alpha\gamma^\alpha\Phi \;\text{ and h.c.e.} \tag{5}$$

The Lagrangian (1) is invariant to the transformation

$$\Phi^{'} = e^{\mathrm{i}\lambda}\Phi \quad \text{where } \lambda \text{ is a constant.} \tag{6}$$

That leads to the scalar constant of motion for the Dirac field

$$Q = qK\int(\Psi^+\Phi + \Phi^+\Psi)\mathrm{d}^3x, \tag{7}$$

(q is a scalar constant). In the case of the EM field, the transformation (6) makes a linear combination of vector and pseudovector and as a consequence the corresponding constant of motion is not a scalar one. So, there arises the question of the existence of a scalar constant of motion of the EM field. In this article, we show that such a constant does exist, but it is connected with an internal structure of the field and not the field as a whole. We also analyse some consequences of its existence.

2. *Scalar constant of motion*

General solution of Eqs. (4) for $b^\alpha = 0$, $G = 0$ (a standard field; G is a scalar) can be written in the form

$$A^\alpha = \sum_{\vec{k}} (A^\alpha_{\vec{k}} e^{-\mathrm{i}k_\beta x^\beta} + c.c.), \quad k_\beta k^\beta = 0 \quad (L^3 = 1). \tag{8}$$

Due to the transversality of the EM field, we assume in the following $A^0 = 0$ ($\vec{k}\vec{A}_{\vec{k}} = 0$).

We now decompose A_α into two parts in a such way that in the energy–momentum and spin vectors there are no mixed terms:

$$\vec{A}_{\vec{k}} = \vec{A}^a_{\vec{k}} + \vec{A}^b_{\vec{k}}, \tag{9}$$

where

$$\vec{A}^a_{\vec{k}} = \frac{1}{1+\mathrm{i}}(A_{k1r} - A_{k1i})\vec{e}_{\vec{k}1} + \frac{1}{1-\mathrm{i}}(A_{k2r} + A_{k2i})\vec{e}_{\vec{k}2},$$

$$\vec{A}^b_{\vec{k}} = \frac{1}{1-\mathrm{i}}(A_{k1r} + A_{k1i})\vec{e}_{\vec{k}1} + \frac{1}{1+\mathrm{i}}(A_{k2r} - A_{k2i})\vec{e}_{\vec{k}2}, \tag{10}$$

and

$$\vec{A}_{\vec{k}} = (A_{k1r} + \mathrm{i}A_{k1i})\vec{e}_{\vec{k}1} + (A_{k2r} + \mathrm{i}A_{k2i})\vec{e}_{\vec{k}2},$$

$$\vec{e}_{\vec{k}i}\vec{e}_{\vec{k}j} = \delta_{ij} \quad (\vec{e}_{\vec{k}i} - \text{polarization vectors}). \tag{11}$$

The indices "r" and "i" denote real and imaginary parts, respectively.

The energy–momentum fourvector and the spin vector are given by

$$P^0 = \frac{K}{c}\int (\vec{E}^2 + \vec{B}^2)\mathrm{d}^3x = P^{a_0} + P^{b_0} + P^{ab_0},$$

$$\vec{P} = \frac{2K}{c}\int \vec{E}\times\vec{B}\mathrm{d}^3x = \vec{P}^a + \vec{P}^b + \vec{P}^{ab}, \tag{12}$$

$$\vec{S} = \frac{2K}{c}\int \vec{E}\times\vec{A}\mathrm{d}^3x = \vec{S}^a + \vec{S}^b + \vec{S}^{ab},$$

where

$$P^{a_0} + P^{b_0} = \frac{4K}{c}\sum_{\vec{k}} k_0^2(\vec{A}^{a*}_{\vec{k}}\vec{A}^a_{\vec{k}} + \vec{A}^{b*}_{\vec{k}}\vec{A}^b_{\vec{k}}),$$

$$\vec{P}^a + \vec{P}^b = \frac{4K}{c}\sum_{\vec{k}} \vec{k}k_0(\vec{A}^{a*}_{\vec{k}}\vec{A}^a_{\vec{k}} + \vec{A}^{b*}_{\vec{k}}\vec{A}^b_{\vec{k}}), \tag{13}$$

$$\vec{S}^a + \vec{S}^b = \frac{4K}{c}\mathrm{i}\sum_{\vec{k}} k_0(\vec{A}^a_{\vec{k}} \times \vec{A}^{a*}_{\vec{k}} + \vec{A}^b_{\vec{k}} \times \vec{A}^{b*}_{\vec{k}}),$$

$$P^{ab_0} = \frac{4K}{c}\sum_{\vec{k}} k_0^2(\vec{A}^{a*}_{\vec{k}}\vec{A}^b_{\vec{k}} + \vec{A}^{b*}_{\vec{k}}\vec{A}^a_{\vec{k}}),$$

$$\vec{P}^{ab} = \frac{4K}{c}\sum_{\vec{k}} \vec{k}k_0(\vec{A}^{a*}_{\vec{k}}\vec{A}^b_{\vec{k}} + \vec{A}^{b*}_{\vec{k}}\vec{A}^a_{\vec{k}}), \tag{14}$$

$$\vec{S}^{ab} = \frac{4K}{c}\mathrm{i}\sum_{\vec{k}} k_0(\vec{A}^a_{\vec{k}} \times \vec{A}^{b*}_{\vec{k}} + \vec{A}^b_{\vec{k}} \times \vec{A}^{a*}_{\vec{k}}).$$

Substitution of Eq. (10) into Eq. (14) gives

$$P^{ab\alpha} = 0, \quad \text{and}$$

$$\vec{S}^{ab} = 0, \tag{15}$$

in agreement with expressed requirement.

Thus

$$P^\alpha = P^{a\alpha} + P^{b\alpha}$$

$$\vec{S} = \vec{S}^a + \vec{S}^b. \tag{16}$$

The fields determined by $\vec{A}^a_k$ and $\vec{A}^b_k$ satisfy also Eqs. (4). Using these equations, one finds

$$\partial_\alpha j^\alpha = 0, \tag{17}$$

where[1]

$$j^\alpha = \frac{2K}{c}\left[(F^{b\alpha\eta} + g^{\alpha\eta}G^b)A^a_\eta - (F^{a\alpha\eta} + g^{\alpha\eta}G^a)A^b_\eta\right]. \tag{18}$$

From Eq. (17) follows the scalar constant of motion

$$Q = q\frac{2K}{c}\int(\vec{E}^b\vec{A}^a - \vec{E}^a\vec{A}^b)\mathrm{d}^3x =$$

[1]The vector j^α is derived in Ref. 2.

$$= q\frac{4K}{c}\mathrm{i}\sum_{\vec{k}} k_0(\vec{A}^b_k\vec{A}^{a*}_k - \vec{A}^a_k\vec{A}^{b*}_k), \quad (q - \text{a scalar constant}). \tag{19}$$

Thus one obtains a scalar constant of motion of the electromagnetic field. This scalar constant of motion does not come from the filed as a whole but from its internal structure given by Eqs. (9) and (10). Consequently, there exists a scalar constant of motion of the electromagnetic field associated with the internal structure of the field.

Let us consider first to the field $A^{a\alpha}$. Decomposition of this field according to Eq. (10) reproduces the field itself:

$$(\vec{A}^a_{\vec{k}})^a = \frac{1}{1+\mathrm{i}}(A^a_{k1r} - A^a_{k1i})\vec{e}_{\vec{k}1} + \frac{1}{1-\mathrm{i}}(A^a_{k2r} + A^a_{k2i})\vec{e}_{\vec{k}2} =$$

$$= \frac{1}{1+\mathrm{i}}(A_{k1r} - A_{k1i})\vec{e}_{\vec{k}1} + \frac{1}{1-\mathrm{i}}(A_{k2r} + A_{k2i})\vec{e}_{\vec{k}2} = \vec{A}^a_{\vec{k}}, \tag{20}$$

$$(\vec{A}^a_{\vec{k}})^b = \frac{1}{1-\mathrm{i}}(A^a_{k1r} + A^a_{k1i})\vec{e}_{\vec{k}1} + \frac{1}{1+\mathrm{i}}(A^a_{k2r} - A^a_{k2i})\vec{e}_{\vec{k}2} = 0.$$

We decompose $\vec{A}^a$ in the following way:

$$\vec{A}^a_{\vec{k}} = \vec{A}^a_{1\vec{k}} + \vec{A}^a_{2\vec{k}}, \tag{21}$$

$$\vec{A}^a_{1\vec{k}} = \frac{1}{2}(A_{k1r} - A_{k1i})\vec{e}_{\vec{k}1} + \frac{\mathrm{i}}{2}(A_{k2r} + A_{k2i})\vec{e}_{\vec{k}2},$$

$$\vec{A}^a_{2\vec{k}} = \frac{-\mathrm{i}}{2}(A_{k1r} - A_{k1i})\vec{e}_{\vec{k}1} + \frac{1}{2}(A_{k2r} + A_{k2i})\vec{e}_{\vec{k}2}. \tag{22}$$

Notice:

$$\vec{A}^a_{2\vec{k}} = -\mathrm{i}\vec{A}^a_{1\vec{k}}. \tag{23}$$

Due to Eq. (23), the mixed terms in P^α and $\vec{S}$ are equal to zero (there is no "interaction" between these fields). Thus, one gets

$$P^{a\alpha} = P^{a\alpha}_1 + P^{a\alpha}_2 = 2P^{a\alpha}_1 = \frac{8K}{c}\sum_{\vec{k}} k^\alpha k_0 \vec{A}^{a*}_{1\vec{k}}\vec{A}^a_{1\vec{k}},$$

$$\vec{S}^a = \vec{S}^a_1 + \vec{S}^a_2 = 2\vec{S}^a_1 = \frac{8K}{c}\mathrm{i}\sum_{\vec{k}} k_0\vec{A}^{a*}_{1\vec{k}}\vec{A}^a_{1\vec{k}}. \tag{24}$$

Using Eqs. (4), one obtains Eq. (17) with

$$j_a^\alpha = \frac{2K}{c}\left[(F_2^{a\alpha\eta} + g^{\alpha\eta}G_2^a)A_{1\eta}^a - (F_1^{a\alpha\eta} + g^{\alpha\eta}G_1^a)A_{2\eta}^a\right] \tag{25}$$

and the scalar constant of motion

$$Q^a = q\frac{2K}{c}\int(\vec{E}_2^a\vec{A}_1^a - \vec{E}_1^a\vec{A}_2^a)\mathrm{d}^3x = q\frac{8K}{c}\sum_{\vec{k}} k_0 \vec{A}_{1\vec{k}}^{a*}\vec{A}_{1\vec{k}}^a. \tag{26}$$

This scalar constant of motion has to be added in Eq. (24).

Introducing new quantities $a_{\vec{k}1}$, $a_{\vec{k}2}$ according to

$$\vec{A}_{1k}^a = \frac{1}{2\sqrt{2Kk_0}}\vec{c}_{\vec{k}} = \frac{1}{2\sqrt{2Kk_0}}(c_{\vec{k}1}\vec{e}_{\vec{k}1} + c_{\vec{k}2}\vec{e}_{\vec{k}2}) =$$

$$= \frac{1}{2\sqrt{2Kk_0}}\left[(a_{\vec{k}1} + a_{\vec{k}2})\vec{e}_{\vec{k}1} + \mathrm{i}(a_{\vec{k}1} - a_{\vec{k}2})\vec{e}_{\vec{k}2}\right] \tag{27}$$

or

$$A_{k1r} - A_{k1i} = \frac{1}{2\sqrt{Kk_0}}(a_{\vec{k}1} + a_{\vec{k}2}),$$

$$A_{k2r} + A_{k2i} = \frac{1}{2\sqrt{Kk_0}}(a_{\vec{k}1} - a_{\vec{k}2}), \tag{28}$$

Eqs. (26) and (24) change to

$$Q^a = \frac{q}{c}\sum_{\vec{k}}(a_{\vec{k}1}^* a_{\vec{k}1} + a_{\vec{k}2}^* a_{\vec{k}2}),$$

$$P^{a\alpha} = \frac{1}{c}\sum_{\vec{k}} k^\alpha(a_{\vec{k}1}^* a_{\vec{k}1} + a_{\vec{k}2}^* a_{\vec{k}2}), \tag{29}$$

$$\vec{S}^a = \frac{1}{c}\sum_{\vec{k}}\frac{\vec{k}}{k}(a_{\vec{k}1}^* a_{\vec{k}1} - a_{\vec{k}2}^* a_{\vec{k}2}),$$

and

$$\vec{S}_{\vec{k}}^a\frac{\vec{k}}{k} = \frac{1}{c}(a_{\vec{k}1}^* a_{\vec{k}1} - a_{\vec{k}2}^* a_{\vec{k}2}).$$

According to Eqs. (28) the new parameters $a_{\vec{k}l}$ are real.

The sum in the first of Eqs. (29) is also a scalar constant. It is determined by the field through Eq. (27). Denoting

$$\sum_{\vec{k}}(a^*_{\vec{k}1}a_{\vec{k}1} + a^*_{\vec{k}2}a_{\vec{k}2}) = \eta^a_{\vec{Q}}, \tag{30}$$

one may introduce the new parameters $\tilde{a}_{\vec{k}l}$,

$$a_{\vec{k}l} = \sqrt{\eta^a_Q}\tilde{a}_{\vec{k}l}, \quad l = 1, 2, \tag{31}$$

so that

$$\sum_{\vec{k}}(\tilde{a}^*_{\vec{k}1}\tilde{a}_{\vec{k}1} + \tilde{a}^*_{\vec{k}2}\tilde{a}_{\vec{k}2}) = 1. \tag{32}$$

Therefore, $\tilde{a}_{kl}$ may be interpreted as a probability amplitude. On the other side, $\tilde{a}_{\vec{k}l}$ are nondimensional quantities. Using $\tilde{a}_{\vec{k}l}$, Eqs. (29) can be written in the form

$$Q^a = \frac{q}{c}\eta^a_Q,$$

$$P^{a\alpha} = \frac{1}{c}\sum_{\vec{k}}\eta^a_Q k^\alpha(\tilde{a}^*_{\vec{k}1}\tilde{a}_{\vec{k}1} + \tilde{a}^*_{\vec{k}2}\tilde{a}_{\vec{k}2}), \tag{33}$$

$$\vec{S}^a_{\vec{k}}\frac{\vec{k}}{k} = \frac{1}{c}\eta^a_Q(\tilde{a}^*_{\vec{k}1}\tilde{a}_{\vec{k}1} - \tilde{a}^*_{\vec{k}2}\tilde{a}_{\vec{k}2}) \tag{34}$$

with

$$\sum_{\vec{k}}(\tilde{a}^*_{\vec{k}1}\tilde{a}_{\vec{k}1} + \tilde{a}^*_{\vec{k}2}\tilde{a}_{\vec{k}2}) = 1. \tag{35}$$

From the second of Eqs. (32) follows that η^a_Q has the physical dimension of the Planck constant. Expressions (33) show a possible physical interpretation in terms of a quantum particle. $a_{\vec{k}1}$ is the probability amplitude of momentum k^α and spin up and $a_{\vec{k}2}$ is the probability amplitude of momentum k^α and spin down. This is entirely a result of classical physics. No additional principle is applied. Therefore, some elements of quantum physics are already in the classical physics. There are two differences from the standard quantum theory. One concerns the constant η^a_Q and the other the number of particles (photons). η^a_Q depends on A^α while in the standard quantum theory it has a unique value equal to the Planck constant. That is not outside of the classical physics. It would be desirable to know the reason for this particular value, i.e. for the selection of that particular field. The conventional quantum theory doesnot restrict the number of photons to one. It may be achieved by writing

$$\sum_{\vec{k}}(a^2_{\vec{k}1}+a^2_{\vec{k}2})=N\beta^a_Q \tag{36}$$

and

$$a_{\vec{k}1}=\sqrt{\beta^a_Q}\tilde{a}_{\vec{k}1}, \tag{37}$$

where N is total number of photons. Then

$$\sum_{\vec{k}}(a_{\vec{k}1}+a_{\vec{k}2})=N. \tag{38}$$

It is also possible (within the classical physics) that $\tilde{a}_{\vec{k}1}$, $\tilde{a}_{\vec{k}2}$ are integers, i.e.

$$\tilde{a}^2_{\vec{k}l}=n_{kl},\quad n_{kl}=0,1,2,...,\quad l=1,2. \tag{39}$$

Then

$$Q^a=\frac{q}{c}N\beta^a_Q,$$

$$P^{a\alpha}=\sum_{\vec{k}}\frac{1}{c}\beta^a_Q k^\alpha(n_{\vec{k}1}+n_{\vec{k}2}), \tag{40}$$

$$\vec{S}^a_{\vec{k}}\frac{\vec{k}}{k}=\frac{1}{c}\beta^a_Q(n_{\vec{k}1}-n_{\vec{k}2}),$$

$$\sum_{\vec{k}}(n_{\vec{k}1}+n_{\vec{k}2})=N.$$

Eqs. (40) are in accordance with the conventional quantum theory (where conservation of photons is mere a matter of arbitrary selection). The selection (39) is essentially a quantization. Naturally, one may ask for its origin. However, there is another approach to this problem.

We may decompose the fields $\vec{A}^a_{1\vec{k}}$ and $\vec{A}^a_{2\vec{k}}$ similarly to the field $\vec{A}_{\vec{k}}$. Writing

$$\vec{A}^a_{1\vec{k}}=\vec{A}^a_{11\vec{k}}+\vec{A}^a_{12\vec{k}}, \tag{41}$$

where

$$\vec{A}^a_{12\vec{k}}=-\mathrm{i}\vec{A}^a_{11\vec{k}} \tag{42}$$

and

$$\vec{A}^a_{11\vec{k}} = \frac{1}{1-\mathrm{i}}\vec{A}^a_{1\vec{k}} = \frac{\mathrm{i}+1}{4}(A_{k1r} - A_{k1i})\vec{e}_{\vec{k}1} + \frac{\mathrm{i}-1}{4}(A_{k2r} - A_{k2i})\vec{e}_{\vec{k}2}, \tag{43}$$

$$Q^a_{1\vec{k}} = q\frac{8K}{c}k_0\vec{A}^{a*}_{11\vec{k}}\vec{A}^a_{11\vec{k}} = q\frac{K}{c}\left[(A_{k1r} - A_{k1i})^2 + (A_{k2r} + A_{k2i})^2\right],$$

$$P^{a\alpha}_{1\vec{k}} = \frac{8K}{c}k^\alpha k_0\vec{A}^{a*}_{11\vec{k}}\vec{A}^a_{11\vec{k}} = \frac{K}{c}k^\alpha k_0\left[(A_{k1r} - A_{k1i})^2 + (A_{k2r} + A_{k2i})^2\right], \tag{44}$$

$$\vec{S}^a_{1\vec{k}}\frac{\vec{k}}{k_0} = \frac{K}{c}2k_0(A_{k1r} - A_{k1i})(A_{k2r} + A_{k2i}).$$

Here the $\vec{k}$-components are also solutions of Eqs. (4). Introducing new parameters $a_{\vec{k}11}$, according to

$$\vec{A}^a_{11\vec{k}} = \frac{1}{4\sqrt{2Kk_0}}(1+\mathrm{i})\left[(a_{\vec{k}11} + a_{\vec{k}12})\vec{e}_{\vec{k}1} + \mathrm{i}(a_{\vec{k}11} - a_{\vec{k}12})\vec{e}_{\vec{k}2}\right] \tag{45}$$

or

$$\begin{aligned} A_{k1r} - A_{k1i} &= \frac{1}{\sqrt{2Kk_0}}(a_{\vec{k}11} + a_{\vec{k}12}), \\ A_{k2r} + A_{k2i} &= \frac{1}{\sqrt{2Kk_0}}(a_{\vec{k}11} - a_{\vec{k}12}), \end{aligned} \tag{46}$$

Eqs. (44) change to

$$\begin{aligned} Q^a_{1\vec{k}} &= \frac{q}{c}(a^2_{\vec{k}11} + a^2_{\vec{k}12}), \\ P^a_{1\vec{k}} &= \frac{1}{c}k^\alpha(a^2_{\vec{k}11} + a^2_{\vec{k}12}), \\ \vec{S}^a_{1\vec{k}}\frac{\vec{k}}{k} &= \frac{1}{c}(a^2_{\vec{k}11} - a^2_{\vec{k}12}). \end{aligned} \tag{47}$$

Analogously, for $\vec{A}^a_{2\vec{k}}$ we have

$$\vec{A}^a_{2\vec{k}} = \vec{A}^a_{21\vec{k}} + \vec{A}^a_{22\vec{k}}, \quad \vec{A}^a_{22\vec{k}} = -\mathrm{i}\vec{A}^a_{11\vec{k}}, \tag{48}$$

where

$$\vec{A}^a_{21\vec{k}} = \frac{1-\mathrm{i}}{4}(A_{k1r}-A_{k1i})\vec{e}_{\vec{k}1}+\frac{1+\mathrm{i}}{4}(A_{k2r}+A_{k2i})\vec{e}_{\vec{k}2}, \quad (\text{notice: } \vec{A}_{21\vec{k}} = \vec{A}^a_{12\vec{k}}) \tag{49}$$

and

$$\begin{aligned} Q^a_{2\vec{k}} &= \frac{q}{c}(a^2_{\vec{k}21} + a^2_{\vec{k}22}), \\ P^{a\alpha}_{2\vec{k}} &= \frac{1}{c}k^\alpha(a^2_{\vec{k}21} + a^2_{\vec{k}22}), \\ \vec{S}^a_{2\vec{k}}\frac{\vec{k}}{k} &= \frac{1}{c}(a^2_{\vec{k}21} - a^2_{\vec{k}22}), \end{aligned} \tag{50}$$

where

$$\begin{aligned} a_{\vec{k}21} + a_{\vec{k}22} &= \sqrt{2Kk_0}(A_{k1r} - A_{k1i}), \\ a_{\vec{k}21} - a_{\vec{k}22} &= \sqrt{2Kk_0}(A_{k2r} + A_{k2i}). \end{aligned} \tag{51}$$

Inspection of Eqs. (51) and (46) shows

$$\begin{aligned} a_{\vec{k}21} &= a_{\vec{k}11}, \\ a_{\vec{k}22} &= a_{\vec{k}12}. \end{aligned} \tag{52}$$

Then

$$\begin{aligned} Q^a_{2\vec{k}} &= Q^a_{1\vec{k}}, \\ P^{a\alpha}_{2\vec{k}} &= P^{a\alpha}_{1\vec{k}}, \\ \vec{S}^a_{2\vec{k}}\frac{\vec{k}}{k} &= \vec{S}^a_{1\vec{k}}\frac{\vec{k}}{k}. \end{aligned} \tag{53}$$

Consequently,

$$\begin{aligned} Q^a_{\vec{k}} &= Q^a_{1\vec{k}} + Q^a_{2\vec{k}} = 2Q^a_{1\vec{k}}, \\ P^{a\alpha}_{\vec{k}} &= P^{a\alpha}_{1\vec{k}} + P^{a\alpha}_{2\vec{k}} = 2P^{a\alpha}_{1\vec{k}}, \end{aligned} \tag{54}$$

$$\vec{S}^a_{\vec{k}}\frac{\vec{k}}{k} = \vec{S}^a_{1\vec{k}}\frac{\vec{k}}{k} + \vec{S}^a_{2\vec{k}}\frac{\vec{k}}{k} = 2\vec{S}^a_{1\vec{k}}\frac{\vec{k}}{k}.$$

In these relations appear integers (here 2). The factor 3 one obtains by decomposition of $\vec{A}_{11\vec{k}}$, $\vec{A}_{12\vec{k}}$, $\vec{A}_{22\vec{k}}$:

$$\begin{aligned}
Q^a_{\vec{k}} &= 3Q^a_{11\vec{k}} + q\frac{8K}{c}k_0\vec{A}^{a*}_{21\vec{k}}\vec{A}^a_{21\vec{k}},\\
P^{a\alpha}_{\vec{k}} &= 3P^{a\alpha}_{11\vec{k}} + \frac{8K}{c}k^\alpha k_0\vec{A}^{a*}_{21\vec{k}}\vec{A}^a_{21\vec{k}},\\
\vec{S}^a_{\vec{k}}\frac{\vec{k}}{k} &= 3\vec{S}^a_{11\vec{k}} + \frac{8K}{c}\mathrm{i}k_0\vec{A}^{a*}_{21\vec{k}}\vec{A}^a_{21\vec{k}},
\end{aligned} \tag{55}$$

where

$$\begin{aligned}
Q^a_{11\vec{k}} &= \frac{q}{c}(a_{\vec{k}111} + a_{\vec{k}112}),\\
P^{a\alpha}_{11\vec{k}} &= \frac{1}{c}k^\alpha(a_{\vec{k}111} + a_{\vec{k}112}),\\
\vec{S}^a_{11\vec{k}}\frac{\vec{k}}{k} &= \frac{1}{c}(a_{\vec{k}111} - a_{\vec{k}112}),
\end{aligned} \tag{56}$$

$$\begin{aligned}
a_{\vec{k}111} + a_{\vec{k}112} &= \sqrt{Kk_0}(A_{k1r} - A_{k1i}),\\
a_{\vec{k}111} - a_{\vec{k}112} &= \sqrt{Kk_0}(A_{k2r} + A_{k2i}).
\end{aligned} \tag{57}$$

Following this procedure, after $n^a_{\vec{k}}$ steps one obtains

$$\begin{aligned}
Q_{\vec{k}} &= n^a_{\vec{k}}Q^a_{n^a_{\vec{k}}} + Q^a_{rest},\\
P^a_{\vec{k}} &= n^a_{\vec{k}}P^a_{n^a_{\vec{k}}} + P^{a\alpha}_{rest},
\end{aligned} \tag{58}$$

where

$$Q_{n^a_{\vec{k}}} = \frac{q}{c}(a^2_{n^a_{\vec{k}}1} + a^2_{n^a_{\vec{k}}2}),$$

$$P^{\alpha}_{n^a_{\vec{k}}} = \frac{1}{c} k^{\alpha} (a^2_{n^a_{\vec{k}}1} + a^2_{n^a_{\vec{k}}2}), \tag{59}$$

$$\vec{S}^a_{n^a_{\vec{k}}} \frac{\vec{k}}{k} = \frac{1}{c} (a^2_{n^a_{\vec{k}}1} - a^2_{n^a_{\vec{k}}2}),$$

$$a_{n^a_{\vec{k}}1} + a_{n^a_{\vec{k}}2} = \left(\frac{1}{\sqrt{2}} \right)^{n^a_{\vec{k}}-1} 2\sqrt{Kk_0} (A_{k1r} - A_{k1i}),$$

$$a_{n^a_{\vec{k}}1} - a_{n^a_{\vec{k}}2} = \left(\frac{1}{\sqrt{2}} \right)^{n^a_{\vec{k}}-1} 2\sqrt{Kk_0} (A_{k2r} + A_{k2i}). \tag{60}$$

Sum over the $\vec{k}$-fields gives

$$Q^a = \sum_{\vec{k}} n^a_{\vec{k}} Q_{n^a_{\vec{k}}} + Q^a_{rest},$$

$$P^{a\alpha} = \frac{1}{c} \sum_{\vec{k}} n^a_{\vec{k}} k^{\alpha} (a^2_{n^a_{\vec{k}}1} + a^2_{n^a_{\vec{k}}2}) + P^{a\alpha}_{rest}, \tag{61}$$

$$\vec{S}^a_{\vec{k}} \frac{\vec{k}}{k} = n^a_{\vec{k}} (a^2_{n^a_{\vec{k}}1} - a^2_{n^a_{\vec{k}}2}) + \vec{S}^a_{\vec{k}rest} \frac{\vec{k}}{k}.$$

Let us now pay attention to the field "b". Everything what has been done for the field "a" can also be done for the filed "b". The previous calculation should only be repeated after $A_{k1r} - A_{k1i}$ is replaced by $A_{k1r} + A_{k1i}$ and $A_{k2r} + A_{k2i}$ by $A_{k2r} - A_{k2i}$, and by taking the complex conjugate values. Thus

$$\vec{A}^b_{2\vec{k}} = \mathrm{i} \vec{A}^b_{1\vec{k}}, \tag{62}$$

$$Q^b = -q \frac{8K}{c} \sum_{\vec{k}} k_0 \vec{A}^{b*}_{1\vec{k}} \vec{A}^b_{1\vec{k}}, \tag{63}$$

and finally

$$Q^b = \sum_{\vec{k}} n^b_{\vec{k}} Q_{n^b_{\vec{k}}} + Q^b_{rest},$$

$$P^{b\alpha} = \frac{1}{c} \sum_{\vec{k}} n^b_{\vec{k}} k^{\alpha} (b^2_{n^b_{\vec{k}}1} + b^2_{n^b_{\vec{k}}2}) + P^{b\alpha}_{rest}, \tag{64}$$

$$\vec{S}^b_{\vec{k}}\frac{\vec{k}}{k} = n^b_{\vec{k}}(b^2_{n^b_{\vec{k}}1} - b^2_{n^b_{\vec{k}}2}) + \vec{S}^b_{\vec{k}rest}\frac{\vec{k}}{k},$$

$$Q_{n^b_{\vec{k}}} = -\frac{q}{c}(b^2_{n^b_{\vec{k}}1} + b^2_{n^b_{\vec{k}}2}),$$

$$b^2_{n^b_{\vec{k}}1} + b^2_{n^b_{\vec{k}}2} = \left(\frac{1}{\sqrt{2}}\right)^{n^b_{\vec{k}}-1} 2\sqrt{Kk_0}(A_{k1r} + A_{k1i}), \tag{65}$$

$$b^2_{n^b_{\vec{k}}1} - b^2_{n^b_{\vec{k}}2} = \left(\frac{1}{\sqrt{2}}\right)^{n^b_{\vec{k}}-1} 2\sqrt{Kk_0}(A_{k2r} - A_{k2i}).$$

For the total field we have

$$Q = Q^a + Q^b = \sum_{\vec{k}}(n^a_{\vec{k}}Q_{n^a_{\vec{k}}} + n^b_{\vec{k}}Q_{n^b_{\vec{k}}}) + Q_{rest},$$

$$P^\alpha = P^{a\alpha} + P^{b\alpha} = \frac{1}{c}\sum_{\vec{k}}\left[n^a_{\vec{k}}(a^2_{n^a_{\vec{k}}1} + a^2_{n^a_{\vec{k}}2}) + n^b_{\vec{k}}(b^2_{n^b_{\vec{k}}1} + b^2_{n^b_{\vec{k}}2})\right] + P_{rest}, \tag{66}$$

$$\vec{S}_{\vec{k}}\frac{\vec{k}}{k} = \vec{S}^a_{\vec{k}}\frac{\vec{k}}{k} + \vec{S}^b_{\vec{k}}\frac{\vec{k}}{k} = \frac{1}{c}\left[n^a_{\vec{k}}(a^2_{n^a_{\vec{k}}1} - a^2_{n^a_{\vec{k}}2}) + n^b_{\vec{k}}(b^2_{n^b_{\vec{k}}1} - b^2_{n^b_{\vec{k}}2})\right] + \vec{S}_{\vec{k}rest}\frac{\vec{k}}{k}.$$

In these relations, the numbers $n^a_{\vec{k}}$, $n^b_{\vec{k}}$ are arbitrary. Introducing the quantity χ, we determine them by the requirements

$$c\chi = a^2_{n^a_{\vec{k}}1} + a^2_{n^a_{\vec{k}}2} + c\chi^a_{\vec{k}rest},$$

$$c\chi = b^2_{n^b_{\vec{k}}1} + b^2_{n^b_{\vec{k}}2} + c\chi^b_{\vec{k}rest}, \tag{67}$$

or according to Eqs. (56) and (62)

$$c\chi = \sqrt{Kk_0}2^{-n^a_{\vec{k}}}\left[(A_{k1r} - A_{k1i})^2 + (A_{k2r} + A_{k2i})^2\right] + c\chi^a_{\vec{k}rest},$$

$$c\chi = \sqrt{Kk_0}2^{-n^b_{\vec{k}}}\left[(A_{k1r} + A_{k1i})^2 + (A_{k2r} - A_{k2i})^2\right] + c\chi^b_{\vec{k}rest}, \tag{68}$$

where $\chi^a_{\vec{k}rest}$ and $\chi^b_{\vec{k}rest}$ denote complements to the solution with the highest integers $n^a_{\vec{k}}$ and $n^b_{\vec{k}rest}$. Specific value of χ is not determined here. With such $n^a_{\vec{k}}$ and $n^b_{\vec{k}}$, the expressions (66) change to

$$Q = q\sum_{\vec{k}}(n^a_{\vec{k}} - n^b_{\vec{k}}) + Q'_{rest},$$

$$P^\alpha = \sum_{\vec{k}}(\chi k^\alpha)(n^a_{\vec{k}} + n^b_{\vec{k}}) + P'^{\alpha}_{rest}, \tag{69}$$

$$\vec{S}_{\vec{k}}\frac{\vec{k}}{k} = \frac{1}{c}\left[n^a_{\vec{k}}(a^2_{n^a_{\vec{k}}1} - a^2_{n^a_{\vec{k}}2}) + n^b_{\vec{k}}(b^2_{n^b_{\vec{k}}1} - b^2_{n^b_{\vec{k}}2})\right].$$

These expressions manifest explicitely granular (particle), quantum–like structure of the field. There are two kinds of granules, "a" and "b". These results are completely within the classical physics. We conclude that the granular properties of the field are already present in the classical physics. Thus, old dilemma about corpuscular or wave nature of the electromagnetic field is not eliminatory one since both properties are present. In modern language, that is called the corpuscular–wave dualism. These granules are essentially photons. The scalar χ corresponds to the Planck constant. The "rest" of the field corresponds to the ground state of the electromagnetic field in quantum electrodynamics (QE). There are no two kinds of photons in QE.

3. Some examples

We give some examples for the illustration of the structure of the electromagnetic granules (particles).

1) $$n^b_{\vec{k}} = 0, \quad n^a_{\vec{k}'} = \delta_{\vec{k}\vec{k}'}, \quad a_{\vec{k}1} = \sqrt{c\chi}, \quad a_{\vec{k}2} = 0.$$

From Eqs. (28), (65), (8), (9) and (10) follows

$$A_{k1i} = -A_{k1r}, \quad A_{k2i} = A_{k2r}, \quad A_{k2r} = -A_{k1r}, \quad A_{k1r} = \sqrt{\frac{c\chi}{2Kk_0}},$$

$$\vec{A} = \vec{A}_{\vec{k}} = \frac{1}{2}\sqrt{\frac{c\chi}{2Kk_0}}\left[(\cos k_\alpha x^\alpha - \sin k_\alpha x^\alpha)\vec{e}_{\vec{k}1} + (\cos k_\alpha x^\alpha + \sin k_\alpha x^\alpha)\vec{e}_{\vec{k}2}\right],$$

$$\vec{A}^a_{\vec{k}} = \vec{A}_{\vec{k}}, \quad \vec{A}^b_{\vec{k}} = 0,$$

$$\vec{A}^a_{1\vec{k}} = \frac{1}{2}\sqrt{\frac{c\chi}{Kk_0}}(\cos k_\alpha x^\alpha \vec{e}_{\vec{k}1} + \sin k_\alpha x^\alpha \vec{e}_{\vec{k}2}),$$

$$\vec{A}^a_{2\vec{k}} = \frac{1}{2}\sqrt{\frac{c\chi}{Kk_0}}(-\sin k_\alpha x^\alpha \vec{e}_{\vec{k}1} + \cos k_\alpha x^\alpha \vec{e}_{\vec{k}2}),$$

$$\vec{A}^a_{1\vec{k}}\vec{A}^a_{2\vec{k}} = 0, \quad |\vec{A}^a_{1\vec{k}}| = |\vec{A}^a_{2\vec{k}}|, \quad \vec{A}^a_{1\vec{k}} \times \vec{A}^a_{2\vec{k}} \sim \vec{k}.$$

Therefore, this "photon" is composed from two right–hand orthogonal and right–hand circularly polarized waves of equal amplitude ($Q = q\chi$, $P^\alpha = \chi k^\alpha$, $\vec{S}_{\vec{k}}(\vec{k}/k) = \chi$).

2) $$n^b_{\vec{k}} = 0, \quad n^a_{\vec{k}'} = \delta_{\vec{k}\vec{k}'}, \quad a_{\vec{k}1} = 0, \quad a_{\vec{k}2} = \sqrt{c\chi}.$$

$$A_{k1i} = -A_{k1r}, \quad A_{k2i} = A_{k2r}, \quad A_{k2r} = A_{k1r}, \quad A_{k1r} = \sqrt{\frac{c\chi}{2Kk_0}},$$

$$\vec{A} = \vec{A}_{\vec{k}} = \frac{1}{2}\sqrt{\frac{c}{2Kk_0}}\left[(\cos k_\alpha x^\alpha + \sin k_\alpha x^\alpha)\vec{e}_{\vec{k}1} + (\cos k_\alpha x^\alpha - \sin k_\alpha x^\alpha)\vec{e}_{\vec{k}2}\right],$$

$$\vec{A}^a_{\vec{k}} = \vec{A}_{\vec{k}}, \quad \vec{A}^b_{\vec{k}} = 0,$$

$$\vec{A}^a_{1\vec{k}} = \frac{1}{2}\sqrt{\frac{c\chi}{Kk_0}}(\cos k_\alpha x^\alpha \vec{e}_{\vec{k}1} - \sin k_\alpha x^\alpha \vec{e}_{\vec{k}2}),$$

$$\vec{A}^a_{2\vec{k}} = \frac{1}{2}\sqrt{\frac{c\chi}{Kk_0}}(-\sin k_\alpha x^\alpha \vec{e}_{\vec{k}1} - \cos k_\alpha x^\alpha \vec{e}_{\vec{k}2}),$$

$$\vec{A}^a_{1\vec{k}}\vec{A}^a_{2\vec{k}} = 0, \quad |\vec{A}^a_{1\vec{k}}| = |\vec{A}^a_{2\vec{k}}|, \quad \vec{A}^a_{1\vec{k}} \times \vec{A}^a_{2\vec{k}} \sim -\vec{k}.$$

This "photon" is composed from two left–hand orthogonal and left–hand circularly polarized waves of equal amplitude ($Q = q\chi$, $P^\alpha = \chi k^\alpha$, $\vec{S}_{\vec{k}}(\vec{k}/k) = -\chi$).

3) $$n^a_{\vec{k}} = 0, \quad n^b_{\vec{k}'} = \delta_{\vec{k}\vec{k}'}, \quad b_{\vec{k}1} = \sqrt{c\chi}, \quad b_{\vec{k}2} = 0.$$

$$A_{k1i} = A_{k1r}, \quad A_{k2i} = -A_{k2r}, \quad A_{k2r} = A_{k1r}, \quad A_{k1r} = \sqrt{\frac{c\chi}{2Kk_0}},$$

$$\vec{A} = \vec{A}_{\vec{k}} = \frac{1}{2}\sqrt{\frac{c\chi}{2Kk_0}}\left[(\cos k_\alpha x^\alpha + \sin k_\alpha x^\alpha)\vec{e}_{\vec{k}1} + (\cos k_\alpha x^\alpha - \sin k_\alpha x^\alpha)\vec{e}_{\vec{k}2}\right],$$

$$\vec{A}^b_{\vec{k}} = \vec{A}_{\vec{k}}, \quad \vec{A}^a_{\vec{k}} = 0,$$

$$\vec{A}^a_{1\vec{k}} = \frac{1}{2}\sqrt{\frac{c\chi}{Kk_0}}(\cos k_\alpha x^\alpha \vec{e}_{\vec{k}1} - \sin k_\alpha x^\alpha \vec{e}_{\vec{k}2}),$$

$$\vec{A}^a_{2\vec{k}} = \frac{1}{2}\sqrt{\frac{c\chi}{Kk_0}}(\sin k_\alpha x^\alpha \vec{e}_{\vec{k}1} + \cos k_\alpha x^\alpha \vec{e}_{\vec{k}2}),$$

$$\vec{A}^b_{1\vec{k}}\vec{A}^b_{2\vec{k}} = 0, \quad |\vec{A}^b_{1\vec{k}}| = |\vec{A}^b_{2\vec{k}}|, \quad \vec{A}^a_{1\vec{k}} \times \vec{A}^a_{2\vec{k}} \sim \vec{k}.$$

This "photon" is composed from two right–hand orthogonal and left–hand circularly polarized waves of equal amplitude ($Q = -q\chi$, $P^\alpha = \chi k^\alpha$, $\vec{S}_{\vec{k}}(\vec{k}/k) = \chi$).

4) $$n^a_{\vec{k}} = 0, \quad n^b_{\vec{k}'} = \delta_{\vec{k}\vec{k}'}, \quad b_{\vec{k}1} = 0, \quad b_{\vec{k}2} = \sqrt{c\chi}.$$

$$A_{k1i} = A_{k1r}, \quad A_{k2i} = -A_{k2r}, \quad A_{k2r} = -A_{k1r}, \quad A_{k1r} = \sqrt{\frac{c\chi}{2Kk_0}},$$

$$\vec{A} = \vec{A}_{\vec{k}} = \frac{1}{2}\sqrt{\frac{c\chi}{2Kk_0}}\left[(\cos k_\alpha x^\alpha + \sin k_\alpha x^\alpha)\vec{e}_{\vec{k}1} - (\cos k_\alpha x^\alpha - \sin k_\alpha x^\alpha)\vec{e}_{\vec{k}2}\right],$$

$$\vec{A}^b_{\vec{k}} = \vec{A}, \quad \vec{A}^a_{\vec{k}} = 0,$$

$$\vec{A}^a_{1\vec{k}} = \frac{1}{2}\sqrt{\frac{c\chi}{Kk_0}}(\cos k_\alpha x^\alpha \vec{e}_{\vec{k}1} + \sin k_\alpha x^\alpha \vec{e}_{\vec{k}2}),$$

$$\vec{A}^a_{2\vec{k}} = \frac{1}{2}\sqrt{\frac{c\chi}{Kk_0}}(\sin k_\alpha x^\alpha \vec{e}_{\vec{k}1} - \cos k_\alpha x^\alpha \vec{e}_{\vec{k}2}),$$

$$\vec{A}^b_{1\vec{k}}\vec{A}^b_{2\vec{k}} = 0, \quad |\vec{A}^b_{1\vec{k}}| = |\vec{A}^b_{2\vec{k}}|, \quad \vec{A}^a_{1\vec{k}} \times \vec{A}^a_{2\vec{k}} \sim -\vec{k}.$$

This "photon" is composed from two left–hand orthogonal and right–hand circularly polarized waves ($Q = -q\chi$, $P^\alpha = \chi k^\alpha$, $\vec{S}_{\vec{k}}(\vec{k}/k) = -\chi$).

4. *Summary*

Any solution of the Maxwell free field equations can be decomposed into parts which possess corpuscular properties as they are known in the modern quantum theory. It is a consequence of the scalar constant of motion that is the result of two Maxwellian solutions. The particles have an internal field structure. This shows that wave–corpuscular dualism of the electromagnetic field is a phenomenon already present in the classical physics.

References

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SKALARNA KONSTANTA GIBANJA I NJEZINE POSLJEDICE

Razmatrano je postojanje skalarne konstante gibanja i analizirani su dobiveni zaključci.