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τ–LEPTON TUNNELING THROUGH STATIC ONE–DIMENSIONAL HIGGS POTENTIAL

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Using the GWS lagrangian for electroweak interactions in the (1+1)–dimensional case, we studied the τ–lepton tunneling through the static one–dimensional Higgs potential. We obtained the exact solution for the Dirac τ–lepton wave functions and the exact expression for the transmission coefficient of τ–leptons.

1. *Introduction*

The GSW model of electroweak interactions was developed for leptons by Glashow [1], Salam [2] and Weinberg [3] and extended to include the quark degrees of freedom in Ref. 4. It is a minimal model of electroweak interactions with respect to the number of required fields and it is characterized by the representation of weak interaction universality as a symmetry under weak isospin $\mathrm{SU}(2)_W$ and weak hypercharge $\mathrm{U}(1)_W$ transformations. In order to generate the mass of the weak gauge bosons, the Higgs mechanism for the spontaneous symmetry breaking of the $\mathrm{SU}(2)_W \times \mathrm{U}(1)_W$ symmetry is used in such a way that the electromagnetic gauge group $\mathrm{U}(1)_{EM}$ remains conserved as a residual symmetry.

The Higgs sector of the GSW model is needed to give mass to the gauge bosons without destroying the renormalizability of the theory. However, the mass and strength of the self–interaction of the Higgs particle can not be given explicitely.

The upper limit of the Higgs mass was estimated in Ref. 5 at $M_H < 10^3$ GeV and the most recent lower limit is quoted at $M_H > 60$ GeV (See Refs. 6 and 7). For a general review of the Higgs mechanism see Refs. 9 and 10.

The coupling of fermions to Higgs particles is proportional to the fermion masses. The light leptons (electrons with a mass of 0.511 MeV and muons with a mass of 105.6 MeV) are, therefore, assumed to play a minor role in the Higgs–particle reactions. The τ–leptons with a mass of 1.87 GeV may, however, be usefull to study the most likely interactions of the Higgs particles probably involve the heavy quark degrees of freedom.

Due to its applications on the macroscopic scale, the Abelian gauge theory of electromagnetism was the subject of extensive classical treatment without the use of field quantization. The classical electromagnetic equations are solved exactly for many particular problems in one, two or three spatial dimensions. On the other hand, the exact classical solution for the non–Abelian gauge theories, such as the GSW model, are much harder to find. In the present paper we present an exact classical solutions for the problem of the τ–lepton tunneling through the static one–dimensional Higgs potential.

2. *The lepton–field equations*

The GSW lagrangian, including only the terms for the free Higgs particles with self–interaction, free τ–leptons and the lepton–Higgs interactions, after the spontaneous symmetry breaking and generation of the Higgs and τ–lepton masses, is given by

$$\mathcal{L} = -\frac{1}{2}\left(\partial_\mu\varphi\partial^\mu\varphi + M_H^2\varphi^2\right) + \frac{\mu M_H}{\sqrt{2}}\varphi^3 - \frac{\mu^2}{4}\varphi^4 -$$

$$-\overline{\tau}\left(\gamma_\mu\partial^\mu + M_\tau\right)\tau - G_\tau\overline{\tau}\tau\varphi, \tag{2.1}$$

where $\varphi = \varphi(\underline{x}, t)$ is the real scalar Higgs field with mass M_H, $\tau = \tau(\underline{x}, t)$ is the complex spinor τ–lepton field with the mass M_τ, μ is the Higgs self–interaction parameter and G_τ is the lepton–Higgs coupling constant.

In the (1+1)–dimensional space the lagrangian (2.1) is given by

$$\mathcal{L} = \frac{1}{2}\left(\frac{\partial\varphi}{\partial t}\right)^2 - \frac{1}{2}\left(\frac{\partial\varphi}{\partial z}\right)^2 - \frac{1}{2}M_H^2\varphi^2 + \frac{\mu M_H}{\sqrt{2}}\varphi^3 - \frac{\mu^2}{4}\varphi^4 -$$

$$-\overline{\tau}\left[\gamma_3\frac{\partial\tau}{\partial z} - \mathrm{i}\gamma_4\frac{\partial\tau}{\partial t} + (M_\tau + G_\tau\varphi)\tau\right], \tag{2.2}$$

where the space axis is chosen to be the z–axis for the sake of simplicity in the particular representation of the Dirac γ–matrices, even though the results obtained are quite general. The field equation for the pure Higgs–field with self–interaction,

$$\frac{\partial^2\varphi}{\partial t^2} - \frac{\partial^2\varphi}{\partial z^2} + M_H^2\varphi - \frac{3\mu M_H}{\sqrt{2}}\varphi^2 + \mu^2\varphi^3 = 0, \tag{2.3}$$

where the lepton source of Higgs–particles is neglected, admits a stable static solution of the form

$$\varphi_\pm = \frac{M_\tau}{G_\tau}(1 \pm \tanh\frac{M_H z}{2}), \quad M_\tau = G_\tau\frac{M_H}{\sqrt{2}\mu}. \tag{2.4}$$

The solution (2.4) is a well–known textbook case known as the kink solution. Although very popular in the literature, it has so far not been useful in any true or hypothetical physical applications.

It should be noted that the GSW theory the scalar Higgs field is a SU(2)–doublet and carries hypercharge. In the unitary gauge, used in the present paper, there is in general a singularity when $\varphi = 0$, and a stable GSW kink–solution is not possible.

Nevertheless, the solution (2.4), applicable to the broken vacuum of the GSW model only, exists as a static solution to the Eq. (2.3). Therefore, a motion of τ–leptons in the potential field proportional to (2.4), analogously to the case of motion of leptons in the static Coulomb potentials in electrodynamics, may well be approximately described by the present interaction model. The physical situation here is similar to that of the hydrogen atom where the static classical solution to the Maxwell equations, i.e. the Coulomb potential, is substituted into classical electron field equation to obtain the electron wave functions.

However, we should keep in mind that the solution (2.4) is not a proper soliton solution to the GSW lagrangian, as such according to the above discussion does not exist.

Finally, it should be noted that in the present paper we use an approximation where the back reaction of τ–leptons on Higgs boson is neglected. If not neglected, the Higgs and the τ–lepton equations should be solved simultaneously. In order to obtain an exact classical solution, we assume that for lower energies the Higgs particles (with an assumed mass of $M_H > 100$ GeV) are relatively little affected by the τ–leptons (with mass of $M_\tau = 1.87$ GeV). The rigorous treatment of the validity of such an approximation is not discussed in the present paper.

The energy density of such a static Higgs solution is strongly concentrated around the spatial origin due to a rather large mass of the Higgs–particle setting the space scale in (2.4).

Substituting (2.4) into the τ–lepton part of the lagrangian (2.2) we obtain the lepton–field equation in the form

$$\gamma_3 \frac{\partial \tau}{\partial z} - \mathrm{i}\gamma_4 \frac{\partial \tau}{\partial t} + [M_\tau + V(z)]\tau = 0, \tag{2.5}$$

where

$$V(z) = \frac{1}{2} V_0 \left(1 + \tanh \frac{z}{z_0}\right), \quad V_0 = 2M_\tau, \quad z_0 = \frac{2}{M_H}. \tag{2.6}$$

This is now Dirac equation with a smooth static potential (2.6) which will be solved exactly for the τ–lepton wave functions. The potential (2.6) has been studied in Ref. 11.

Similar studies on fermion–kink interactions, without reference to GSW model, have been reported in Ref. 8 and in some references therein. The static τ-lepton wave function has the form

$$\tau(z,t) = u(z)\exp(-\mathrm{i}Et) \tag{2.7}$$

where E is the energy of the τ–leptons and

$$u(z) = [u_1(z)u_2(z)u_3(z)u_4(z)]^T \tag{2.8}$$

is the static four–component spinor. Substituting Eq. (2.7) into Eq. (2.5), we obtain the equation

$$\gamma_3 \frac{\mathrm{d}u}{\mathrm{d}z} + [M_\tau + V(z)]u = \gamma_4 E u. \tag{2.9}$$

In the present paper we use the standard representations for the matrices γ_3, γ_4, α_3 and σ_3.

We assume that the τ–lepton has the positive helicity such that the condition $\sigma_3 u = +u$ is satisfied. From the particular representation of γ–matrices we see that it implies $u_2(z) = u_4(z) = 0$. Therefore, the system of four equations (2.9) reduces to a system of two equations for $u_1(z)$ and $u_3(z)$, i.e.

$$-\mathrm{i}\frac{\mathrm{d}u_3}{\mathrm{d}z} + [M_\tau + V(z)]u_1 = Eu_1, \tag{2.10a}$$

$$-\mathrm{i}\frac{\mathrm{d}u_1}{\mathrm{d}z} - [M_\tau + V(z)]u_3 = Eu_3. \tag{2.10b}$$

Introducing the fields ϕ_1 and ϕ_2 instead of u_1 and u_3, in such a way that

$$\phi_1(z) = u_1(z) + \mathrm{i}u_3(z), \quad \phi_2(z) = u_1(z) - \mathrm{i}u_3(z), \tag{2.11}$$

we obtain the following uncoupled second–order differential equations for the fields ϕ_1 and ϕ_2

$$\frac{\mathrm{d}^2\phi_1}{\mathrm{d}z^2} + \left\{ E^2 - [M_\tau + V(z)]^2 - \frac{\mathrm{d}V}{\mathrm{d}z} \right\} \phi_1 = 0, \tag{2.12a}$$

$$\frac{\mathrm{d}^2\phi_2}{\mathrm{d}z^2} + \left\{ E^2 - [M_\tau + V(z)]^2 + \frac{\mathrm{d}V}{\mathrm{d}z} \right\} \phi_2 = 0. \tag{2.12b}$$

The charge density of the τ–leptons with the positive helicity is

$$\rho = -\mathrm{i}\overline{\tau}\gamma_4\tau = \tau^+\tau = u^+u = |u_1|^2 + |u_3|^2 = \frac{1}{2}(|\phi_1|^2 + |\phi_2|^2), \tag{2.13}$$

while the current density is

$$s = \overline{\tau}\gamma_3\tau = \tau^+\alpha_3\tau = u^+\alpha_3 u = u_1 u_3^* + u_3 u_1^* = \frac{\mathrm{i}}{2}(\phi_1^*\phi_2 - \phi_1\phi_2^*). \tag{2.14}$$

3. *The exact solutions of the lepton–field equations*

The equations (2.12) can be reduced to the hypergeometric equations and solved exactly. The exact solutions are given by

$$\phi_1 = x^p(1-x)^q F(p+q+r, p+q-r+1, 2p+1; x) \tag{3.1a}$$

$$\phi_2 = x^p(1-x)^q \frac{(p+q+r)(p-q+r)}{r^2} F(p+q+r+1, p+q-r, 2p+1; x) \tag{3.1b}$$

where

$$x = \frac{1}{1+\exp(2z/z_0)} \tag{3.2}$$

and

$$p^2 = \frac{-E^2 + 9M_\tau^2}{M_H^2}, \quad q^2 = \frac{-E^2 + M_\tau^2}{M_H^2}, \quad r = 2\frac{M_\tau}{M_H}. \tag{3.3}$$

In the asymptotic region $z \to +\infty (x \to 0)$, we obtain from Eq. (3.1)

$$\phi_1(+\infty) \to \exp(\mathrm{i}k^{'}z), \tag{3.4a}$$

$$\phi_2(+\infty) \to \frac{(p+q+r)(p-q+r)}{r^2} \exp(\mathrm{i}k^{'}z), \tag{3.4b}$$

where

$$k^{'} = \sqrt{E^2 - 9M_\tau^2}. \tag{3.5}$$

In the asymptotic region $z \to -\infty (x \to 1)$, we obtain from Eq. (3.1)

$$\phi_1(-\infty) \to \frac{\Gamma(2p+1)\Gamma(-2q)}{\Gamma(p-q-r+1)\Gamma(p-q+r)} \exp(-\mathrm{i}kz)+$$

$$+\frac{\Gamma(2p+1)\Gamma(2q)}{\Gamma(p+q-r+1)\Gamma(p+q+r)} \exp(\mathrm{i}kz), \tag{3.6a}$$

$$\phi_2(-\infty) \to \frac{(p+q+r)(p-q+r)}{r^2} \frac{\Gamma(2p+1)\Gamma(-2q)}{\Gamma(p-q-r+1)\Gamma(p-q+r)} \exp(-\mathrm{i}kz)+$$

$$+\frac{(p+q+r)(p-q+r)}{r^2} \frac{\Gamma(2p+1)\Gamma(2q)}{\Gamma(p+q-r+1)\Gamma(p+q+r)} \exp(\mathrm{i}kz), \tag{3.6b}$$

where

$$k = \sqrt{E^2 - M_\tau^2}. \tag{3.7}$$

4. *The transmission coefficient*

From Eqs. (3.5) and (3.7) we see that when $E > 3M_\tau$, which is the most interesting case from the point of view of the high–energy physics, both p and q are purely imaginary. Substituting Eq. (3.4) into Eq. (2.14), we obtain the transmitted τ–lepton current as follows

$$s(+\infty) = \frac{2\lambda}{r}, \quad \lambda = \mathrm{i}p. \tag{4.1}$$

Substituting the second terms of Eqs. (3.6a) and (3.6b), which describe the incident τ-lepton waves (we denote them by ϕ_1^+ and ϕ_2^+, respectively), into Eq. (2.14), we obtain the incident τ–lepton current as follows

$$s^{+}(-\infty)=\frac{2\kappa}{r}|\phi_1^{+}|^2, \quad \kappa=\mathrm{i}q. \tag{4.2}$$

The square of the absolute value of the second term of Eq. (3.6a) is easily calculated keeping in mind that both p and q are purely imaginary. We obtain

$$s^{+}(-\infty)=\frac{2\kappa}{r}\left|\frac{\Gamma(2p+1)\Gamma(2q)}{\Gamma(-p-q-r+1)\Gamma(p+q+r)}\right|^2. \tag{4.3}$$

The transmission coefficient is equal to

$$T=\frac{s(+\infty)}{s^{+}(-\infty)}=\frac{\lambda}{\kappa}\left|\frac{\Gamma(-p-q-r+1)\Gamma(p+q+r)}{\Gamma(2p+1)\Gamma(2q)}\right|^2. \tag{4.4}$$

Using now Eqs. (4.1) and (4.2) and the formulae

$$\Gamma(\chi+1)=\chi\Gamma(\chi),\ \ \Gamma(\chi)\Gamma(1-\chi)=\frac{\pi}{\sin(\pi x)},\ \ |\Gamma(1-\mathrm{i}\chi)|^2=\frac{\pi\chi}{\sinh(\pi\chi)}, \tag{4.5}$$

after some algebra we obtain a fairly simple final result for the transmission coefficient

$$T=\frac{\sinh(2\pi\lambda)\ \sinh(2\pi\kappa)}{\sinh^2[\pi(\lambda+\kappa)]+\sin^2(\pi r)} \tag{4.6}$$

where, in accordance with Eqs. (3.3), (4.1) and (4.2), we have

$$\lambda^2=\frac{E^2-9M_\tau^2}{M_H^2},\ \ \kappa^2=\frac{E^2-M_\tau^2}{M_H^2},\ \ r=2\frac{M_\tau}{M_H}. \tag{4.7}$$

In the limit $E \gg 3M_\tau$, it is easily seen that the transmission coefficient is equal to unity as expected. The numerical result for the transmission coefficient as a function of the incident τ–lepton energy in units of the τ–lepton mass, for the value of the Higgs mass of $M_H = 100$ GeV, is shown in Fig. 1. Furthermore, it should be noted that for larger Higgs masses ($M_H \to \infty$ and $z_0 \to 0$) the potential (2.6) approaches the step function at the spatial origin and the transmitted fraction of τ–leptons, and indeed the transmission coefficient, decreases.

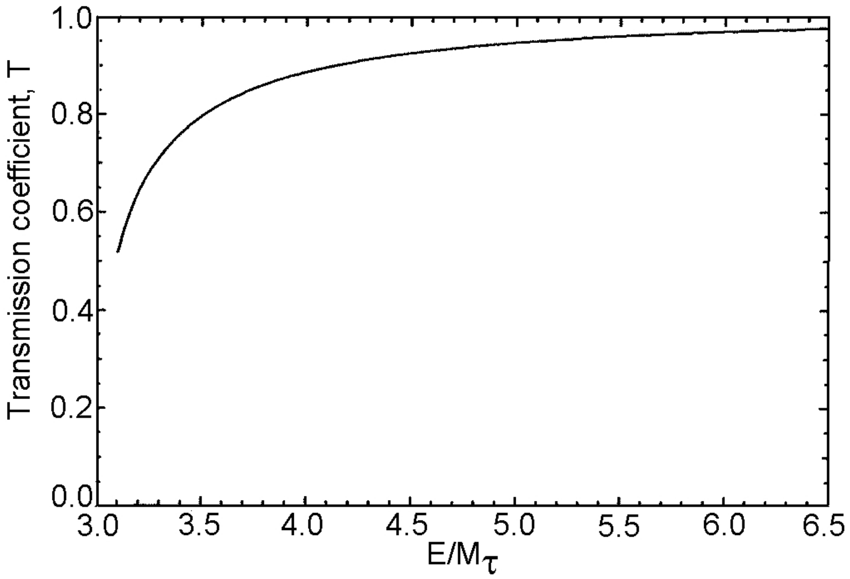

Fig. 1. The transmission coefficient T as a function of the incident τ–lepton energy E in units of the τ–lepton mass M_τ, with the assumed Higgs–particle mass of $M_H = 100$ GeV.

5. Conclusions

We indicated the possibility of finding the exact solutions for the non–linear Higgs–particle lagrangian in the (1+1)–dimensional case and to use such solutions as potential barriers for the exact classical studies of interactions between the Higgs and lepton sectors of the GSW model of electroweak interactions. The interesting question for further studies is whether the self–interaction terms of the Higgs sector lagrangian can be modified to allow such solutions in the physically more interesting (3+1)–dimensional case. In view of the Derrick theorem, such self–interaction terms may have to include higher order derivative terms of some kind of stabilization methods which are at the same time consistent with the GSW–model and provide correct masses of the gauge and lepton fields.

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TUNELIRANJE τ–LEPTONA KROZ STATIČKI JEDNODIMENZIJSKI HIGGSOV POTENCIJAL

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Primjenom GSW lagranžijana elektroslabog međudjelovanja u (1+1)–dimenziji proučava se tuneliranje τ–leptona kroz statički jednodimenzijski Higgsov potencijal. Dobiveno je egzaktno rješenje za Diracovu valnu funkciju τ–mezona i egzaktan izraz za koeficijent transmisije τ–leptona.