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THE SU(2) ASYMMETRY IN THE LIGHT QUARK SEA

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We obtain an analytical expression for the quantity, $\overline{d}(x) - \overline{u}(x)$, which measures the SU(2) asymmetry in the light quark sea, by solving analytically the (non–singlet) evolution equation of QCD, for which Debye's steepest descent method in its orginal rigorous form is used. Our solution is consistent with the recent NMC data concerned, and Regge–behaved as well.

1. *Introduction*

While the Gottfried sum rule (GSR) [1] had predicted that

$$S_G = \int_0^1 \frac{\mathrm{d}x}{x} [F_2^{ep}(x) - F_2^{en}(x)] = \frac{1}{3}, \tag{1}$$

the recent NMC data have been interpreted [2] to yield a significantly lower value, viz.

$$S_G = 0.24 \pm 0.016. \tag{2}$$

To explain the above discrepancy there has been a profusion of theoretical papers in recent times [3-6], which invoke either the SU(2) symmetry–breaking in the light quark sea [3,4] or the i–spin symmetry–breaking between the proton and the neutron [5,6] as the two alternative theories of GSR–violation. Since the GSR itself implies respecting the symmetries concerned, some kind of symmetry–breaking has been, naturally, introduced into its violation as indicated by Eq. (2) above. For short, we would often refer to the sea symmetry–breaking by the acronym, SSB, not related to the Goldstone theorem. The i–spin symmetry–breaking between the neutron and the proton will analogously be designated NPSB.

We mention, in passing, the fact that the cited papers have not made use of the QCD, which, incidentally, is the reason why we seek here to investigate the discrepancy, in question, in terms of the QCD evolution equation.

It is, however, of interest to recall that the SSB, in the very sense in which it occurs in the cited references, corresponds to an inequality occurring in a relatively early paper of Field and Feynman [7], which reads

$$\overline{d}(x) - \overline{u}(x) > 0 \tag{3}$$

and which followed from their assumption,

$$u^v > d^v \tag{4}$$

with the subscript v denoting “valence”. It is also to be noted that Eq. (4) also led, owing to the exclusion principle, to alternative form of Eq. (3), viz.

$$d\overline{d} - u\overline{u} > 0. \tag{5}$$

The calculated value of S_G in Ref. 7 to whit,

$$S_G = 0.27 \tag{6}$$

seems to be rather close to the one given in Eq. (2). For the data [8] avialable in 1977 were dated by present standards.

So much for the elements of SSB as it occurs in some recent references too [4,6]. Now, the other kind of symmetry–breaking, NSPB, is defined [6] by the inequality,

$$u^{\mathrm{p}} \neq d^{\mathrm{n}} \tag{7}$$

where the suffices (p,n) denote proton and neutron, respectively. As a measure of symmetry–breaking, we define

$$D(x) = \overline{d}(x) - \overline{u}(x) \tag{8}$$

When the contending models are examined, a substratum common to them looms large. In fact, Eqs. (4) and (7) are not quite incongruous (except for very small x, which makes the sea distribution more important). Even the deep inelastic scattering (DIS) cross–sections for several important processes turn out to be almost identical whether the SSB–model or the NPSB–model is employed to modify the naive quark model which is resorted to by either model calculation. The question that arises in view of the above state affairs is this: is the dovetailing of some asymmetry to a symmetric (quark) model a compensation for the neglect of possible hadronic processes ? Such hadronic processes can be entirely ignored when the DIS is counted at energies which are high enough for interactions to be purely of the fundamental type. Unfortunately, this energetic criterion is scarcely fulfilled till now. The sequel to the above question is another riddle. How are we to decide which is the more viable model ? A futuristic hope of some decision, if and when more reliable data would be avialable has, however, been expressed by some authors [6].

2. The SU(2) *asymmetry concerned in the light of QCD*

Meanwhile, we try a tertium quid – a third something, QCD. It entails no dovetailing of a symmetric model and a "corrective" asymmetry. Besides, we calculate no DIS cross–section here. We just examine, analytically, the x–dependence of discrepancy itself which undoubtedly is the crux of the matter under consideration. Our choice of QCD for the DIS stems from the precedent set by gluonic corrections to interpret scaling violations in similar processes. Moreover, gluon emission from a valence quark and its conversion into sea quark pair would let QCD admit both kinds of symmetry–breaking referred to above. Hence, our incorporation of any asymmetry of the kind would not be prejudical as it would be to a free quark model. For ours is a QCD–based model. Altough, both kinds of asymmetry presumably coexist, we choose the SU(2) sea asymmetry simply for definiteness and without any predilection.

Accordingly, we recast the inequality of Eq. (3) into an equation, where D measures the discrepancy referred to above. Note that the Q^2–dependence, too, of the quantities of Eq. (8) will be shown explicitly later on. Note also that we consider the asymmetry, D, to be a practically–minded measure of the GSR–NMC discrepancy.

As for the x–dependence of $\overline{d}$ and $\overline{u}$ of Eq. (8), empirical expression have been tried by several authors. In Ref. 7 itself we find one such attempt. We investigate the x–dependence of D instead, and that also by analytically solving the nonsinglet evolution equation of QCD. During the calculation of the inverse transform of the Mellin transformed version of the evolution equation, we use the Debye method of steepest descent, maintaining all its rigour. Our calculation is entirely free from the "convergence–crisis" at small x, which crisis has long been known to impair the non–rigorous but simplified variant of the Debye's method [9]. It is amusing to note that rigour has lent a simplicity to our working formula which the over–

simplified approach lacked. Our earlier success in treating the singlet case of the coupled evolution Eq. (10) has prompted us to try the following method.

Without any further ado, we write the nonsinglet kind of evolution equation for the D–function of Eq. (8),

$$\frac{\mathrm{d}D(x,t)}{\mathrm{d}t} = a(t) \int\limits_x^1 \frac{\mathrm{d}y}{y} P_{DD}(x/y) D(y,t), \tag{9}$$

where we consider the linear evolution equation valid for D, since it is so for $\overline{d}$ and $\overline{u}$ of Eq. (8) and where we have shown the Q^2–dependence of D explicitely by stipulating

$$D = D(x,t) \tag{9a}$$

with

$$t = \ln(Q^2/Q_0^2), \tag{9b}$$

Q_0^2 being the usual (fixed) reference momentum. Note that we have written $a(t)$ in Eq. (9) for $a(t)/2\pi$ to facilatate the writing. To solve Eq. (9) we proceed as follows: With S for the Mellin transformation variable, the Mellin transformed version of Eq. (9) becomes

$$\frac{\mathrm{d}M_D}{\mathrm{d}t} = a(t) M_P(S) M_D(S,t) \tag{10}$$

where

$$M_D(S,t) = \int\limits_0^1 \mathrm{d}x x^s D(x,t) \tag{10a}$$

and

$$M_P(S) = \int\limits_0^1 \mathrm{d}z z^s P(z). \tag{10b}$$

We had to use the convolution theorem of the Mellin transforms to obtain Eq. (10). We have also assumed

$$\int a(t)\mathrm{d}t \simeq at, \tag{10c}$$

which is permissible when experimental data to be interpreted belong to a region of approach to the asymptotic realm. Now, in view of Eq. (10c), the solution of Eq. (10) is trivially found to be

$$M_D(s,t) = A \exp(M_P(s)at) \tag{11}$$

where the t–independent object, A, can be eliminated as follows: we assume an imput distribution,

$$D(x,0) \simeq (1-x)^d, \tag{12}$$

which yields, in view of (10a),

$$M_D(S,0) \simeq \int\limits_0^1 \mathrm{d}x x^s (1-x)^d = B(s+1,d+1), \tag{13}$$

where B is the usual symbol for the beta function. Comparing Eqs. (11) and (13), we obtain

$$A = B(s+1,d+1). \tag{14}$$

Hence, Eq. (11) turns out to be free from A, to wit,

$$M_D(s,t) = B(s+1,d+1)\exp[M_P(s,0)]. \tag{15}$$

But $M_P(s)$, which is defined in Eq. (10b), involves $P(Z)$. To compute $P(Z)$ in QCD, one must consider all elementary interactions which would contribute to

$$\gamma^* + p \rightarrow all \tag{16}$$

where, for definiteness, the DIS is envisaged as an ep–process. The process dictated by the free quark model are supplemented in QCD by corrections due to gluonic process. Incidentally, the gluonic corrections were not incorporated in the calculation of the DIS–cross–sections in cited references [4,6]. It appears that the hypotetical symmetry–breaking has been a pragmatic compensation for the emission of corrections. It is hoped that gluonic corrections mitigate the fault of neglecting hadronics interactions, which are apt to complicate the simple picture of fundamental interactions as long as our DIS data corresponds to the present energy limits, which are rather low for the interaction to be exclusively fundamental.

To return to the question of the calculation of $P(Z)$, we note that the free quark model results are significanntly modifed in QCD owing to (i) the propagator correction and (ii) the vertex correction. We need not go into the details of these things, the relevant calculation being standard [11]. Yet, the following points deserve mention:

First, the gluon being real, the polarization sum is given by

$$\sum_{\lambda} e_\alpha(k,\lambda) e^*_\beta(k,\lambda) = -g_{\alpha\beta} + \frac{k_\alpha u_\beta + k_\beta u_\alpha}{k \cdot n} \tag{17}$$

where for a lightlike gauge, we also have

$$k \cdot \epsilon = u \cdot \epsilon = 0,$$

$$u^2 = 0. \tag{17a}$$

Second, one is to consider terms of equal in Q^2 which yields

$$\Phi^{\mu\nu} = -2\pi \int_{-1}^{1} \mathrm{d}(\cos\Theta) \int_{0}^{\infty} \frac{\mathrm{d}k^0 \cdot k^0}{2} \frac{1}{\nu} \delta(\rho - x) \left(\frac{1+\rho^2}{1-\rho}\right) \times$$

$$\times \frac{\mathrm{Tr}\gamma^\mu (\rho \not{k}_f + \not{q}) \gamma^\nu \not{p}_f}{2k^0 p^0_f \cos\Theta - (Q_0^2 + 2k^0 p^0_f)} \tag{18}$$

where

$$\rho = 1 - \frac{Qk^0}{\nu}. \tag{18a}$$

Third, the singularity at $\rho = 1$ of Eq. (18) is just cancelled as complete corrections for the vertex and the propagator are incorporated. With these three points for a cue, one can obtain by some straightforward calculation the expression for $P(Z)$, which reads

$$P(Z) = \frac{4}{3} \frac{1+Z^2}{1-Z}, \tag{19}$$

where a three–flavour model is employed.

Combining Eqs. (10b) and (19), one is led to the expression for $M_p(s)$, viz.,

$$M_p(s) = c_2(R) \left[\frac{3}{2} + \frac{1}{(s+1)(s+2)} - 2\Psi(s+2) - 2\gamma\right], \tag{20}$$

where $c_2(R)$ is a Cassimir operator corresponding to the adjoint representation of the colour group, Ψ the logarithmic gamma function, and γ the Euler–Mascheroni constant ≈ 0.577.

The quantity of interest D, which is the intended solution of EQ. (9), is given by the inverse Mellin transform of Eq. (15), which is

$$D(x,t) = \frac{1}{2\pi \mathrm{i}} \int_C \exp[hf(s,x)B(s+1,d+1)]\mathrm{d}s, \tag{21}$$

where

$$h \simeq at, \tag{21a}$$

$$f(s,x) = M_p(s) - \frac{s+1}{h} \ln x, \tag{21b}$$

and C is the integration contour to be specified presently. While formulating the steepest descent method, Debye had specified C such that on a part C_0 thereof, the following conditions – the so–called Debye conditions – hold:

(1) Im $f(s)$ = constant along C.

(2) There exist some $S_0 \in C_0$ satisfying $\frac{\mathrm{d}f}{\mathrm{d}s}\big|_{s=s_0} = 0$.

(3) Re $f(s)$ is a relative maximum at $s = s_0$, s_0 being the saddle–point. (Note that a local maximum is inadmissible inside the domain of analyticity of Re $f(s)$).

It is to be noted that the customary approach would simplify matters by an illegitimate assumption that the minimum of Re $f(s)$ corresponds to the maximum of Im $f(s)$. Actually, Im $f(s)$ is not a maximum but merely a constant there. A further oversimplification, preferred in that nonrigorous approach, consisted in integration along the imaginary axis. This had the apparent advantage of real part of the integration variable being a constant. But, then, it had no way to incorporate the Debye conditions (1) and (2) which determine the contour in our case. Moreover, a second order Taylor expansion sufficed for the simplified method of steepest descent; by contrast, we employ a full expansion of $f(s)$ as we would presently see. While the customary oversimplification of the steepest descent method sacrificed rigour in more ways than the few we have mentioned, we can nevertheless realize the reason of its failure to ensure covergence of the solution, particularly at small values of x. While numerical solutions have been in vogue, for reasons discussed in Ref. 9, we present this rigorous version of the same Debye method with no pretension to precision, since numerical methods are excellent in that regard. Our idea is to assert that the evolution equation can have a well–behaved solution if hasty oversimplifications are not imposed on the steepest descent method employed during Mellin inversion.

To return to the calculation, we put

$$f(s) = f(s_0) - u^2, \tag{22}$$

where u^2 is real by virtue of Debye conditions set forth above, s_0 being the saddle–point as defined threin. The selfsame conditions on C_0 would make Re $f(s)$ register a steep fall outside a small neighbourhood of s_0. (A small neighbourhood being intended, the question of indiscrete topology does not arise at all). Now, Eq. (22) allows a good approximation to Eq. (21), which reads

$$D(x,t) \approx \exp[hf(s_0)] \frac{1}{2\pi \mathrm{i}} \int\limits_{-\infty}^{\infty} \exp(-hu^2) B(s+1), d+1) \frac{\mathrm{d}s}{\mathrm{d}u}(u) \mathrm{d}u. \tag{23}$$

We now employed the power series expansion,

$$\frac{1}{2\pi \mathrm{i}} B(s+1, d+1) \frac{\mathrm{d}s}{\mathrm{d}u}(u) = \sum_{n=0}^{\infty} a_n u^n. \tag{24}$$

Combining Eqs. (23) and (24), we write

$$D(x,t) \approx \exp[hf(s_0)] \sum_{n=0}^{\infty} a_n \int\limits_{-\infty}^{\infty} \exp(-hu^2) u^n \mathrm{d}u. \tag{25}$$

Let us introduce the notation,

$$G_n = \int\limits_{-\infty}^{\infty} \exp(-hu^2) u^n \mathrm{d}u. \tag{25a}$$

whence

$$G_n = -G_n, \quad \text{for } n = 1, 3, 5, \ldots ,$$

implying

$$G_n = 0, \quad \text{for } n = 1, 3, 5, \ldots .$$

But

$$G_0 = \int\limits_{-\infty}^{\infty} \exp(-hu^2) u^n \mathrm{d}u = \sqrt{\frac{\pi}{t}}. \tag{25b}$$

Whence, noticing that

$$G_{2n} = -\frac{\partial}{\partial h} G_{2n-2} \tag{25c}$$

we obtain

$$G_{2n} = \frac{(\pi)^{1/2} \cdot 1 \cdot 3 \cdot 5 \cdot ... \cdot (2n-1)}{2^n (h)^{(2n+1)/2}} \tag{25d}$$

for $n > 0$.

We obtain from above a series representation for $D(x,t)$ of Eq. (25), to wit

$$D(x,t) = \exp[hf(s_0)] \sum_{n=0}^{\infty} a_{2n} \Gamma(n+1/2) h^{-(n+1/2)}. \tag{26}$$

But Q^2 for a DIS process is large, too, in view of Eqs. (9b) and (21a). The factor $h^{-(n+1/2)}$ of the general solution (26) ensures that even a lowest order approximation to Eq. (26), corresponding to $n = 0$, would be a good enough working formula for our purpose. It reads

$$D(x,t) = a_0 \sqrt{\frac{\pi}{t}} \exp[atM_p(s_0)] \left(\frac{1}{x}\right)^{s_0+1} \tag{27}$$

where $M_p(s_0)$ follows from Eq. (20).

3. Numerical results and discussion

It is, however, of interest to judge the ratioinale of the validity of Eq. (27) by the hind–sight of Ref. 9, where small x caused some difficulty. The higher order terms in Wilson coefficients and the anomalous dimensions which cause such difficulty, as one solves the renormalization group equation written for Wilson coefficients in accordance with the formal method, clearly correspond to higher n–terms of our series solution. This is so because the Altarelli–Parisi approach, which we are following, corresponds to the formal approach under the stipulation,

$$\int_0^1 \mathrm{d}z \; z^{n-1} P_{NS}^{(0)}(z) = \gamma_{NS}^{(0)}(n), \tag{28}$$

where $\gamma_{NS}^{(0)}(n)$ is the one which occurs in the context of the formal moment equation of QCD. Since our integer variable n of Eq. (26) makes no contribution for higher values, our solution is inherently exempted from the danger of divergence which was there in the solution given in Ref. 7 as mentioned above. The reason why our solution is flawless even at small x lies in our meticulous avoidance of any oversimplification of Debye's rigorous method of steepest descent.

It follows from the calculations done in Ref. 4 and 6 that if the value of the integral

$$\int_0^1 \left[\, \overline{d}(x) - \overline{u}(x)\right] \mathrm{d}x = \int_0^1 D(x)\mathrm{d}x, \tag{29}$$

be taken to be equal to 0.140, S_G would be given by its NMC value, namely, ≈ 0.24. But Eq. (29) involves $D(x)$, which, via Eq. (27) involves the unknown parameter a_0. We first fix a_0 by way of putting the integral of Eq. (29) equal to 0.140 and keeping t fixed. Once we fixed a_0, we go to compute six different S_G values by varying t. We obtain for S_G the set of values 0.24700, 0.24248, 0.24240, 0.24120, 0.24050 and 0.23805, where the average departure from the NMC value of 0.24 is about 1.2% , the maximum departure being 3% . The consistency obtained with the simple QCD model may be considered satisfactory.

The numerical computation made use of the relation,

$$0 = \frac{\partial M_P}{\partial s_0} - \frac{\ln x}{h} \tag{30}$$

which follows when Debye conditions are combined with Eq. (21b). We next find s_0 and $M_p(s_0)$, which occur in Eq. (27), by the help of the computer.

We note also a striking similarity between the Regge behaviour found in Ref. 4 and that predicted by the present work.

We conclude that our QCD – based model can explain the asymmetry problem disscused in Ref. 7 and can reproduce the actual x–dependence of that asymmetry reasonably well.

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SU(2) ASIMETRIJA U MORU LAKIH KVARKOVA

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Dobiven je analitički izraz za veličinu $\overline{d}(x) - \overline{u}(x)$, koja mjeri SU(2) asimetriju u moru lakih kvarkova.