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HINDRANCE FACTORS FOR THE ^{14}C–CLUSTER DECAY OF ^{223}Ra WITHIN THE ENLARGED SUPERFLUID MODEL

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Within the one level R–matrix approach several hindrance factors for the radioactive decays are calculated in which ^{14}C nuclei are emitted. The interior wave functions are supposed to be given by the shell model with effective residual interactions i.e. the enlarged superfluid model (ESM). The exterior wave functions are calculated from a cluster–nucleus double–folding model potential obtained with the Michigan 3–Yukawa (M3Y) interaction. As example of the cluster decay fine structure we analysed the case of ^{14}C–decay of ^{223}Ra. Good agreement with the experimental data is obtained.

1. *Introduction*

The spontaneous emission of nuclear fragments heavier than alpha particles and lighter than the most probable fission fragments, termed as exotic or cluster decays, theoretically predicted by Sandulescu, Poenaru and Greiner [1], has been experimentally confirmed [2-5]. Moreover, Hourani and his co–workers [6] experimentally discovered the fine structure radioactivity which has been theoretically predicted by Greiner and Schied [7], opening in this way a new area of research. Most of theoretical models of heavy cluster decay [5] are based, essentially on Gamov's theory [8]. The differences in approaches are related to the way of calculating the potential barrier defined by the interaction potential acting between the emitted cluster and the residual nucleus.

In Ref. 9 the formal expression for the theoretical hindrance factors are derived.

In the present paper we continue this work and calculate several hindrance factors for the ^{14}C radioactivity. The calculations will be performed by using the enlarged superfluid model (ESM) [10].

2. *Enlarged superfluid model*

The ESM Hamiltonian for nonrotational states of deformed nuclei includes an average field of neutron and proton system in the form of the axial–symmetric Saxon–Woods potential, monopole pairing, isoscalar and isovector particle–hole and particle–particle multipole and spin–multipole interactions between quasiparticle as well as the so–called alpha–like four nucleon interaction [10] of the rank $N > 1$. It is given by:

$$H = H_0 + H^{'} \tag{1}$$

where

$$H_0 = \sum_{\tau} (H^{av}_{s.p.}(\tau) - G_\tau P^+_\tau P_\tau) + H_4 \tag{2}$$

in which

$$H^{av}_{s.p.}(\tau) = \sum_{s,\sigma} E_s a^+_{s,\sigma} a_{s,\sigma}, \tag{3}$$

$$P_\tau = \sum_{s} a_{s^-} a_{s^+}, \tag{4}$$

$$H_4 = -G_4 P^+_{\mathrm{p}} P^+_{\mathrm{n}} P_{\mathrm{n}} P_{\mathrm{p}}, \tag{5}$$

and

$$H^{'} = \sum_{\tau} \Bigg\{ -\frac{1}{2} \sum_{\lambda,\mu,\sigma} \sum_{n=1}^{N} \Bigg[\sum_{\eta=\pm 1} (k^{\lambda\mu}_{0\tau} + \eta k^{\lambda\mu}_{1\tau})$$

$$\times Q^+_{n,\lambda,\mu,\sigma}(\tau) Q_{n,\lambda,\mu,\sigma}(\eta\tau) + G^{\lambda\mu}_\tau P^+_{n,\lambda,\mu,\sigma}(\tau) P_{n,\lambda,\mu,\sigma}(\tau) \Bigg] -$$

$$-\frac{1}{2}\sum_{L\lambda,\mu,\sigma}\sum_{n=1}^{N}\Bigg[\sum_{\eta=\pm 1}(k_{0\tau}^{L\lambda\mu}+\eta k_{1\tau}^{L\lambda\mu})$$

$$\times T_{n,L,\lambda,\mu,\sigma}^{+}(\tau)T_{n,L,\lambda,\mu,\sigma}(\eta\tau)+G_{\tau}^{L\lambda\mu}P_{n,L,\lambda,\mu,\sigma}^{+}(\tau)P_{n,L,\lambda,\mu,\sigma}(\tau)\Bigg]\Bigg\}. \qquad (6)$$

Here $\tau=-1/2$ stands for the proton system and $\tau=+1/2$ stands for the neutron system, $a_{s,\sigma}^{+}$ $(a_{s,\sigma})$ are the fermion operators which create (destroy) a nucleon in (from) the single particle state $|s_\tau\sigma_\tau>$, where σ_τ is the sign of the projection of the angular momentum of the state onto the nuclear symmetry axis, s_τ being the rest of the quantum numbers that label the single particle energy levels. The term H_4 from Eq. (5) is an effective coherent two–pairs (four–nucleon) interaction term, which induces the dynamical alpha–like four nucleon correlations in the superfluid phases of atomic nuclei [11]. G_τ are the pairing coupling strengths, $G_\tau^{\lambda\mu}$ and $G_\tau^{L\lambda\mu}$ are the coupling constants of the particle–particle interactions [10], $k_{0,\tau}^{\lambda\mu}$, $k_{1,\tau}^{\lambda\mu}$ and $k_{0,\tau}^{L\lambda\mu}$, $k_{1,\tau}^{L\lambda\mu}$ are the isoscalar and isovector coupling constants of the particle–hole multipole and spin–multipole interactions [12]. G_4 is the four–nucleon interaction constant and $\sigma=\pm1$.

To find the ground and excitation spectrum and corresponding wave functions we used the recepe from Refs. 12 and 13.

3. The cluster spectroscopic factor

The ^{223}Ra nucleus belongs [14] to the well known–region of soft nuclei with $Z\simeq 88$ and $N\simeq 134$, with strong octupole correlations in the ground and low lying excited states. The hindrance factors for both the alpha– and ^{14}C–decays of the ground state of ^{223}Ra are very difficult to be calculated at present, due to the lack of accurate structures of the mother and daughter nuclei. The structure of the ground state of the ^{223}Ra is not as simple as e.g. the ^{225}Fm case [15], and it may contain many more or less equal components of single quasiparticle or quasiparticle–phonon structure. It is not sufficient to have a description of these states within an independent particle model only [16,17]. Residual interactions could play an important role [13].

With the ESM–model the expression for the spectroscopic amplitude entering the hindrance factor can be factorised [15] according to:

$$\Theta_{NlK_i-K_f}^{(K_i^{\pi_i}K_i\to K_f^{\pi_f}K_f)}=C_{\Omega_i}C_{\Omega_f}a_{N_il_ij_i}^{\Omega_i=K_i}a_{N_fl_fj_f}^{\Omega_f=K_f}\sqrt{2I_f+1}\begin{pmatrix} I_il & I_f\\ K_iK & K_f\end{pmatrix}\Theta_{\mathrm{core}}^{(j_i\pi_i\to j_f\pi_f)}$$

where $C_{\Omega_{i(f)}}$ are the weights of the single quasiparticle state in the structure of the $i(f)$–state, a^{Ω}_{nlj} are the Nilsson–like amplitudes. $\Theta_{\rm core}$ acts as a spectroscopic amplitude between many body spherical core $|j_{i(f)}\pi_{i(f)}>$ states, including both the cluster overlaps and the intrinsic overlap integrals [15].

$\begin{pmatrix} I_i l & I_f \\ K_i K & K_f \end{pmatrix}$ stands for the 3-j simbol. Within the EMS–model we calculated the hindrance factors for ^{223}Ra(g.s) $\to ^{14}$C $+ ^{209}$Pb–cluster transitions (see Table 1).

TABLE 1.

The calculated, within ESM [10], hindrance factors for the ^{14}C *transition:* ^{223}Ra(g.s) $\to ^{14}$C $+ ^{209}$Pb. *The abbreviation DSPC means the dominant single particle configuration.*

E_f (keV)	$I_f^{\pi_f}$ (DSPC)	$[Nn_z\Lambda]_i$	$[Nn_z\Lambda]_f$	$a^{\Omega_i}_{nlj}$	$a^{\Omega_f}_{nlj}$	C_{Ω_i}	C_{Ω_f}	HF_{exp}	HF_{ESM}
0.	$\frac{9}{2}^+ (2g_{9/2})$	[631]	[615]	0.8	1.0	2%	98%	600	668
779.	$\frac{11}{2}^+ (1i_{11/2})$	[642]	[606]	0.8	1.0	78%	97%	3	28

In calculating the ^{223}Ra ground state structure (see Table 2) the EMS parameters used [18] are: $G_p = 0.14$ MeV, $G_n = 0.10$ MeV, $G_4 = 0.26$ keV. The parameters of the average field [19] are: $V_{op} = 55.53698$ MeV, $r_{op} = 1.30975$ fm, $a_p = 0.70071$, $k_{s-o,p} = 5.56479$ MeV, $V_{on} = 37.78683$ MeV, $r_{on} = 1.39628$ fm, $a_n = 0.70071$, $k_{s-o,n} = 7.3197$ MeV. The deformation parameters are $\beta_{20} = 0.15$, $\beta_{40} = 0.10$. The particle–hole quadropule and octupole parameters used [10] are: $k^{\lambda\mu}_{n\tau} = k^{2\mu}_{o\tau} = 0.67$ keV fm^{-4}; $k^{\lambda\mu}_{n\tau} = k^{2\mu}_{1\tau} = 0.06$ keV fm^{-4}; $k^{\lambda\mu}_{n\tau} = k^{3\mu}_{o\tau} = 0.01$ keV fm^{-6}; $k^{\lambda\mu}_{n\tau} = k^{3\mu}_{1\tau} = 1$ eV fm^{-6}. The particle–particle quadrupole parameters used [10] are: $G^{\lambda\mu}_{n\tau} = G^{2\mu}_{\tau} = 15$ eV fm^{-4}.

TABLE 2.

The calculated, within ESM [10], structure of the ground and one excited state entering the ^{14}C *cluster transition:* ^{223}Ra*(g.s)* $\to$ ^{14}C + ^{209}Pb.

Nucleus	$I^\pi K$	E_{exp} (MeV)	E_{theo} (MeV)	Structure
^{223}Ra	$\frac{3}{2}^+ \frac{3}{2}$	0.	0.	$78.21\%\ [642]\frac{3}{2}^+ + 2.04\%\ [631]\ \frac{3}{2}^+ +$ $+13.1\%\ [752]\frac{5}{2}^- Q_{31} + 2.5\%\ [761]\frac{3}{2}^- Q_{30+}$ $+3.1\%\ [631]\frac{1}{2}^+ Q_{22} + 2.5\%\ [501]\frac{1}{2}^- Q_{31}$
^{209}Pb	$\frac{9}{2}^+ \frac{9}{2}$	0.	0.	$92.92\%\ [615]\frac{9}{2}^+ + 1.09\%\ [624]\ \frac{9}{2}^+ +$ $+1.04\%\ [615]\frac{9}{2}^+ Q_{20} + 2.5\%\ [624]\frac{9}{2}^+ Q_{20}$

4. Conclusions

In this work we reported some calculations performed within the enlarged superfluid model [10], for some selected ^{14}C transitions of the ^{223}Ra nucleus. We calculated also the quasiparticle–phonon structure of the ^{223}Ra nucleus, that emits ^{14}C clusters and one excited state entering the ^{14}C–cluster transition: ^{223}Ra(g.s) $\rightarrow$ ^{14}C + ^{209}Pb.

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FAKTOR USPORAVANJA RASPADA ^{223}Ra EMISIJOM GROZDA ^{14}C U OKVIRU PROŠIRENOG SUPRAFLUIDNOG MODELA

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U okviru R matričnog modela s jednim nivoom izračunato je nekoliko faktora usporavanja za radioaktivne raspade u kojima su emitirane jezgre ^{14}C. Pretpostavljeno je da su unutarnje valne funkcije opisane modelom ljuske s efektivnom rezidualnom interakcijom tj. s proširenim suprafluidnim modelom. Vanjske valne funkcije računate su pomoću dvostruko–preklopnog potencijala grozd–jezgra izvedenog na osnovi Michigan–Yukawa (M3Y) interakcije. Kao primjer fine strukture raspada grozda analiziran je raspad ^{223}Ra emisijom grozda ^{14}C. Dobiveno je dobro slaganje s eksperimentalnim rezultatima.