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A NON–HEDGEHOG SOLUTION FOR THE CHIRAL BAG

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The chiral sigma model, embedded in the chiral-bag environment, is solved by an ansatz which conserves isospin and spin separably. This chiral ansatz is treated in two ways: i) as a set of operator equations of motion solved between quark states and ii) the hamilton operator is averaged between suitable hadron states, and the equations of motion are derived for these mean fields. The second approach is completely analogous to the usual one which employs hedgehog quarks, which is also reproduced here. It turns out that the energy minimum (i.e. hadron masses) can be found with chiral quarks as well as with hedgehog quarks. Model predictions for the axial-vector coupling constant and for the nucleon magnetic moment obtained with chiral quarks are of the same quality, or better than those obtained using the usual hedgehog-based approximation.

1. Introduction

Various successful semiempirical descriptions for hadrons have emerged from some chiral-bag model (CBM) [1,2]. In such models, the physical space is divided in two regions: the bag (inside) and the surroundings (the bag outside). The inside quarks move freely and the quark-gluon interaction is considered to be saturated with the first-order gluon exchange. Quarks interact with the surroundings by means of the surface interaction with soliton objects which might carry quark quantum operators [2,3]. Some of their spatial properties resemble the observable mesons. By calculating currents, energy and masses, one is able to reproduce the basic features of baryon and meson mass spectroscopy, as well as magnetic moments and the axial-vector coupling constant.

The most common ansatz for the two phases, i.e. inside and outside, is the well-known hedgehog form [1,4,5] which also leads to an energy minimum [6]. It was applied both to the linear and to the non-linear (Skyrme) sigma models. Notwhitstanding the practical success and the theoretical support for hedgehog forms, it is a matter of some curiosity to find whether an alternative ansätze might work at all and how good its predictions of static and semistatic (g_A, magnetic moment) nucleon properties are.

It turns out that using the linear sigma model and the simple product of the spin and isospin parts of the quarks wave functions, $\chi(\text{spin}) \otimes \chi(\text{isospin})$, one can find a stable solution which conserves $\vec{J}$ and $\vec{I}$ independently. The bag boundary condition then induces a sort of quantization for meson fields in which quark-operator pairs appear. As shown in detail in Sect. 3 of this paper, this meson phase contains parts which depend either on the product of quark-quark (antiquark) operators or on the mixed product of quark-antiquark operators. The first part is the continuation of the quark density, for example the current, outside the confinement (bag) region. The second part is an analogon of the quantized boson field which appears in the coherent-state description. All this follows quite naturally from the formalism and was used previously in an (approximate) chiral-bag-model calculation of non-leptonic decays [3]. An important difference with the more usual hedgehog version of the model [1,4,5] is the presence of the s-wave component in the pion field. As shown in Sect. 5 the s-wave components vanishes when the $\chi(\text{spin}) \otimes \chi(\text{isospin})$ part of the quark wave function is replaced by the hedgehog combination.

In our ansatz the s-wave component is multiplied by the combination of particle-antiparticle creation (annihilation) operators. This product has the same correct parity as the p-wave component which is multiplied by two particle (antiparticle) operators. Although the s-wave component does not contribute directly to the baryon form factors (it would to mesons) its presence in the non-linear system of equations changes its solutions and thus leads to a better agreement with the experiment.

The hedgehog version of the model is presented in Sect. 5. The hedgehog ansätze [1,4,5] are described and the corresponding equations are derived. The meson phase either can contain quark operators or can be quantized as an elementary boson field. The second choice, which uses the coherent states [1,7], leads to the same results as the first one. One hopes that the considerations given in Sect. 5 could facilitate a comparison of the chiral-quark solution presented in this paper, with the well-known methods and results.

Section 6 contains comments on the numerical procedure of integration of a non-linear system of ordinary differential equations containing mesonic degrees of freedom.

Results and conclusions are presented in Sect. 7.

The linear σ-model continues to be present in the literature where the colour dielectric model is among the recent contributions [8]. The non-linear version of the σ-model is part of the Skyrme conjecture as well as part of other different topological, non-topological, bag and quark models of hadrons [5]. The new ansätze presented here might also lead to a corresponding treatment of the non-linear σ-model .

2. *The linear sigma model and the bag formalism*

The lagrangian containing the linear sigma model embedded in the bag environment has the usual form [1,8]:

$$\mathcal{L} = \mathcal{L}_\psi \Theta + \mathcal{L}_{int}\delta_S + [\mathcal{L}_{\sigma\pi} - U(\sigma, \vec{\pi})]\overline{\Theta}, \tag{2.1}$$

where

$$\mathcal{L}_\psi = \frac{\mathrm{i}}{2}(\bar{\psi}(x)\gamma^\mu \partial_\mu \psi(x) - \partial_\mu \bar{\psi}(x)\gamma^\mu \psi(x)) - B,$$

$$\mathcal{L}_{int} = \frac{g}{2}\bar{\psi}(x)(\sigma(x) + \mathrm{i}\vec{\tau}\vec{\pi}(x)\gamma_5)\psi(x),$$

$$\mathcal{L}_{\sigma\pi} = \frac{1}{2}\partial^\mu \sigma(x)\partial_\mu \sigma(x) + \frac{1}{2}\partial^\mu \vec{\pi}(x)\partial_\mu \vec{\pi}(x), \tag{2.2}$$

$$U(\sigma, \vec{\pi}) = \frac{\lambda^2}{4}(\sigma^2(x) + \vec{\pi}^2(x) - \nu^2)^2 - f_\pi m_\pi^2 \sigma(x)$$

and $f_\pi = 0.093$ GeV. The $\Theta(x)$ equals zero for $x < 0$, i.e. $\mathcal{L}_\psi$ is different from zero inside the bag ($r < R_{bag}$). The surface δ-function δ_S gives the surface quark$-\pi$ (or σ) interaction, and $\overline{\Theta}$ ensures that the potential U and the $(\sigma, \vec{\pi})$ kinetic-energy terms exist (only) outside the bag. In the *spherical* bag, $\Theta(x)$ and $\overline{\Theta}$ become $\theta(R_{bag} - r)$ and $\theta(r - R_{bag})$, respectively. The self-interaction potential U contains the symmetry-breaking (SB) term $c\sigma(x) \equiv -f_\pi m_\pi^2 \sigma(x)$. The values of other constants are fixed by the creation of mass terms for the $\vec{\pi}$ and σ fields, by the PCAC and by the requirement $U^{(min)} = 0$. Their values are given in Sect. 6. In the framework of this particular model, m_σ and m_π are not necessarily equal to the physical sigma and pion mass, but play the role of model parameters.

In order to extract the physical content of the theory from the lagrangian given by Eq. (2.1), one can use two basically different methods (the first of which is further applied in two different ways):

1) the chiral-quark approach (Sect. 3) where the quark fields contain the standard spinor-isospinor product (see Eqs. (3.1) and (3.2)). The meson fields are given in terms of these quark fields by the ansätze which reflect their flavour and space-time properties. In one version of this non-hedgehog method,

   (a) the equations of motion are obtained from the lagrangian (2.1) using the standard variational methods and the quantized ansätze (see Eqs. (3.1)–(3.4)) are used. The bosonic entities (3.3) and (3.4) are not the elementary π or σ fields but solitons possessing some meson-like transformation properties and satisfying the boundary conditions (3.7) and (3.9). This results in the operator equations of motion and the operator boundary conditions. The required non-operator relations are then "projected" by "sandwiching" those relations between suitably chosen initial and final quark states. The end results (non-linear coupled differential equations) involve the classical profile functions. In the other version of the same method,

   (b) the classical profile functions are obtained by *first* "sandwiching" the lagrangian (2.1) between the chosen hadron states.The fields in this lagrangian are replaced by the quantized ansätze (3.1)–(3.4). The equations of motion are then obtained using the variational method. This is analogous to the *mean-field approximation* (MFA) [1].

2) In the *hedgehog-quark model*, the lagrangian (hamiltonian) is expressed in terms of (quantized) hedgehog quarks and the MFA, as in case (1.2) , is employed to get the (classical) profile functions. The "sandwiching" is accomplished by the hedgehog initial/final baryon states. The equations of motion for the classical fields (profile functions) are obtained using the standard variational methods. The coherent states are also discussed.

3. *The chiral quarks - The non-hedgehog method 1*

The ansatz for the quark field is

$$\psi_f^c(x) = \frac{N}{\sqrt{4\pi}} \begin{pmatrix} j_0 \\ \mathrm{i}(\vec{\sigma}\hat{r})j_1 \end{pmatrix} \chi_m^f b_{m,f}^c + \frac{N}{\sqrt{4\pi}} \begin{pmatrix} (\vec{\sigma}\hat{r})j_1 \\ \mathrm{i}j_0 \end{pmatrix} \chi_m^f d_{m,f}^{c\dagger}. \tag{3.1}$$

Here c is a quark colour and f is a quark flavour, whereas m is the spin projection. $b_{m,f}^c$ and $d_{m,f}^c$ are quark and antiquark annihilation operators, respectively. The quantities $j_{0,1}(r)$ are spherical Bessel functions of the order (0,1) and χ_m^f is the quark isospinor ($\tilde{\chi}^f$)–spinor (χ_m) product

$$\chi_m^f = \tilde{\chi}^f \cdot \chi_m. \tag{3.2}$$

The σ-field ansätze are given by the $s-$wave component, and in terms of *chiral*-quark operators together with the symmetry-breaking term (f_π):

$$\sigma(r) = \sigma_s(r) \cdot (b^{c\dagger}_{m,f} b^c_{m,f} + d^{c\dagger}_{m,f} d^c_{m,f}) - f_\pi. \tag{3.3}$$

The pion field contains the s- and p-wave components

$$\pi^a(r) = \pi_s(r)(b^{c\dagger}_{m,f} d^{c\dagger}_{m',f'} + d^c_{m,f} b^c_{m',f'}) \cdot [\chi^\dagger_{m,f} \tau^a \chi_{m',f'}]$$

$$+\pi_p(r)(b^{c\dagger}_{m,f} b^c_{m',f'} + d^c_{m,f} d^{c\dagger}_{m',f'}) \cdot [\chi^\dagger_{m,f} (\vec{\sigma}\hat{r}) \tau^a \chi_{m',f'}]. \tag{3.4}$$

The ansätze (3.1)–(3.4) are formally related to the perturbation treatment of a quantum field theory in the Heisenberg picture [9]. Quark fields are solutions of a complicated system of non-linear equations. One can start by expanding the quark field operator ψ :

$$\psi = \psi^{(0)} + \psi^{(1)} + \dots \tag{3.5}$$

In this expansion each term depends on various combinations of the quark field operators $b,\ d$. In a more realistic situation, using full QCD, gluon operators should also be included. In the model application, semiempirical features, such as bag and mesonic phases, are introduced. Nevertheless, it is obvious that the ansätze (3.1, 3.2) correspond just to the term $\psi^{(0)}$ in Eq. (3.5). In addition, in resolving a complex non-linear theory, one (in principle) encounters the full set of all possible Fock states. This is here approximated by the lowest (first) Fock state, built out of valence quarks in keeping with the retention of $\psi^{(0)}$ from the expansion (3.5). The ansätze (3.3) and (3.4) also have some relation to the solution presented in Ref. 10. There they also introduce an isospin dependence which is a spatial constant, which differs from the hedgehog ansätze (5.1)–(5.4) below.

The Euler-Lagrange (E-L) equation for the σ field which stems from varying $\mathcal{L}$ with respect to σ is

$$\partial^\mu \partial_\mu \sigma(r) + \lambda^2 \sigma(r)[\sigma(r)^2 + \vec{\pi}^2 - \nu^2] + f_\pi m_\pi^2 = 0. \tag{3.6}$$

From the variation of the derivative terms one obtains the boundary conditions imposed on the σ field:

$$(\partial^\mu \sigma(r)) n_\mu \delta_S - \frac{g_\sigma}{2} \overline{\psi} \psi \delta_S = 0. \tag{3.7}$$

The E-L equation for the pion field reads

$$\partial^\mu \partial_\mu \pi^a(r) + \lambda^2 \pi^a(r)[\sigma(r)^2 + \vec{\pi}(r)^2 - \nu^2] = 0. \tag{3.8}$$

In the same way as for the σ field, one obtains the boundary condition for π^a:

$$(\partial^\mu \pi^a(r)) n_\mu \delta_S - \frac{g_\pi}{2} \overline{\psi}(r) \mathrm{i} \tau^a \gamma_5 \psi(r) \delta_S = 0. \tag{3.9}$$

The ansätze (3.1)–(3.4) have been introduced into the above equations.

To extract the equations for the s- and p-wave components from the operator equations of motion the equations (3.6)–(3.9) are "sandwiched" between the final state $\langle f| = \langle q^c_{f,t}| = \langle 0| b^c_{f,t}$ and the initial state $|i\rangle = |q^c_{i,u}\rangle = b^{c\dagger}_{i,u}|0\rangle$. This choice yields the equation for $\sigma_s(r)$:

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}r^2} + \frac{2}{r}\frac{\mathrm{d}}{\mathrm{d}r}\right]\sigma_s(r) = \lambda^2\left[\sigma_s(r) - f_\pi\right]\left[(\sigma_s(r) - f_\pi)^2 + 3\pi_p^2(r) - \nu^2\right] + f_\pi m_\pi^2 \quad (3.10)$$

and for $\pi_p(r)$:

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}r^2} + \frac{2}{r}\frac{\mathrm{d}}{\mathrm{d}r} - \frac{2}{r^2}\right]\pi_p(r) = \lambda^2\pi_p(r)\left[(\sigma(r) - f_\pi)^2 + 3\pi_p^2(r) - \nu^2\right]. \quad (3.11)$$

The other choice, i.e. $\langle f| = \langle 0|$ and $|i\rangle = |q^c_{i,u}\overline{q}^c_{i',u'}\rangle = d^{c\dagger}_{i',u'} b^{c\dagger}_{i,u}|0\rangle$, gives the pion s-wave component

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}r^2} + \frac{2}{r}\frac{\mathrm{d}}{\mathrm{d}r}\right]\pi_s(r) = \lambda^2\pi_s(r)\left[f_\pi^2 + 36\pi_s^2(r) - \nu^2\right]. \quad (3.12)$$

It is now easy to specify the boundary conditions (3.8) and (3.9) using the ansätze (3.1)–(3.4) and the same initial/final-states combinations:

$$\frac{\partial}{\partial r}\sigma_s(r)\Big|_{r=R_{bag}} = -\frac{N^2}{4\pi}\frac{g_{\sigma/s}}{2}[j_0^2(\omega) - j_1^2(\omega)],$$

$$\frac{\partial}{\partial r}\pi_s(r)\Big|_{r=R_{bag}} = -\frac{N^2}{4\pi}\frac{g_{\pi/s}}{2}[j_0^2(\omega) + j_1^2(\omega)],$$

$$\frac{\partial}{\partial r}\pi_p(r)\Big|_{r=R_{bag}} = -\frac{N^2}{4\pi}\frac{g_{\pi/p}}{2}[j_0(\omega)\cdot j_1(\omega)]. \quad (3.13)$$

At spatial infinity the σ and π "fields" (i.e. solitons) have to vanish:

$$\sigma_s(r)\Big|_{r\to\infty} = 0 \qquad \pi_s(r)\Big|_{r\to\infty} = 0 \qquad \pi_p(r)\Big|_{r\to\infty} = 0. \quad (3.14)$$

Varying $\mathcal{L}$ with respect to the fermion field and its derivative and collecting the corresponding surface terms, one obtains the boundary condition

$$\mathrm{i}(\vec{\gamma}\hat{r})\psi(r)\Big|_{r=R_{bag}} = \mathrm{i}g_\sigma\sigma(r)(\vec{\gamma}\hat{r})\psi(r)\Big|_{r=R_{bag}} - g_\pi\vec{\tau}\vec{\pi}(r)(\vec{\gamma}\hat{r})\gamma_5\psi(r)\Big|_{r=R_{bag}}. \quad (3.15)$$

This boundary condition is "sandwiched" between quark (Fock) states, as done with the equations of motion. Between $\sigma - \psi$ and between $\vec{\pi} - \psi$ one inserts the complete set of states. Depending on the type of states, one obtains relations between the coupling constants and radial functions evaluated at $r = R_{bag}$. This is a straightforward but somewhat lengthy procedure. As an example, here are some details: With the ansätze (3.1)–(3.4), the boundary condition (3.15) takes the following form:

$$\begin{pmatrix} j_0 \\ \mathrm{i}(\vec\sigma\hat r)j_1 \end{pmatrix} \chi_m^f\, b^c_{m,f} + \begin{pmatrix} (\vec\sigma\hat r)j_1 \\ \mathrm{i}j_0 \end{pmatrix} \chi_m^f\, d^{c\,\dagger}_{m,f} =$$

$$-g_{\pi/p}\pi_p(R)\begin{pmatrix} (\vec\sigma\hat r)j_0 \\ -\mathrm{i}j_1 \end{pmatrix} \chi_m^f\, b^{d\,\dagger}_{m_1,f_1}(\vec\tau\cdot\vec\tau)\, b^d_{m_2,f_2}[\chi_{m_1}^{f_1\,\dagger}(\vec\sigma\hat r)\chi_{m_2}^{f_2}]\, b^c_{m,f}$$

$$-g_{\pi/p}\pi_p(R)\begin{pmatrix} (\vec\sigma\hat r)j_0 \\ -\mathrm{i}j_1 \end{pmatrix} \chi_m^f \text{đ}(\vec\tau\cdot\vec\tau)\, d^{d\,\dagger}_{m_2,f_2})[\chi_{m_1}^{f_1\,\dagger}(\vec\sigma\hat r)\chi_{m_2}^{f_2}]\, b^c_{m,f}$$

$$-g_{\pi/p}\pi_p(R)\begin{pmatrix} j_1 \\ -\mathrm{i}(\vec\sigma\hat r)j_0 \end{pmatrix} \chi_m^f\, b^{d\,\dagger}_{m_1,f_1}(\vec\tau\cdot\vec\tau)\, b^d_{m_2,f_2}[\chi_{m_1}^{f_1\,\dagger}(\vec\sigma\hat r)\chi_{m_2}^{f_2}]\, d^{c\,\dagger}_{m,f}$$

$$-g_{\pi/p}\pi_p(R)\begin{pmatrix} j_1 \\ -\mathrm{i}(\vec\sigma\hat r)j_0 \end{pmatrix} \chi_m^f \text{đ}(\vec\tau\cdot\vec\tau)\, d^{d\,\dagger}_{m_2,f_2}[\chi_{m_1}^{f_1\,\dagger}(\vec\sigma\hat r)\chi_{m_2}^{f_2}]\, d^{c\,\dagger}_{m,f}$$

$$+\mathrm{i}g_\sigma\sigma_s(R)\begin{pmatrix} \mathrm{i}j_1 \\ -(\vec\sigma\hat r)j_0 \end{pmatrix} \chi_m^f\, b^{d\,\dagger}_{m_1,f_1}\, b^d_{m_2,f_2}\, b^c_{m,f}$$

$$+\mathrm{i}g_\sigma\sigma_s(R)\begin{pmatrix} \mathrm{i}j_1 \\ -(\vec\sigma\hat r)j_0 \end{pmatrix} \chi_m^f\, d^{d\,\dagger}_{m_1,f_1}\, d^d_{m_2,f_2}\, b^c_{m,f}$$

$$-\mathrm{i}g_\sigma f_\pi\begin{pmatrix} \mathrm{i}j_1 \\ -(\vec\sigma\hat r)j_0 \end{pmatrix} \chi_m^f\, b^c_{m,f}$$

$$+\mathrm{i}g_\sigma\sigma_s(R)\begin{pmatrix} \mathrm{i}(\vec\sigma\hat r)j_0 \\ -j_1 \end{pmatrix} \chi_m^f\, b^{d\,\dagger}_{m_1,f_1}\, b^d_{m_2,f_2}\, d^{c\,\dagger}_{m,f}$$

$$+\mathrm{i}g_\sigma\sigma_s(R)\begin{pmatrix} \mathrm{i}(\vec\sigma\hat r)j_0 \\ -j_1 \end{pmatrix} \chi_m^f\, d^{d\,\dagger}_{m_1,f_1}\, d^d_{m_2,f_2}\, d^{c\,\dagger}_{m,f}$$

$$-\mathrm{i}g_\sigma f_\pi\begin{pmatrix} \mathrm{i}(\vec\sigma\hat r)j_0 \\ -j_1 \end{pmatrix} \chi_m^f\, d^{c\,\dagger}_{m,f}$$

$$-g_{\pi/s}\pi_s(R)\begin{pmatrix} (\vec\sigma\hat r)j_0 \\ -\mathrm{i}j_1 \end{pmatrix} \chi_m^f\, b^{d\,\dagger}_{m_1,f_1}(\vec\tau\cdot\vec\tau)\, d^{d\,\dagger}_{m_2,f_2}\, b^c_{m,f}$$

$$-g_{\pi/s}\pi_s(R)\begin{pmatrix}(\vec{\sigma}\hat{r})j_0\\ -\mathrm{i}j_1\end{pmatrix}\chi_m^f d^d_{m_1,f_1}(\vec{\tau}\cdot\vec{\tau})b^d_{m_2,f_2}b^c_{m,f}$$

$$-g_{\pi/s}\pi_s(R)\begin{pmatrix}j_1\\ -\mathrm{i}(\vec{\sigma}\hat{r})j_0\end{pmatrix}\chi_m^f b^{d\,\dagger}_{m_1,f_1}(\vec{\tau}\cdot\vec{\tau})d^{d\,\dagger}_{m_2,f_2}d^{c\,\dagger}_{m,f}$$

$$-g_{\pi/s}\pi_s(R)\begin{pmatrix}j_1\\ -\mathrm{i}(\vec{\sigma}\hat{r})j_0\end{pmatrix}\chi_m^f d^d_{m_1,f_1}(\vec{\tau}\cdot\vec{\tau})b^d_{m_2,f_2}d^{c\,\dagger}_{m,f}. \qquad (3.16)$$

This boundary conditios can be sandwiched between the final anti-quark state $\langle f| = \langle \overline{q}^a_{p,r}|$ and the initial vacuum state $|i\rangle = |0\rangle$. It is easy to see by inspection that many terms drop out, so that one ends up with very simple relations. On the LHS one has

$$\mathrm{LHS} = \begin{pmatrix}(\vec{\sigma}\hat{r})j_1\\ ij_0\end{pmatrix}\chi_m^f\langle 0|d^a_{p,q}d^{c\,\dagger}_{m,f}|0\rangle. \qquad (3.17a)$$

On the RHS one has to insert the complete set of intermediate states $|s\rangle\langle s|$:

$$\mathrm{RHS} = ig_\sigma\sigma_s(R)\begin{pmatrix}\mathrm{i}(\vec{\sigma}\hat{r})j_0\\ -j_1\end{pmatrix}\chi_m^f\langle 0|d^a_{p,r}d^d_{m_2,f_2}|s\rangle\langle s|d^{c\,\dagger}_{m,f}|0\rangle$$

$$-g_\sigma f_\pi\begin{pmatrix}\mathrm{i}(\vec{\sigma}\hat{r})j_0\\ -j_1\end{pmatrix}\chi_m^f + 3g_{\pi/s}\pi_p(R)\begin{pmatrix}j_1\\ -\mathrm{i}(\vec{\sigma}\hat{r})j_0\end{pmatrix}\chi_m^f(\vec{\sigma}\hat{r}). \qquad (3.17b)$$

Thus one obtains

$$\begin{pmatrix}(\vec{\sigma}\hat{r})j_1\\ \mathrm{i}j_0\end{pmatrix} = \mathrm{i}g_\sigma\sigma_s(R)\begin{pmatrix}\mathrm{i}(\vec{\sigma}\hat{r})j_0\\ -j_1\end{pmatrix} - \mathrm{i}g_\sigma f_\pi\begin{pmatrix}\mathrm{i}(\vec{\sigma}\hat{r})j_0\\ -j_1\end{pmatrix}$$

$$+3g_{\pi/p}\pi_p(R)\begin{pmatrix}(\vec{\sigma}\hat{r})j_1\\ -\mathrm{i}j_0\end{pmatrix}. \qquad (3.18)$$

Two equations follow from the above expression (here $R = R_{bag}$:)

$$j_0(R)g_\sigma(f_\pi - \sigma_s(R)) - j_1(R)(1 - 3g_{\pi/p}\pi_p(R)) = 0,$$

$$j_0(R)(1 + 3g_{\pi/p}\pi_p(R)) - j_1(R)g_\sigma(f_\pi - \sigma_s(R)) = 0. \qquad (3.19)$$

These two equations constitute a homogeneous system for the (unknown) functions $j_{0,1}$, so the determinant of the system should vanish.

The other projection between the vacuum and the one-quark state $|i\rangle = |q^a_{p,q}\rangle$ gives a system similar to that above:

$$j_0(R) - j_1(R)(g_\sigma f_\pi + 3g_{\pi/s}\pi_s(R)) = 0,$$

$$j_0(R)(g_\sigma f_\pi - 3g_{\pi/s}\pi_s(R)) - j_1(R) = 0. \tag{3.20}$$

The quark eigenenergy ω will be determined from the compatibility of the boundary conditions (3.13) and (3.15). In this case, instead of a common meson coupling constant g (Eq. (2.2)) flavour- and angular-momentum dependent couplings $g_{\sigma/s}$, $g_{\pi/s}$ and $g_{\pi/p}$ appear. This reflects chiral symmetry breaking. As shown in (3.21) below, this appears naturally when the non-linear system (2.2) is solved using the ansätze (3.1)-(3.4). One can solve the system of equations (3.19) and (3.20). One solution for $g_{\pi/p} = \pi_p(R)/3$ gives a trivial solution for g_σ, i.e. $g_\sigma = 0$. The other gives

$$g_\sigma = \frac{J^2+1}{2f_\pi J},$$

$$g_{\pi/s} = \frac{1-J^2}{6J\pi_s(R)},$$

$$g_{\pi/p} = \frac{J^2-1}{3(J^2+1)\pi_p(R)},$$

$$\sigma_s(R) = f_\pi \frac{J^4-4J^2+1}{(1+J^2)^2},$$

$$J = j_1(R)/j_0(R). \tag{3.21}$$

The problem is to find a set of solutions of the differential equations (3.6), (3.11) and (3.12), $\{\sigma(r), \pi_s(r), \pi_p(r)\}$, which satisfy the *mathematical* boundary conditions (3.13) and (3.14). These solutions must be compatible with Eq. (3.21) which is independent of r. Of course, J contains information on the system of differential equations, so one has a strongly correlated algebraic system (3.19) and (3.20) and the system of differential equations.

The parameters (λ, ν) which enter $\mathcal{L}$ (2.2) are restricted by the symmetry-breaking behaviour of the theory. Usually [1,11], the σ particle is considered to be a 1.2 GeV resonance, whereas the pion "mass" is a parameter which, for simplicity (and lack of knowledge), is assigned the value of the physical pion mass (0.137 GeV). In the present application, these values have also been used, although m_σ and m_π can, in principle, be considered as additional parameters.

The magnetic moment operator is

$$\vec{\mu}(\vec{r}) = \frac{1}{2}(\vec{r} \times \vec{j}_{EM}(\vec{r})). \tag{3.22}$$

Here

$$j^\mu_{EM}(r) = \overline{\psi}(r)\gamma^\mu Q\psi(r) + \epsilon_{3ij}\pi_i(r)\partial^\mu\pi_j(r) \tag{3.23a}$$

and

$$Q = \frac{2}{3} \cdot \frac{1+\tau_3}{2} - \frac{1}{3} \cdot \frac{1-\tau_3}{2}. \tag{3.23b}$$

The quark contribution to μ_N is

$$\mu^{(Q)} = \frac{2}{3} \cdot \frac{R}{\omega^4} \cdot \frac{(\omega/2) - (3/8)\sin 2\omega + (\omega/4)\cos 2\omega}{j_0^2(\omega) + j_1^2(\omega) - 2j_0(\omega)j_1(\omega)/\omega}. \tag{3.24}$$

The meson contribution is

$$\mu_p^{(M)} = \frac{16\pi}{3} \cdot \frac{11}{3} \int_{R_{bag}}^{\infty} r^2\,\mathrm{d}r [\pi_p(r)]^2 \mu_p. \tag{3.25}$$

The proton magnetic moment is given by

$$\mu_p = \mu^{(Q)} + \mu_p^{(M)}. \tag{3.26}$$

The axial-vector coupling constant g_A is the matrix element of the component $A_3^3(\vec{r})$ of the isovector axial-vector current (2.3) sandwiched between nucleon states and integrated over all space [1,12]. The quark contribution is

$$\begin{aligned} g_A^{(Q)} &= \langle p \uparrow | \int \mathrm{d}^3\vec{r}\overline{\psi}(\vec{r})\gamma^3\gamma^5 \frac{\tau^3}{2}\psi(\vec{r}) | n \uparrow \rangle \\ &= \frac{5}{3} \cdot \frac{1}{3} \cdot \frac{j_0^2(\omega) + j_1^2(\omega)}{j_0^2(\omega) + j_1^2(\omega) - 2j_0(\omega)j_1(\omega)/\omega}. \end{aligned} \tag{3.27}$$

For the proton one obtains the meson contribution:

$$g_A^{(M)} = \frac{5}{3} \cdot \frac{4\pi}{3} \cdot \int_{R_{bag}}^{\infty} \mathrm{d}r\, r^2 \big[(\sigma_s(r) - f_\pi)\big[\pi_p'(r) + \frac{2\pi_p(r)}{r}\big] - \pi_p(r)\sigma_s'(r)\big]. \tag{3.28}$$

Finally:

$$g_A^{(p)} = g_A^{(Q)} + g_A^{(M)}. \tag{3.29}$$

4. The chiral quarks - The non-hedgehog mean-field method 2

This approach has numerous features analogous to the hedgehog ansätze presented in the next section. One retains only the p-wave component for the pion field and derives the equations of motion from the classical hamiltonian, which is obtained by averaging the quantized hamiltonian over a baryon (proton, delta). The baryon wave functions in terms of chiral quarks belong to the conventional **56** representations of $SU(6)$.

The ansätze for the sigma and pion fields are

$$\vec{\pi}(r) = \pi_p(r)\big[\chi_{s'}^{a'}\vec{\tau}(\vec{\sigma}\hat{r})\chi_s^a\big] \cdot \mathrm{b}_{s'}^{\dagger\, a'}\, \mathrm{b}_s^a,$$

$$\sigma(r) = \frac{\sigma_s(r)}{3} \cdot \mathrm{b}_s^{\dagger\, a}(m)\, \mathrm{b}_s^a(n) - f_\pi. \tag{4.1}$$

The hamiltonian is

$$\mathcal{H} = \int \mathrm{d}^3x \big\{\psi^\dagger[-\mathrm{i}\vec{\alpha}\vec{\partial} + g\gamma^0(\sigma + \mathrm{i}\gamma_5\vec{\tau}\vec{\pi})]\psi + (\vec{\partial}\sigma)^2 + \frac{1}{2}(\vec{\partial}\vec{\pi}_a)^2 + U(\sigma,\vec{\pi})\big\}. \tag{4.2}$$

For the proton, the expectation value of $\mathcal{H}$ has the following form:

$$\langle p|H|p\rangle = \mathcal{H}_p = 4\pi \int_R^\infty \mathrm{d}r\, r^2 \Big\{ \frac{(\sigma_s')^2}{2} + \frac{1}{2}\cdot\frac{\Sigma_p}{3}\left(\pi_p^2 + \frac{2\pi_p^2}{r^2}\right)$$

$$+ f_\pi m_\pi^2(\sigma_s - f_\pi) + \frac{\lambda^2}{4}\left[(\sigma_s - f_\pi)^2 + \pi_p^2\frac{\Sigma_p}{3} - \nu^2\right]^2 \Big\}. \tag{4.3}$$

For Δ , one finds

$$\langle\Delta|\mathcal{H}|\Delta\rangle = \mathcal{H}_\Delta = 4\pi \int_R^\infty \mathrm{d}r\, r^2 \Big\{ \frac{(\sigma_s')^2}{2} + \frac{1}{2}\cdot\frac{\Sigma_p}{3}\left((\pi_p')^2 + \frac{2\pi_p^2}{r^2}\right) + f_\pi m_\pi^2(\sigma - f_\pi)$$

$$+ \frac{\lambda^2}{4}\left[(\sigma_s - f_\pi)^2 + \pi_p^2\frac{\Sigma_\Delta}{3} - \nu^2\right]^2 + \frac{\lambda^2}{4}\pi_p^4 \cdot 16 \Big\}. \tag{4.4}$$

Here $\Sigma_{p,\Delta}$ are the matrix elements of the spin-isospin operators averaged over spinor-isospinor part of the p/Δ wave function [2]; for example,

$$\Sigma_p = \langle p|(\sigma_i\tau_j)(\sigma_i\tau_j)|p\rangle. \tag{4.5}$$

The equations of motion (corresponding to the proton) are

$$\sigma_s'' + \frac{2}{r}\sigma_s' = \lambda^2(\sigma_s - f_\pi)\left[(\sigma_s - f_\pi)^2 + \frac{\Sigma_p}{3}\pi_p^2 - \nu^2\right],$$

$$\pi_p'' + \frac{2}{r}\pi_p' - \frac{2}{r^2}\pi_p = \lambda^2 \pi_p \left[(\sigma_s - f_\pi)^2 + \frac{\Sigma_p}{3}\pi_p^2 - \nu^2 \right] + \lambda^2 \pi_p^3 \frac{48}{\Sigma_\Delta}. \tag{4.6}$$

The boundary conditions for the meson profile functions are

$$\frac{\partial}{\partial r}\sigma_s(r)\Big|_{r=R_{bag}} = -\frac{3N^2}{4\pi}\frac{g}{2}[j_0^2(\omega) - j_1^2(\omega)],$$

$$\frac{\partial}{\partial r}\pi_p(r)\Big|_{r=R_{bag}} = -\frac{3N^2}{4\pi} g[j_0(\omega) \cdot j_1(\omega)], \tag{4.7}$$

and

$$\sigma_s(r)\Big|_{r\to\infty} = 0, \qquad \pi_p(r)\Big|_{r\to\infty} = 0. \tag{4.8}$$

Using the same method as in the preceding section one obtains the consistency condition for the quark eigenenergies from

$$j_0\, g(f_\pi - \sigma_s(R)) + j_1(1 - \frac{11}{3} g\pi_p(R)) = 0,$$

$$j_0(1 + \frac{11}{3} g\pi_p(R)) - j_1 g(f_\pi - \sigma_s(R)) = 0. \tag{4.9}$$

Thus the expression for the coupling constant g is (see Eq. (3.21))

$$g = \frac{1}{\sqrt{(\sigma_s(R) - f_\pi)^2 + (11/9)\pi_p(R)}}. \tag{4.10}$$

Here the number 11 arises from the matrix element Σ (4.5). The other equation analogous to Eq. (3.21) is

$$\frac{1}{J} = \frac{1 - g \cdot (11/9) \cdot \pi_p(R)}{g(f_\pi - \sigma_s(R)} = \frac{1 - (\Sigma_\Delta/9) \cdot \pi_p(R)/3}{g(f_\pi - \sigma(R))}. \tag{4.11}$$

The electromagnetic properties are calculated taking into account the electromagnetic current, [12] Eqs. (3.23 a, b) .

The quark contribution to the magnetic moment retains the form (3.24) but with the ω determined from (4.9).

For the proton, one finds that $\mu_p^{(M)}$ again has the form (3.25).

The axial-vector coupling constant g_A has the quark contribution (3.27) and the meson contribution (3.28). As already mentioned, the ω value and all parameter values corresponds to the model defined by (4.3)–(4.11).

5. *The hedgehog ansätze*

This section is intended to provide a detailed comparison between the ansätze used in the precceding section and the hedgehog ansätze.

At the classical level there is not much difference between the results obtained in this section and the results presented in Sect. 4. The equations of motion are similar and their (classical) solutions are almost identical (see Sect. 6). There is a slight difference in the quantization procedure. Usually [1,8], one quantizes (hedgehog) quarks and (hedgehog) mesons as elementary fermion and boson fields. Coherent states are used [1,7,8] to provide a quantum representation of the boson fields.

In the example provided here the bosonic phase is quantized in the same way as used in the ansätze (4.1). The end result, see Eq. (5.14) below, is the same as that obtained using coherent states.

The baryons are given in the form of *hedgehog*

$$|h\rangle = b_1^\dagger b_2^\dagger b_3^\dagger |0\rangle; \qquad \langle h|h\rangle = 1. \tag{5.1}$$

The pion state is a p-wave and it assumes a hedgehog form as well,

$$\pi_a(\vec{r}) = \hat{r}_a \pi(r) \cdot b_i^\dagger b_i \tag{5.2}$$

and σ is given by the scalar component and the symmetry-breaking term

$$\sigma(\vec{r}) = \sigma(r) \cdot b_i^\dagger b_i - f_\pi. \tag{5.3}$$

The hedgehog baryon is neither a nucleon nor a Δ, and it has to be projected into a spin-isospin eigenstate [1,11].

The hedgehog form (5.2) is closely related to the ansätze (4.1). If the isospinor-spinor combination χ_m^f in Eq. (4.1) is replaced by the hedgehog combination, i.e.

$$\chi_m^f = \tilde{\chi}^f \chi_m \to \frac{1}{\sqrt{2}} \left(\tilde{\chi}^{f=u} \cdot \chi_{m=-1/2} - \tilde{\chi}^{f=d} \cdot \chi_{m=1/2} \right) = \chi_h, \tag{5.4a}$$

then one finds

$$\chi_h^\dagger (\vec{\sigma}\hat{r}) \tau^a \chi_h = \hat{r}^a. \tag{5.4b}$$

Thus the mapping (5.4) transforms the ansätze (4.1) into the corresponding ones (5.2) and (5.3). It is not surprising that the (classical) equations of motion barely change. The change comes from the fact that with the hedgehog ansätze there is only one universal baryon $|h\rangle$ (5.1), whereas the chiral ansätze distinguishes N and Δ baryons. However the s-wave components (3.4) do vanish when the replacement (5.4a) is effected. One obtains

$$\chi_h^\dagger \tau^a \chi_h = 0. \tag{5.4c}$$

The solution presented in this section differs in an essentied way from the one discussed in Sect. 3. The method (1.1) leads to better g_A values than the methods (1.2) and (2).

The expectation value of the normal-order hamiltonian (4.2) is

$$\langle h|\mathcal{H}|h\rangle = \mathcal{H}_h$$

$$= 3 \cdot \int_0^R \mathrm{d}r\, r^2 \left(u\frac{\partial v}{\partial u} - v\frac{\partial u}{\partial r} \right)$$

$$+4\pi \int_R^\infty \mathrm{d}r\, r^2 \frac{1}{2} \left[\left(\frac{\partial \sigma}{\partial r} \right)^2 + \left(\frac{\partial \pi}{\partial r} \right)^2 + \frac{2}{r^2}\pi^2 \right]$$

$$+\frac{\lambda^2}{4}\left(\sigma^2 + \pi^2 - \nu^2\right)^2 + f_\pi m_\pi^2 \sigma. \tag{5.5}$$

The Euler-Lagrange equations are given in terms of mean fields approximated by the static expectation values.

Instead of Eq. (4.6) one finds

$$\sigma'' + \frac{2}{r}\sigma' = \lambda^2(\sigma(r) - f_\pi)[(\sigma(r) - f_\pi)^2 + (\pi(r))^2 - \nu^2] + f_\pi m_\pi^2 \tag{5.6}$$

and

$$\pi'' + \frac{2}{r}\pi' - \frac{2}{r^2}\pi = \lambda^2 \pi(r) \left[(\sigma(r) - f_\pi)^2 + (\pi(r))^2 - \nu^2 \right]. \tag{5.7}$$

The boundary condition for $\sigma(r)$ is

$$\frac{d\sigma}{dr} = -\frac{3g}{8\pi} N^2 [j_0^2(\omega) - j_1^2(\omega)], \tag{5.8}$$

where g is calculated from the fields at the boundary

$$g = \frac{1}{\sqrt{(\sigma(R_{bag}))^2 + (\pi(R_{bag}))^2}}. \tag{5.9}$$

For the pion phase one gets

$$\frac{\mathrm{d}\pi}{\mathrm{d}r} = -\frac{3g}{4\pi} N^2 [j_0(\omega) j_1(\omega)]. \tag{5.10}$$

The eletromagnetic properties are calculated using Eqs. (3.22) and (3.23a, b).

The quark contribution to the proton magnetic moment retains the form (3.24). Also, $\mu_n^{(Q)} = -\frac{2}{3}\mu_p^{(Q)}$. With the hedgehog ansätze for meson fields one finds [1]

$$\mu^{(M)} = \frac{4\pi}{3} \int_{R_{bag}}^{\infty} r^2 \, \mathrm{d}r [\pi(r)]^2. \tag{5.11}$$

The quark contribution to the nucleon axial-vector coupling constant g_A retains the form (3.27).

The meson part of the axial-vector constant is [1]

$$g_A^{(M)} = \frac{8\pi}{3} \int r^2 \, \mathrm{d}r \left[(\sigma(r) - f_\pi)\pi'(r) - \pi(r)\sigma'(r) + \frac{2(\sigma(r) - f_\pi)\pi(r)}{r} \right]. \tag{5.12}$$

The difference in constant factors between Eqs. (3.25), (3.28) and (5.11), (5.12), respectively, can be traced to averaging over Eq. (5.1) rather than over the proton wave function, as done in Sect. 4.

The quantum properties (5.2) and (5.3) of boson solitons follow from the hedgehog version of the boundary condition (3.15). Thus our baryon (5.1) differs from the usual form [1] which uses the coherent states. However, with the hedgehog ansätze, both methods lead to an identical expression for the energy $\mathcal{H}_p$ (4.3).

Using the trial wave function of Ref. 1

$$|h_{coh}\rangle = \exp(A_\sigma^+) \exp(A_h^+) |h\rangle \tag{5.13}$$

one easily finds

$$\mathcal{H}_h = \frac{\langle h_{coh} | \mathcal{H} | h_{coh} \rangle}{\langle h_{coh} | h_{coh} \rangle}. \tag{5.14}$$

Here A_σ^+ contains the elementary sigma-field operator $a_0^+(k)$, i.e.

$$A_\sigma^+ = \int \mathrm{d}^3k [\frac{\omega_{\sigma\,k}}{2}] \tilde{F}(k) a_0^+(k) \sigma(r) = \frac{2}{(2\pi)^3} \int \mathrm{d}^3k e^{i\vec{k}\vec{r}} \tilde{F}(k), \tag{5.15}$$

and analogously for A_h^+.

Variation with respect to $v(r), \pi(r)$ and $\sigma(r)$ leads to the above equations of motion.

A possible generalization of the coherent state for the chiral-quark ansätze (3.1) is considerably more complicated than (5.13) [1, 7]. Even the one pion approximation [13], which includes the coherent sigma-field state, is quite involved. It seems that the ansätze (3.3) and (3.4) lead to a simpler procedure that is analogous to the hedgehog approach (5.3) and (5.4) used here.

6. The numerical procedure

Numerics will be illustrated here for a non-linear system of coupled ordinary differential equations which have been derived in Sect. 3. The other two approaches, chiral quarks with hadron averaging and hedgehog quarks, lead to very similar systems which differ only in some superficial details.

A sequence of approximations led to a still quite complicated system. First, the very complex QCD field-theory dynamics was modelled by the chiral bag. Then, this model field theory, non-linear and complex, was approximated by the leading terms in the expansion in free-field operators.

This resulted in a system which determined fermion and boson radial functions appearing in the ansätze, for example in (3.1), (3.2), (3.3) and (3.4).

The boson radial functions had to satify Eqs. (3.10), (3.11) and (3.12).

These equations were supplemented by the boundary conditions given by Eqs. (3.13) and (3.14).

The conditions (3.14) were dictated by the (physical) requirement that the (massive) field solitons should vanish at infinity.

In Eq. (3.13) the normalization constant N can be expressed in terms of Bessel functions and quark eigenfrequencies ω:

$$N^2 = \frac{1}{R^3}\left[j_0^2(\omega) + j_0^2(\omega) - \frac{2j_0(\omega)j_1(\omega)}{\omega}\right]. \tag{6.1}$$

The radial parts of the quark wave functions appearing in Eq. (3.1) are Bessel functions $j_\ell(\omega r/R)$ for any spherical bag with radius R. At the bag boundary, where $r = R$, these functions have to satisfy the relations (3.19) and (3.20) which combine the quark frequency ω with the coupling constants g_σ, g_π, f_π etc. The algebraic relations among the coupling constants stem from the requirement that the homogeneous system of linear equations should have the vanishing determinant. Therefore, the coupling constants have to satisfy the consistency conditions given by Eq. (3.21)

The linear σ-model parameters satisfy the following relations derived from the symmetry breaking pattern (see Sect. 2) [1,8,12]:

$$\lambda^2 = \frac{m_\sigma^2 - m_\pi^2}{2f_\pi^2}, \qquad \nu^2 = f_\pi^2 - \frac{m_\pi^2}{\lambda^2}, \qquad d = \frac{1}{2}f_\pi^2 m_\pi^2 \frac{2m_\sigma^2 - 3m_\pi^2}{m_\sigma^2 - m_\pi^2}. \tag{6.2}$$

Here the value of d is determined by the requirement that $U(\sigma, \vec{\pi})$ should have zero minima. The σ meson is expected to have a mass of about 1 GeV [11]. Thus the *parameter* masses m_σ and m_π are selected to be 1.2 GeV and 0.139 GeV, respectively.

One has to solve simultaneously the system containing non-linear differential equations (3.10),(3.11) and (3.12), Eqs. (3.13) and (3.14) and the algebraic relations (3.21) and (6.2). This determines the meson functions $\sigma(r)$, $\pi_s(r)$ and $\pi_p(r)$, the quark frequency ω and various coupling (g_π, g_σ, etc.).

This complex system has been solved using the code COLSYS, the **col**location **sy**stem **s**olver, developed by U. Asher, J. Christiansen and R.D. Russel from the *University of British Columbia* and *Simon Fraser University, Canada* [14]. The boundary conditions are set at $[R_{bag}, R]$, where R is set to be so large that the fields can be approximated by zero at R. The initial guesses have been supplied. From the asymptotic behaviour and some earlier experience the input was rather simple and convergence has been achieved quickly.

The problem turns out to be rather sensitive to the derivative boundary conditions which in all cases involve the coupling constant(s). Although the asymptotic behaviour of the solutions can be inferred from the system itself (see also Ref. 15), the COLSYS is able to handle rather general initial (guess) solutions.

Upon return the routine gives error estimates for components and its derivatives. The problem parameters can be gradually changed (increased) by using a continuation method in COLSYS which is left to choose the initial mesh points, and in the continuation procedure it refines and redistributes the (former) mesh.

There are additional chiral-bag-model parameters, the same as those used in the MIT bag, i.e. B, Z_0 and α_s [1-4,10,11]. They are connected with the bag properties (B, Z_0) and with the effective gluon exchange (α_s) which removes the nucleon (N)–resonance (Δ) mass degeneracy. Some earlier experience (see Ref. 3) suggested that these parameters would remain within typical chiral-bag-model values. Here these parameters are used to fix the N and Δ masses within 1% accuracy. The numerical values depend on the particular ansätze used. Thus for example for solution described in Sect. 3 (see Table 1, below) one finds: $R = 6.0$, $\omega = 1.80$, $Z_0 = 0.12$, $B^{\frac{1}{4}} = 0.14$ and $\alpha_s = 0.12$ or $R = 5.0$, $\omega = 2.10$, $Z_0 = 0.3$, $B^{\frac{1}{4}} = 0.15$ and $\alpha_s = 0.25$.

The solutions are compared against the consistency conditions (3.21) and the iterative procedure is continued until the matching is obtained. The iteration consists in performing a self-consistent calculation: the coupling constants for the chiral quarks–non-hedgehog method are set to be the same at the beginning (their value is set to be equal to 10.00) and after every iteration new coupling constants are calculated from Eq. (3.21). These new values are replaced in the boundary conditions to calculate new solutions. The procedure converges rather rapidly. When the matching is achieved, the magnetic moment and the axial constant are calculated from the obtained solutions, i.e from either $\{\sigma(r), \pi_s(r), \pi_p(r)\}$ for the chiral quarks or $\{\sigma(r), \pi(r)\}$ for the hedgehog quarks.

7. *Results, comments and conclusion*

The non-hedgehog method 1 (Sect. 3) leads to the results which depend strongly on the quark eigenfrequency ω, as shown in Table 1. There are several sets of the coupling constants g_i which satisfy the consistency condition (3.21), thus producing several sets of g_A and μ values. One should, possibly, achieve some fine tuning by playing with other parameters, such as λ, ν, m_σ ...

TABLE 1.
The results for the chiral-quark non-hedgehog variant (1.1) of the model (Sect. 3). The bag radius is in GeV^{-1} *units.*

R	ω	magnetic moment			axial constant g_A		
		μ_Q	μ_m	μ_{tot}	g_A/q	g_A/M	g_A/tot
4.5	1.70	1.58	1.09	2.67	1.26	0.12	1.38
5.0	1.88	1.84	0.52	2.36	1.17	0.15	1.32
5.5	1.89	2.03	0.50	2.53	1.16	0.16	1.32
6.0	1.88	2.21	0.54	2.75	1.17	0.18	1.35
6.5	1.89	2.40	0.53	2.93	1.17	0.19	1.36
7.0	1.90	2.59	0.48	3.07	1.16	0.20	1.36
7.5	1.91	2.78	0.44	3.22	1.16	0.20	1.36
The parameters							
$\lambda = 9.062$		$m_\sigma = 1.2$ GeV		$\mu_{\mathrm{exp}} = 2.79$		$m_\pi^{\mathrm{exp}} = 0.139$ GeV	
$\nu = 0.092$		$f_\pi = 0.093$ GeV		$g_A/\mathrm{exp} = 1.26$		$m_\pi = 0.140$ GeV	

However, one is more interested here in comparison of methods. As shown in Table 2, the non-hedgehog mean-field method (1.2) gives consistently too large g_A values and somewhat better μ values. All predictions obtained using the method 1.2 are very similar to those found using the hedgehog mean-field method 2 (Sect. 5).

TABLE 2.
The chiral-quark-bag-model calculation - the non-hedgehog mean-field method has been used to project the physical states. The bag radius is in GeV^{-1} *units. The bag parameters are explained in the main text.*

R	ω	g	magnetic moment			axial constant g_A		
			μ_Q	μ_m	μ_{tot}	g_A/q	g_A/M	g_A/tot
4.97	1.0238	9.299	1.20	0.83	2.02	1.51	0.39	1.90
5.00	0.979	9.311	1.155	1.377	2.531	1.53	0.53	2.06
6.00	1.285	9.799	1.741	1.116	2.857	1.42	0.51	1.93
7.00	1.78	10.799	2.52	0.09	2.61	1.22	0.29	1.50
The parameters								
$\lambda = 9.062$			$m_\sigma = 1.2$ GeV			$\mu_{\mathrm{exp}} = 2.79$		
$\nu = 0.092$			$f_\pi = 0.093$ GeV			$g_A/\mathrm{exp} = 1.26$		

The hedgehog-based [1] results are displayed in Table 3. Here they were obtained by using parameters comparable with those used in Tables 1 and 2 , which facilitates the comparison. It is not suprising that the values in Tables 2 and 3 are similar. Eqs. (4.6), (5.6) and (5.7) are not very different. The same goes for the theoretical expressions for g_A and μ . The values of μ in Table 2 look somewhat better than those in Table 3. However, this could be just an accidental effect of a particular parametrisation.

TABLE 3.
The chiral-bag-model calculation – the hedgehog mean-field method has been used to project the physical states. The bag radius is in GeV^{-1} *units.*

R	ω	g	magnetic	moment		axial	constant	g_A
			μ_Q	μ_m	$\mu_{\rm tot}$	g_A/q	g_A/M	g_A/tot
5.00	1.280	11.250	1.45	0.27	1.72	1.43	0.42	1.85
6.00	1.637	10.878	2.060	0.144	2.204	1.28	0.33	1.61
7.00	1.783	10.799	2.519	0.092	2.610	1.22	0.29	1.504

The parameters		
$\lambda = 9.062$	$m_\sigma = 1.2$ GeV	$\mu_{\rm exp} = 2.79$
$\nu = 0.092$	$f_\pi = 0.093$ GeV	$g_A/\exp = 1.26$

In Table 1 the g_A values are generally better. In the method (1.1) the quark– and meson–phase equation of motion are treated as operator equations, which are approximatively solved. The meson-soliton solutions (i.e. classical profile functions) display all the required characteristics. The $\pi_p(r)$, $\pi_s(r)$ and $\sigma_s(r)$ are smoothly decreasing with distance, as required by the boundary conditions. The large μ values in Table 1 are always associated with smaller g_A values, thus both being simultaneously closer to the experimental date. In Table 1 one can see that such behaviour is caused by the meson-phase contributions. They are proportionally much larger in the case of μ, as it should be.

It is interesting that one can find non-hedgehog ansätze which solve the CBM based on the linear sigma model. However, except for g_A values in Table 1, it is difficult to give strong preference to any of the used methods. The results are also comparable with the Skyrme model [5], where, typically, $\mu = 2.48$, $g_A = 0.61$, or with the Nambu-Jona-Lasinio-model [12], where $\mu = 2.76$ and $g_A = 1.86$.

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NE–JEŽEVSKA RJEŠENJA ZA KIRALNU VREĆU

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Kiralni sigma model, smješten u okolišu kiralne vreće je riješen pomoću uvrštenja ("ansatz") koji čuva izospin i spin, svaki posebno. To kiralno uvrštenje je obrađeno na dva načina: i) kao skup operatorskih jednadžbi, koje se riješe među kvarkovskim stanjima i ii) Hamiltonijan se usrednji između odgovarajućih hadronskih stanja, pa se jednadžbe gibanja izvedu za ta prosječna polja. Drugi pristup je potpuno analogan uobičajenom koji upotrebljava ježevske kvarkove i koji je ovdje također reproduciran. Pokazalo se kako se energijski minimumi (tj. hadronske mase) mogu naći i na kiralnim i na ježevskim kvarkovima. Modelska predviđanja za aksijalno–vektorsku konstantu vezanja i za nukleonski magnetski moment su jednako dobra ili bolja nego ona koja su dobijena u uobičajenoj ježevskoj aproksimaciji.