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ON THE STUDIES OF THE STRUCTURE FUNCTION OF NUCLEON

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The square modulus of the wave function of the proton has been derived in the framework of the statistical model. The presence of antiquarks in the sea, which simulates the screening effect, has been explicitly taken into account. The resulting momentum space wavefunction has also been used to estimate the structure function for the free proton as well as the difference in structure function for the proton and neutron. The results obtained are in agreement with the corresponding experimental data and other theoretical estimates. The colour transparency effect has also been studied with interesting consequences.

1. Introduction

There has been a number of attempts to investigate the structure function of nucleons. The European Muon Collaboration (EMC) effect [1] has attracted much attention due to the fact that it describes the deviation of the nucleon structure function inside a nucleus. The structure function of a free nucleon has been investigated by Jaffe [2], De Rujula and Martin [3], Signal and Thomas [4] and others [5-9]. De Rujula and Martin [3] have investigated the structure functions of pion and nucleon and, from the shape of a nuclear structure function, have shown that the shape of the pion structure function can be calculated from the nucleon structure function using the MIT bag model wavefunction. They have also considered the effect of the hyperfine splitting due to the one-gluon exchange interaction and argued that it produces significant effects especially in the ratio of the structure functions. Modarres [6] has studied the effect of the nuclear structure on the deep inelastic scattering and the EMC effect in the context of the quark cluster model. They have incorporated the cluster property by increasing the MIT-bag radius for various nuclei that will be related to the two-body correlation function via a local density approximation. They have also calculated the free nucleon structure function such that it fits the experimental data and the EMC ratio.

Recently New Muon Collaboration (NMC) [10] suggested differences between the up and down sea-quark distributions in the proton from the measurement of the neutron and proton F_2 structure functions. They measured F_2^n/F_2^p over a wide range of the Bjorken parameter x. If the sea were perturbatively generated from gluons which create $\mathrm{q}\overline{\mathrm{q}}$ pairs, then near equality of masses ($u = d$) suggests that the sea should be SU(2) symmetric. Kumano and Londergen [11] have shown that prediction for F_2^{up} - F_2^{un} at momentum transfer $Q^2 = 4$ GeV^2 from the set B of Harriman et al. [12] and the set of Eichten et al. [13] and have shown the discrepancy between theory and experiment in the region $x < 0.4$. They have suggested that the discrepancy is confined to the small region of x where sea and antiquark contribution are significant. Barone and Predazzi [14] also plotted F_2^{lp} - F_2^{ln} in the context of discussing the EMC effect which suggests that valence quark distribution has a maximum around $x \approx 1/3$.

Though the discussions an colour transparency (CT) have started in the eighties, it has recently become an exciting subject since it is a new testing ground of models of nuclei and QCD. It was recently observed that final state interaction (FSI) does not occur in certain high momentum transfer reactions involving nuclear targets. Absence of such interaction is attributed to the cancellation of colourfield produced by a system of closely spared quarks and gluons. The charge screening effects of QCD occurs due to the colour transparency. The occurence of CT depends on two factors, the formation of small-size wave packets in high momentum transfer reactions and the suppression of interaction between such wavepacket and nucleons. CT was investigated by several authors [15-17]. An excellent review on CT has been made by Frankfurt et al. [15]. Jain and Ralston [16] have shown that the fully interacting hadronic basis, which consist of eigenstates of the exact Hamiltonian in the presence of the nucleus, provides a natural basis to study CT. They also pointed out that CT ratio can be constant with energy but not at variance with perturbative QCD. Bott [17] has presented a simple model to predict the centre of mass energies at which a target nucleus will become transparent to hard and tripple (Landshoff) scattered proton.

In the present work we have derived a wavefunction for a nucleon in the framework of the statistical model. In the present investigation, the effect of the colour screening of the valence quark by the corresponding antiquark sea has been explicitely taken into account, unlike the previous works [18,19]. We have also found the momentum space wavefunction by Fourier transform of the radial wavefunction. Using this wavefunction we have also estimated difference between F_2 structure functions for proton and neutron. The colour transparency ratio has also been estimated in the context of the model. The results obtained are in good agreement with the EMC data and other theoretical estimates.

2. *Formalism*

In the statistical model, a baryon is asumed to consist of three valence quarks q_1, q_2, q_3 in adition to a sea of virtual cloud $q_1\overline{q}_1$, $q_2\overline{q}_2$ and $q_3\overline{q}_3$ pairs. Only the valence quarks determine the quantum number of a baryon. The real and virtual quarks, corresponding to a particular flavour, are assumed to have the same colour, so that they may be regarded as identical and indistinguishable. Hence, a statistical ensamble can be formed resulting in a continuous distribution with each valence quark in the configuration space corresponding to maximum momentum according to Fermi statistics. This brings about uncertainty in the position of the quark within the baryon. To take into account the many-body interactions that a valence quark experiences in its encounter with the large number of corresponding virtual quarks and antiquarks within the cloud in the baryon, we assume that each valence quark moves in an average smooth potential (background) of the type $V = ar + b$ or $V = ar^2 + b$, i.e. linear or harmonic oscillator type. a and b are the interaction parameters. It is now well known that the constituent quarks inside a baryon are approximately in a free state of motion within the cloud. Thus each real valence quark can be regarded as moving almost independently and without any correlation with the other two real valence quarks. Consequently, we treat each individual quark separately in our subsequent analysis.

If p represents the maximum momentum of a type of quark (say q_1) in the small volume $\mathrm{d}\tau$ about r, then it is related to the number density of quarks corresponding to q_1 in a baryon through the relation

$$n_{q_1}(r) = \frac{p^3}{3\pi^2}, \tag{1}$$

where we assume $n_{q_1}(r)$ is the probability density.

As the maximum momentum of the valence quark depends on the average concentration of the quark inside the cloud and not on its intrinsic mass, we adopt the non-relativistic picture in our discussion to investigate the structure of the baryon.

The valence quark q_1 experiences an effective background potential $V_{q_1}(r)$ in its encounter with virtual quarks and antiquarks (q_1 and $\overline{q}_1$) in the cloud we have,

$$\frac{\mathrm{d}n_{q_1}(r)}{\mathrm{d}r} = \frac{3}{2}\left[\frac{\mathrm{d}}{\mathrm{d}r}\left(U_{q_1}(r)\right)/U_{q_1}(r)\right] n_{q_1}(r), \tag{2}$$

where $U_{q_1}(r) = V_{q_1}(r_0) - V_{q_1}(r)$ and r_0 is unknown radius parameter for baryons. Considering the background potential $V_{q_1}(r)$ to be smooth linear type, i.e. $V_{q_1}(r) = ar + b$, we have from (2),

$$n_{\mathrm{q}_1}(r) = A(r_0 - r)^{3/2}. \tag{3}$$

For the antiquark ($\overline{\mathrm{q}}_1$), we also expect an expression like (2) with similar consideration as in the case of quark (q_1), so that

$$\frac{\mathrm{d}n_{\overline{\mathrm{q}}_1}(r)}{\mathrm{d}r} = \frac{3}{2}\left[\frac{\mathrm{d}}{\mathrm{d}r}\left(U_{\overline{\mathrm{q}}_1}(r)\right)/U_{\overline{\mathrm{q}}_1}(r)\right] n_{\overline{\mathrm{q}}_1}(r). \tag{4}$$

In a baryon, all antiquark in the sea of $\mathrm{q}\overline{\mathrm{q}}$ pairs are virtual. We consider that the antiquark are moving in an average smooth background potential which is supposed to be unknown and we represent the potential as a finite power series in r,

$$V(r) = a_0 + a_1 r + a_2 r^2 + \cdots + a_n r^n,$$

so that the redefined smooth potential $U(r)$ becomes,

$$U(r) = V(r_0) - V(r) = a_0' + a_1 r + a_2 r^2 + \cdots + a_n r^n.$$

Hence we get

$$U'(r) = a_1 + 2a_2 r + \cdots + n a_n r^{n-1}.$$

Therefore,

$$U'(r)/U(r) = (a_1 + 2a_2 r + \cdots + n a_n r^{n-1})(a_0' + a_1 r + a_2 r^2 + \cdots + a_n r^n)^{-1} \tag{5}$$

which, on expansion, yields

$$U'_{\overline{\mathrm{q}}_1}(r)/U_{\overline{\mathrm{q}}_1}(r) = \lambda + \mu r + \gamma r^2 + \cdots, \tag{6}$$

where λ, μ and γ are constants.

Recasting right-hand side of (6) in terms of the number density $n_{\overline{\mathrm{q}}_1}(r)$ we obtain

$$f(r) = \lambda + a' n_{\overline{\mathrm{q}}_1}^{1/3} + b' n_{\overline{\mathrm{q}}_1}^{2/3}, \tag{7a}$$

where a' and b' are constants. Assuming high density expansion, the expression (7a) becomes,

$$f(r) \approx \lambda$$

so that from (4) we arrive at,

$$\frac{1}{n_{\overline{\mathrm{q}}_1}(r)}\frac{\mathrm{d}n_{\overline{\mathrm{q}}_1}(r)}{\mathrm{d}r} = \frac{3}{2}\lambda = \lambda' \tag{7b}$$

and obviously $\lambda' < 0$. From (7b) we get

$$n_{\bar{q}_1}(r) = B e^{-\lambda' r} = B(1 - \lambda' r), \tag{8}$$

where B is a unknown constant and λ' is small. With the boundary condition $n_{\bar{q}_1}(r_0) = 0$ at $r = r_0$, we get

$$n_{\bar{q}_1}(r) \approx \frac{B}{r_0}(r_0 - r) = C(r_0 - r). \tag{9}$$

Hence the effective number density of q_1 type of quarks is given by

$$n^{eff}_{q_1}(r) = n_{q_1}(r) - n_{\bar{q}_1}(r) = A(r_0 - r)^{3/2} - C(r_0 - r). \tag{10}$$

Similar relations are obtained for $n^{eff}_{q_2}(r)$ and $n^{eff}_{q_3}(r)$.

It is to be noted that $n^{eff}_{q_\alpha}(r)$ represents the effective number density of an α-type (where α = 1,2 and 3) quarks, due to the screening by its corresponding virtual antiquarks. As there is a continuous distribution of each type of (effective) valence quark, there is a continuous distribution of colour for each valence quark. So for a colourless baryon consisting of three valence quark q_1, q_2 and q_3, we have the normalization condition

$$\int_0^{r_0} \left[n_{q_\alpha}(r) - n_{\bar{q}_\alpha}(r)\right] 4\pi r^2 \mathrm{d}r = 1. \tag{11}$$

It is obvious that $n^{eff}_{q_1}(r) = n^{eff}_{q_2}(r) = n^{eff}_{q_3}(r)$, because the analytical forms of $n^{eff}_{q_\alpha}(r)$ have the same normalization condition given by (11). Consequently, three constraints in (11) reduce to one.

If $\Psi(r)$ represents the usual Schrödinger type wavefunction describing the baryon, we have the normalization condition

$$\int_0^{r_0} |\Psi(r)|^2 4\pi r^2 \mathrm{d}r = 1. \tag{12}$$

Comparing (10), (11) and (12) we get

$$|\Psi(r)|^2 = n^{eff}_{q}(r) = A(r_0 - r)^{3/2} - C(r_0 - r). \tag{13}$$

Hence, we come across a modified version of the nucleon wavefunction unlike in the previous works [18,19], where the distributions of virtual clouds were assumed to be of similar type as that of real valence quarks. The expression (13) suggests that at each point within the baryon, the number density corresponding to each type of quarks has been made equal to $|\Psi(r)|^2$, which represents the probability density of finding a baryon at r. Although the effective number density in principle contains three constants, C, A and r_0, we have effectively only two parameters: A and r_0, because of the (aditional) normalization condition (12).

The parameters of proton can be estimated by adjusting against the experimentally measured value of the electric form factor of the proton. The proton form factor may be expressed for $Q^2 \to 0$, as

$$F(Q^2) = \int \mathrm{e}^{\mathrm{i}qr} |\Psi(r)|^2 \mathrm{d}^3 r = 1 - (0.006 - 0.00685 A r_0^{9/2}) r^2 q^2. \tag{14}$$

The relation between the proton form factor and charge radius of proton is, $Q^2 \to 0$

$$F(Q^2) = 1 - \frac{1}{6} \langle r_{ch}^2 \rangle q^2 \tag{15a}$$

with $\langle r_{ch}^2 \rangle = 0.81$ fm [20]. Comparing (14) and (15) we get

$$A = \frac{9.635}{r_0^{9/2}} - \frac{15.96}{r_0^{13/2}}. \tag{16a}$$

We have fitted r_0 to the experimental results for $F(Q^2)$ at the intermediate value $Q^2 = 0.194$ GeV^2, against $F(Q^2)$ estimated from (14); best fit is found to be at r_0 = 1.5 fm which yields $A = 0.410$ fm$^{-9/2}$ and $C = 0.117$ fm^{-4}.

The momentum space wavefunction $\Psi(K)$ is derived by the Fourier transform of the wavefunction $\Psi(r)$ as

$$\Psi(K) = \frac{C_1}{K} \int_0^{r_0} r\ \Psi(r)\ \mathrm{d}r\ \sin Kr, \tag{15b}$$

where C_1 is the normalization constant and $\Psi(K)$ is found from (13) ignoring an undetermined phase factor.

To evaluate the integral on the right-hand side of (15b), assuming the above mentioned values of A and C, it may be noted that $\Psi(r) \approx 0.76$ at $r = 0$, $\Psi(r) \approx 0.42$ at $r = r_0/2$ and $\Psi(r) \approx 0$ at $r = r_0$. Hence it may be asserted that $\Psi(r)$ varies almost uniformly in the interval of r from 0 to r_0. To evaluate the integral in (16a) approximately, we replace $\Psi(r)$ by Ψ_{av} in the integrand which corresponds to $r = r_0/2$.

Consequently we get,

$$\Psi(K) = \frac{C_1}{K} \Psi(r_0/2) \int_0^{r_0} r\ \mathrm{d}r\ \sin Kr. \tag{16b}$$

From (16b) we obtain an analytical expression for the momentum space wavefunction as

$$\Psi(K) = \frac{C_1}{K} j_1(Kr_0). \tag{17}$$

After normalization, $\Psi(r)$ becomes

$$\Psi(k) = 2\sqrt{3\pi r_0} K^{-1} j_1(Kr_0). \tag{18}$$

It is to be noted that $\Psi(K)$ depends only on r_0, the size parameter of the nucleons. We have investigated the nucleon structure functions with the above $\Psi(K)$ as an input.

3. *Free nucleon structure function*

Following the argument of De Rujula and Martin [3], the free nucleon structure functions in the non-relativistic limit is defined as

$$F(\kappa) = \frac{M}{8\pi^2} \int\limits_{K_{min}}^{\alpha} |\Psi(K)|^2 \mathrm{d}K^2, \tag{19}$$

where M is the mass of the nucleon and $K_{min} = M|\kappa - 1/3|$.

With our $\Psi(K)$ in (18) as an input, $F(x)$ has been evaluated and the resulting graph is shown in Fig. 1. The plot of $F(x)$ suggests that the maximum value is peaked around $x = 1/3$. Our computed results closely agree with the results of De Rejula and Martin [3] and of Signal and Thomas [4].

It is also interesting to estimate the difference in F_2 structure function for proton and neutron using the analytical form of $\Psi(k)$ in Eq. (18). Neutron radius is taken as 1 fm [21]. Pursuing the work by Hoodbhoy and Jaffe [22], we have estimated $F_2^p - F_2^n$ and the results are tabulated in Table 1, along with the experimental results given by EMC group [23] and BCDMS Collaboration [24]. It is to be noted

TABLE 1.
Estimated values of $F_2^p - F_2^n$ along with the experimental values of EMC *[23] and* BCDMS *[24].*

		$F_2^p - F_2^n$	
x	Present calculation	EMC	BCDMS
0.175	0.0060	$0.071 \pm 0.009 \pm 0.027$	–
0.25	0.0615	$0.088 \pm 0.007 \pm 0.020$	–
0.275	0.0772	–	$0.1029 \pm 0.0025 \pm 0.0116$
0.35	0.1078	$0.082 \pm 0.007 \pm 0.012$	$0.0923 \pm 0.0016 \pm 0.0090$
0.45	0.1084	$0.077 \pm 0.007 \pm 0.009$	$0.0668 \pm 0.0014 \pm 0.0057$
0.55	0.0717	$0.046 \pm 0.006 \pm 0.004$	$0.0451 \pm 0.0010 \pm 0.0032$
0.65	0.0133	$0.025 \pm 0.005 \pm 0.003$	$0.2340 \pm 0.0006 \pm 0.0023$

that maximum value of the difference $F_2^p - F_2^n$ in the present calculation is also in a good agreement with the NMC data [10]. The discrepancies in the low x-region may be attributed to the approximation involved in the calculation. It is also to be noted that the different experimental results are somewhat inconclusive. It would not be irrelevant to point out that some authors [11] also get similar type of results and suggested that the discrepancies at small x-region are due to the sea and anti-quark distributions where they have significant contributions. The present work is based entirely on theoretical considerations, unlike the parametrisations assumed by some authors [6,13,14].

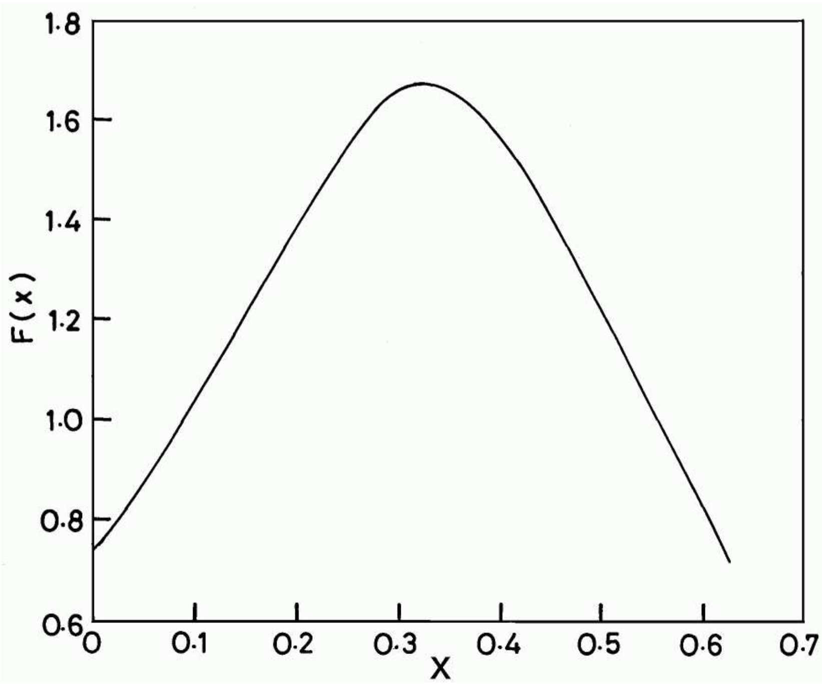

Fig. 1. Free nucleon structure function F(x).

4. Colour transparency

The colour transparency (CT) phenomenon depends basically on two factors. As discussed in Sect. 1, the formation of a small-sized wavepacket, which is a coherent superposition of physical states, is one of the basic conditions for the occurence of the CT in a high momentum transfer. In the context of studying the CT, Frankfurt et al. [15] have pointed out the wavepackets have no significant expansion during the reaction process, so one may concentrate a b^2 dependence of wavepacket-nucleon scattering amplitude, where b^2 is the transverse-size configuration characterised by a length b. In the case of non-relativistic constituent models of hadrons, the nucleon form factor is the matrix element of the electromagnetic current, at high momentum transfer. This hard scattering operator (T_H) acts on a nucleon to form a wavepacket. The importance of the final state interaction is measured by the colour transparency ratio which is defined as,

$$b^2(Q^2) = \frac{1}{F(Q^2)} \int \mathrm{d}^3 r \; \Psi^*(r) \, b^2 \, \mathrm{e}^{\mathrm{i}qr} \Psi(r), \tag{20}$$

where Q^2 is the squared four-momentum transfer.

With our model wavefunction from the expression (13), we have calculated $F(Q^2)$ for proton. Then the CT ratio was also calculated from (20) and $b^2(Q^2)/b^2(0)$ has been displayed in Fig. 2 along with the similar calculations in the cloudy bag model [25].

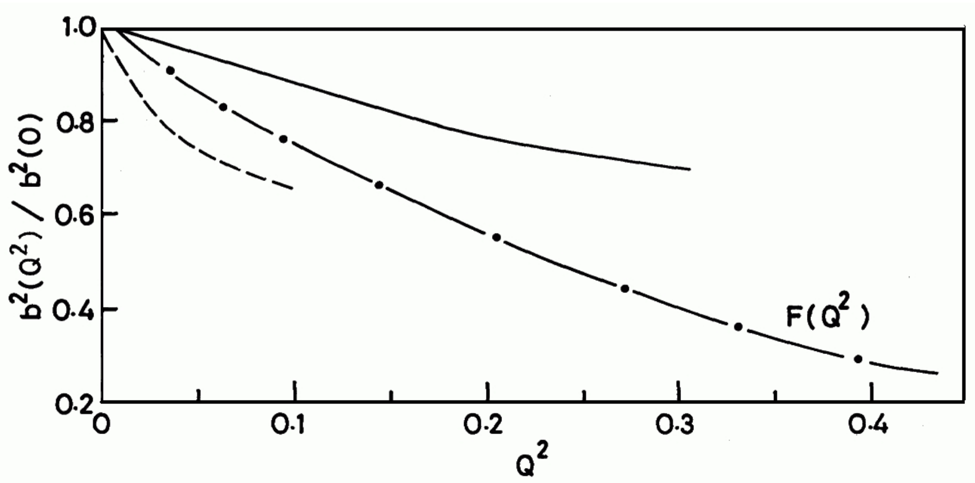

Fig. 2. Colour transparency ratio $b^2(Q^2)/b^2(0)$ is shown. Solid line is the present work. Dotted line is from the cloud bag model calculation of Ref. 25. Dotted-dashed line is $F(Q^2)$ in the present model.

It is to be noted that our wavefunction provides CT in the range $Q^2 = 0$ to 0.35 GeV^2. It is pertinent to note here that the existing models for hadrons provide very diverse predictions for the emergence of the colour transparency. Is has been observed that the CT occurs naturally in realistic models like Skyrmion model [26], cloud bag model [27] etc., whereas some other models [28] provide colour opacity.

5. *Conclusion*

The square modulus of the wavefunction for baryon, as suggested and derived in the context of the statistical model, is used to investigate the structure function for a free nucleon. It reproduces the experimental observation to a considerable extent. The nature of the wavefunction allows to replace $\Psi(r)$ by Ψ_{av}, which is a good approximation as evident from the results we obtain. The difference in proton-neutron structure function, which is estimated in the context of the model, also exhibits a good agreement with experimental suggestions in the range starting from $x > 0.25$. The wavefunction reproduces CT in the range $Q^2 \to 0$ to 0.35 GeV^2. In view of the simplicity of the model, the results are very encouraging and the model is found to be reasonably successful in describing some interesting structural properties of nucleons.

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ANALIZA STRUKTURNE FUNKCIJE NUKLEONA

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U okviru statističkog modela izveden je izraz za kvadrat modula valne funkcije. Eksplicitno je uzeta u obzir prisutnost antikvarkova iz pozadine koja simulira efekt zasjenjenja. Pomoću Fourierovog transformata valne funkcije određena je strukturna funkcija slobodnog protona i razlika strukturnih funkcija protona i neutrona. Dobiveni rezultati slažu se s odgovarajućim eksperimentalnim vrijednostima i rezultatima drugih teorijskih ocjena. Proučava se, također, efekt transparencije boje i izvedene su neke zanimljive posljedice.