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THE PREDICTION OF THE HINDRANCE FACTORS FOR THE ^{20}O-CLUSTER TRANSITION OF ^{255}Fm WITHIN THE ENLARGED SUPERFLUID MODEL

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Within the one-level R-matrix approach several hindrance factors for a particular radioactive decay, the emission of ^{20}O-nuclei, have been calculated. The interior wave functions are supposed to be given by shell model with effective residual interactions. The exterior wave functions are calculated from a cluster-nucleus double-folding potential obtained with the M3Y interaction. As an example of the cluster decay fine structure, we analized the ^{20}O-radioactivity of ^{255}Fm. Our results for the half-lives indicate that observation of ^{20}O-decay would be very difficult.

1. Introduction

Recently Hourani et al. [1] discovered fine structure in the ^{14}C radioactivity [2]. The theoretical studies of alpha [6,7] (see also the review papers [3-5] and the references therein) and heavy cluster (e.g. ^{14}C) decay [7] (see also the recent review paper [8] and the references therein) have very much in common. The process of emission of a cluster of nucleons is assumed to occur in two main steps: The parent nucleus makes a kind of phase transition from the initial state, which could be of any structure, to the final state composed of at least one cluster, which is going to be emitted, and the residual nucleus, which may have also any structure. One mechanism of such a transition could be the cluster condensation, or, what is usually assumed, a formation of the cluster from the already formed condensates of smaller clusters [9,10]. Next, the two daughter nuclei tunnel through the potential barrier in their relative motion without further change in shape.

Most theoretical models of heavy cluster decay [8] are based, essentially, on Gamov's theory [11], i.e. a detailed description of the second step - the tunneling through the potential barrier. Various approaches differ in the ways of calculating the potential barrier defined by the interaction potential acting between the emitted cluster and the residual nucleus. For favoured cluster transitions all these theoretical treatments yield a law analogous to the Geiger-Nuttal law [12] for favoured alpha-decay. It emerges directly from simplest Jeffreys-Wentzel-Kramers-Brillouin expression for the penetrability determined by the square-well plus Coulomb interaction potential.

The unfavoured transitions do not follow the Geiger-Nuttal law, because of the large variations of the reduced widths [3-7]. The reduced widths have a key role in our understanding of the decay process and require a precise knowledge of the structures of the initial and final quantum states. From such transitions we can learn much about the structure of atomic nuclei and about the mechanism of the decay phenomenon. For instance, when treating the favoured cluster decay, one assumes that the nucleons which will form the cluster are more or less strongly correlated in the initial state. This fact leads to small hindrance factors [3,4,7]. On the contrary, the unfavoured transitions, with large hindrance factors, are characterized by the fact that the nucleons assumed to form the cluster are collected from different mutually strongly correlated groups of nucleons entering the structure of the initial state. In the latter case it is necessary to breakup the correlated groups of nucleons and then to form the cluster which is going to be emitted.

In Ref. 7 the formal expression for theoretical hindrance factors have been derived.

In the present paper we continue this work and calculate several hindrance factors for ^{20}O-radioactivity. The calculations will be performed by using the approach given in Ref. 7.

2. *Enlarged superfluid model*

The enlarged superfluid model (ESM) Hamiltonian for nonrotational states of deformed nuclei includes an average field of neutron and proton systems in the form of the axial-symmetric Saxon-Woods (or Hartree-Fock), monopole pairing, isoscalar and isovector particle-hole and particle-particle multipole and spin-multipole interactions between quasiparticles as well as the so-called alpha-like four nucleon interaction [3] of the rank $N > 1$:

$$H = H_0 + H' \tag{1}$$

where

$$H_0 = \sum_{\tau} \left(H^{av}_{s.p.}(\tau) - G_\tau P^\dagger_\tau P_\tau \right) + H_4 \tag{2}$$

in which

$$H^{av}_{s.p.}(\tau) = \sum_{s\sigma} E_s a^\dagger_{s\sigma} a_{s\sigma} \tag{3}$$

$$P_\tau = \sum_{s} a_{s-} a_{s+} \tag{4}$$

$$H_4 = -G_4 P^\dagger_p P^\dagger_n P_n P_p \tag{5}$$

and

$$\begin{aligned} H' = \sum_{\tau} \Bigg\{ & -\frac{1}{2} \sum_{\lambda\mu\sigma} \sum_{n=1}^{N} \Bigg[\sum_{\eta=\pm 1} \left(k^{\lambda\mu}_{0\tau} + \eta k^{\lambda\mu}_{1\tau} \right) \times \\ & \times Q^\dagger_{n\lambda\mu\sigma}(\tau) Q_{n\lambda\mu\sigma}(\eta\tau) + G^{\lambda\mu}_\tau P^\dagger_{n\lambda\mu\sigma}(\tau) P_{n\lambda\mu\sigma}(\tau) \Bigg] - \\ & -\frac{1}{2} \sum_{L\lambda\mu\sigma} \sum_{n=1}^{N} \Bigg[\sum_{\eta=\pm 1} \left(k^{L\lambda\mu}_{0\tau} + \eta k^{L\lambda\mu}_{1\tau} \right) T^\dagger_{nL\lambda\mu\sigma}(\tau) T_{nL\lambda\mu\sigma}(\eta\tau) + \\ & + G^{L\lambda\mu}_\tau P^\dagger_{nL\lambda\mu\sigma}(\tau) P_{nL\lambda\mu\sigma}(\tau) \Bigg] \Bigg\} . \end{aligned} \tag{6}$$

Here $\tau = -\frac{1}{2}$ stands for the proton system and $\tau = +\frac{1}{2}$ stands for the neutron system, $a^\dagger_{s\sigma}$ ($a_{s\sigma}$) are the fermion operators which create (destroy) a nucleon in (from) the single particle state $|s_\tau \sigma_\tau\rangle$, where σ_τ is the sign of the projection of the angular momentum of the state onto the nuclear symmetry axis, s_τ being the rest (N_τ, n_{z_τ}, Ω_τ, Π_τ) of the quantum numbers that label the single particle energy levels. The term H_4 from Eq. (5) is an effective coherent two-pairs (four-nucleon) interaction term, which includes the dynamical alpha-like four nucleon correlations in the superfluid phases of atomic nuclei [13]. G_τ are the pairing coupling strenghts, $G^{\lambda\mu}_\tau$ and $G^{L\lambda\mu}_\tau$ are the coupling constants of the particle-particle interaction [3], $k^{\lambda\mu}_{0\tau}$, $k^{\lambda\mu}_{1\tau}$ and $k^{L\lambda\mu}_{0\tau}$, $k^{L\lambda\mu}_{1\tau}$ are isoscalar and isovector coupling constants of the particle-hole multipole and spin-multipole interactions [14], G_4 is the four-nucleon interaction constant and $\sigma = \pm 1$.

To find the ground and excitation spectrum and corresponding wave functions we used the recipe from Refs. 10, 14 and 16.

3. ^{20}O-cluster decay

We use the enlarged superfluid model (ESM) [10] and calculate the quasiparticle-phonon structure of the ground state of the ^{255}Fm nucleus. We calculated the favoured and the weakly unfavoured ^{20}O-transitions from the ^{255}Fm-nucleus to excited states in the ^{235}U-nucleus, using the approximation suggested in Ref. 7. The results are reproduced in Tables 1 and 2.

TABLE 1.

Structure of some ground and excited states for the ^{255}Fm *(ground state)* $\rightarrow$ ^{20}O + ^{235}U *transitions, calculated within* ESM *[10].*

Nucleus	I^{π}	K	E_{exp} (MeV)	E_{theo} (MeV)	Structure
^{255}Fm	$7/2^+$	7/2	0.	0.	97.91%[613] $7/2^+$ +
					2.1%[624] $7/2^+$ +
					2.1%[613] $7/2^+$ Q_{20} +
					2.5%[611] $3/2^+$ Q_{22}
^{235}U	$7/2^+$	7/2	0.492	0.458	71.03%[624] $7/2^+$ +
					7.09%[613] $7/2^+$ +
					9.04%[743] $7/2^-$ Q_{30} +
					1.05%[725] $11/2^-$ Q_{32}
^{235}U	$7/2^+$	7/2	1.236	1.458	61.02%[613] $7/2^+$ +
					9.9%[624] $7/2^+$ +
					19.04%[624] $7/2^+$ Q_{20} +
					1.05%[725] $11/2^-$ Q_{32}

TABLE 2.

Hindrance factors for the ^{255}Fm *(ground state)* $\rightarrow ^{20}$O + ^{235}U *transitions, calculated within* ESM *[10].*

E_f (keV)	$I_f^{\pi_f}$	HF$_{\rm ESM}$	E_f (keV)	$I_f^{\pi_f}$	HF$_{\rm ESM}$
492.	$7/2^+$	≈ 185	1236.	$7/2^+$	≈ 5
	$9/2^+$	≈ 428		$9/2^+$	≈ 11
	$11/2^+$	≈ 729		$11/2^+$	≈ 18
	$13/2^+$	≈ 1224		$13/2^+$	≈ 31

In calculating the structure of the ^{235}U-excited states we used the ESM parameters: the pairing coupling strenghts $G_p = 0.14$ MeV, $G_n = 0.1$ MeV and the four-nucleon interaction $G_4 = 0.26$ keV. The assumed particle-particle quadrupole parameter (see Ref. 10) are: $G_{n\tau}^{\lambda\mu} = G_{\tau}^{2\mu} = 15$ eV fm^{-4}. The particle-hole quadrupole and octupole parameters (see Ref.10) were assumed as follows: $k_{n\tau}^{\lambda\mu} = k_{0\tau}^{2\mu} = 0.667$ keV fm^{-4}; $k_{n\tau}^{\lambda\mu} = k_{1\tau}^{2\mu} = 0.062$ keV fm^{-4}; $k_{n\tau}^{\lambda\mu} = k_{0\tau}^{3\mu} = 0.011$ keV fm^{-6}; $k_{n\tau}^{\lambda\mu} = k_{1\tau}^{3\mu}$ = 0.001 keV fm^{-6}. The assumed deformation parameters were: $\beta_{20} = 0.23$ and β_{40} = 0.08. The parameters of the average field were taken from Ref. 16.

From the Table 1 we learn that within the ESM-model [10] the structure of the ^{255}Fm ground state contains contributions from two single quasiparticle states, namely, 97.9 % - [613] $7/2^+$ and 2.1 % - [624] $7/2^+$, emerging from the $2g_{9/2}$ and $1i_{11/2}$ subshells, respectively. These states occur also in the structure of the ^{235}U excited states lying at excitation energies of 492 keV and 1236 keV, respectively (see Table 1 and Ref. 16).

4. *Hindrance factors*

The experimental hindrance factor (HF) of any cluster decay is defined as a ratio between the Geiger-Nuttal [12] width divided by the width of the radioactive transition we are interested in [3]:

$$HF = \frac{\Gamma_{GN}(Q)}{\Gamma(Q)} \tag{7}$$

where Q stands for the energy release of the studied decay and [12]

$$\log \Gamma_{GN}(Q) = A + \frac{B}{\sqrt{(Q)}}. \tag{8}$$

The theoretical hindrance factor is defined by Eq. (7) in which the widths are replaced by their theoretical expressions. In the case of heavy deformed nuclei [4]:

$$HF = \frac{P_0(Q)\gamma_0^2}{\sum_l P_l(Q)\gamma_0^2} = \frac{\gamma_0^2}{\sum_l F_l \gamma_l^2} \tag{9}$$

where in the Jeffreys-Wentzel-Kramers-Brillouin approximation

$$F_l = \exp\{-\frac{2}{\hbar}\int_{R_c}^{r_0}(q_{l=0}(r) - q_l(r))\mathrm{d}r\}. \tag{10}$$

The factor γ_l^2 is the reduced width [3,4], while $P_l(Q)$ stands for the penetrability. r_0 and R_c stand for outer ond inner turning points, respectively, and:

$$q_l(r) = \sqrt{2m_0 A_{red}(V_l^{Coul+nucl} - Q)} \tag{11}$$

$V_l^{Coul+nucl}$ is the sum of the Coulomb and nuclear one-body potential acting between the α-cluster and the daughter nucleus when studying the radial part of the Schrödinger equation. Usually [3,8] the Coulomb part of this potential is replaced by point-like Coulomb potential while the nuclear part by a Saxon-Woods one.

To illustrate the contribution of different terms which appear in the final expression of the hindrance factor, let us analyse the ESM structure of the states for odd nuclei ^{255}Fm and ^{235}U given in Table 1. The numerical structure of their wave functions will select the most important of these [14,17,18]. Thus, the ground state of the parent nucleus ^{255}Fm is mainly a simple favoured quasiparticle state determined by the Nilsson orbital 98%[613] with a dominant contribution, $c^2_{\rho_i} = 98\%$. The same state can be found with an important weight $c^2_{\rho_f} = 61\%$, in the structure of the state of the daughter nucleus ^{235}U, lying at 1236 keV excitation energy. The other orbitals corresponding to unfavoured quasiparticle - phonon transitions can be neglected in comparison with the favoured quasiparticle transitions.

Therefore, by using the ESM structure for the initial and final odd-mass nuclei, in the case of favoured or weakly unfavoured radioactive decay with emission of a spherical double even cluster (e.g. ^{20}O), we may write for the hindrance factor the following expression [7]:

HF (parent nucleus $(I_i^{\pi_i} K_i) \rightarrow ^{20}$O + daughter nucleus $(I_f^{\pi_f} K_f)) \approx$

$$\approx \left\{ \sum_l F_l \left| C^{I_i l I_f}_{K_i K K_f} C_{\rho_i} C_{\rho_f} (RSA)^{(i \rightarrow f)} \right|^2 \right\}^{-1} . \tag{12}$$

$C_{\rho_{i(f)}}$ are the weights of the single quasiparticle state in the structure of the initial (final) states. The quantities (RSA) replace essentially the ratio of the favoured intrinsic spectroscopic amplitude [7] corresponding to the transition between odd-mass and doubly-even nuclei. The intrinsic spectroscopic amplitude (Θ_{in}) is defined by

$$\Theta_{in} = \sum_{\nu_1 \cdots \nu_8} \sum_{\omega_1 \cdots \omega_{12}} A^{LM}(\nu_1 \cdots \nu_8 | \omega_1 \cdots \omega_{12}) \xi^{fav}(\nu_1 \cdots \nu_8 | \omega_1 \cdots \omega_{12}) \tag{13}$$

that is analogous to the contributions of quasiparticles in the matrix element in Eq. (11) of Ref. 17 for the alpha-decay rate of axially deformed odd-A nuclei. The only difference in the treatment of odd-mass and doubly-even nuclei is that in the first case the sum in the above equation excludes the common quasiparticle state of both the parent and daughter nuclear states (e.g. [613] 7/2$^+$ for the ^{255}Fm $\rightarrow ^{20}$O + ^{235}U). In our estimations (see Table 2), the approximation $(RSA) \approx 0.4$ was used, determined mainly by the overlap integral between the odd-nucleon orbital wave functions in the parent and daughter nuclei.

Unfortunately, the above discussed ^{20}O radioactivity cases have half-lives larger than the maximum half-life ($10^{25.75}$ s) among the experimentally measured [5] cluster decay half-lives and, hence, will be difficult to measure.

5. *Conclusions*

The enlarged superfluid model [10] has been used to study some selected (favoured and weakly hindered) ^{20}O transitions from the ^{255}Fm-nucleus to excited states in the ^{235}U nucleus. In the estimations (see Table 2), the approximation

$(RSA) \approx 0.4$ has been used. It is determined mainly by the overlap integral between the odd-nucleon orbital wave functions in the parent and daughter nuclei.

We calculated also the quasiparticle-phonon structure of the ground state of the ^{255}Fm nucleus which emits ^{20}O cluster and the ground and several excited states of the daughter nucleus ^{235}U (see Table 1).

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PREDVIĐANJE FAKTORA USPORAVANJA ZA PRIJELAZ ^{255}Fm EMISIJOM ^{20}O GROZDA NA OSNOVI PROŠIRENOG MODELA SUPERTEKUĆINE

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Na osnovi jednorazinske R-matrične teorije izračunati su faktori usporavanja radioaktivnog raspada, emisije jezgri ^{20}O. Pretpostavljene su unutarnje valne funkcije ljuskastog modela s rezidualnim interakcijama. Vanjske su valne funkcije izračunate na osnovi dvostruko-preklopnog potencijala za grozdastu jezgru, izvedenog koristeći M3Y interakciju. Kao primjer fine strukture raspada emisijom grozda analizirali smo ^{20}O-radioaktivnost ^{255}Fm. Dobiveni rezultati za vremena poluživota ukazuju da bi opažanje ^{20}O-raspada bilo vrlo teško.