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QUARK OFF-SHELLNESS IN K, $B \to \gamma\gamma$ DECAYS

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We demonstrate the importance of the quark off-shellness on examples of the radiative pseudoscalar meson decays, characterized by the simplest hadronic matrix elements. We point out that roughly a quarter of the empirical $K_L \to \gamma\gamma$ amplitude originates in the quark off-shellness in kaon. For $B_S \to \gamma\gamma$ we also find non-negligible contributions that increase the decay rate by a factor of 1.5 to 3.

1. Introduction

In attempts to account for weak hadronic decays, one faces the problem of overbridging the quark world (where the W-induced flavour change sets in) and the real world (in which the physical process occurs). The analyses starting from the high-energy side evolved from the traditional Feynman diagram technique to an implementation of the operator-product expansion (OPE) [1]. When studying non-leptonic decays of order $\sim G_F$, and weak radiative decays of order eG_F or e^2G_F, one normally writes down an effective Lagrangian including operators that contribute to a given process and operators that mix with these under QCD renormalization. Within this standard procedure, one usually omits operators containing

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$(i\gamma \cdot D \; - \; m_q)$, by appealing to the equations of motion (EOM) for quark fields [2,3]:

$$(i\gamma \cdot D - m_q) \to 0 \; , \tag{1}$$

where D_μ is the covariant derivative containing the gluon and the photon fields. This procedure corresponds to going on-shell with external quarks in quark operators. Certainly, quarks are not exactly on-shell in hadrons, especially not in the octet (would-be Goldstone) mesons π, K, η. We will see that the naive use of (1) is not correct in general, and that the bound-state interactions within mesons might be understood as a change of the equations of motion.

The purpose of this paper is to shed more light on the role of the off-shellness of quarks in mesons. We start with two cases where the effective Lagrangian containing the factor $(i\gamma \cdot D - m_q)$ has been studied [4]. First, we consider the circumstances under which the renormalized $s \to d$ self-energy transition becomes potentially relevant to the $K \to 2\pi$ decays. Then we turn to Lagrangians obtained from quark diagrams for $s \to d\gamma$ and $s \to d\gamma\gamma$ relevant to $\overline{K^0} \to \gamma\gamma$, and to the similar Lagrangians obtained from quark diagrams for $b \to s\gamma$ and $b \to s\gamma\gamma$ relevant to the $B_s \to \gamma\gamma$ decay.

2. *A quark analogue to the Lamb shift and the chiral quark model*

The proved example of an off-shell effect is the Lamb shift $-$ the tiny difference in the self-energy of the *free* electron and the self-energy of an electron *bound* in the H-atom. It incorporates as the main contribution[2] the Bethe low-frequency part $\Delta\nu_{low} = 1047$ MHz (for virtual photons between the electron Compton wavelength and the size of the atom). One might expect more significant analogous effects for much more strongly bound quarks – especially in approaches where quark masses are generated dynamically. Still, since one can hardly speak of the referent free-quark self-energy, one expects that there might be a better chance of finding an observable effect in the flavour-changing, non-diagonal $s \to d$ self-energy transition. Because there are no direct $s \to d$ transitions in the original Lagrangian, the renormalization is carried out so that $s \to d$ transitions are absent for on-shell s- and d-quarks. The renormalized self-energy corresponds to an effective Lagrangian

$$\mathcal{L}^R_{ds} = -A\,\bar{d}(i\gamma \cdot D - m_d)(i\gamma \cdot DR + M_R R + M_L L)(i\gamma \cdot D - m_s)s \; , \tag{2}$$

where M_L, M_R are constants depending on quark masses, and $A(p^2)$ is a slowly varying (logarithmic) function. In the pure electroweak case, the CP-conserving part of A is of order $G_F m_c^2/M_W^2$, [6]. However, the CP-violating part of A has no such suppression for a t-quark with a mass of the same order as the W-boson

[2]The experimental value is $E_{2S_{1/2}} - E_{2P_{1/2}} = 1057.862(20)$ MHz, requiring a theoretical precision at kHz level. For a recent account, see Ref. 5.

($m_t \simeq M_W$). Moreover, adding perturbative QCD to lowest order, one obtains in any case an unsuppressed contribution $\sim G_F \alpha_s \log\ (m^2)$, where $m = m_c$ and $m \simeq m_t \simeq M_W$ in the CP-conserving and CP-violating cases, respectively [7-9].

If one applies the EOM as in (1), $\mathcal{L}^R_{ds} \to 0$. Then, according to the standard procedure, there will be no contribution from $\mathcal{L}^R_{ds}$ to physical amplitudes, such as $K \to 2\pi$. However, if (1) is violated for off-shell bound quarks in π and K, physical effects could be obtained, and one should explore possible consequences for the $\Delta I = 1/2$ rule and for ϵ'/ϵ in $K \to 2\pi$ decays. Some time ago Donoghue [10] considered possible effects of $s \to d$ transitions and concluded that some non-zero effect might persist. Thereby one has to distinguish the possible short-distance (SD) and long-distance (LD) effects, which deserves some more explanation:

The $K \to \pi\pi$ amplitude proceeds in the second order in the weak coupling

$$S^{(2)}_{K\to 2\pi} = \langle \pi\pi | \int \mathrm{d}^4 x D_F(x, M_W) T(J(x) J^+(0)) | K \rangle \ , \tag{3}$$

where the currents J are dressed (to all orders in the strong QCD coupling). Equation (2) appears in the expansion of Eq. (3), whereby the contraction of the quark field q in the current-current product

$$\bar{s}(x)\gamma^\mu L q(x)\ \bar{q}(0)\gamma_\mu L d(0) \tag{4}$$

results in the two-quark operator at hand. This operator, involved in Shabalin's consideration [7] and its critique [8,9], presents the piece in the OPE for the product of the two currents that vanishes by the EOM and that does not mix under renormalization with the four-quark operators. Technically [2], the renormalization matrix Z acquires a triangular form, and the matrix elements at the partonic level involve only operators that do not vanish by EOM [11]. Thus, the *short-distance* aspects of the $\bar{d}s$ self-energy are unimportant for the $K \to 2\pi$ transition amplitude [8]. Let us note that in the case of the Lamb shift it was not possible to develop an OPE approach[3].

An explicit calculation of the contribution to $K \to 2\pi$ from $\mathcal{L}^R_{ds}$ gives a non-zero but negligible contribution in the CP-conserving case [4,9]. This does not mean that contributions that resemble self-energy are unimportant for the $\Delta I = 1/2$ rule. It is likely that the $\Delta I = 1/2$ enhancement comes principally from terms in Eq. (3) where the two u-quarks in the four-fermion operators are contracted and are festooned by myriads of soft gluons. However, these long-distance contributions (eye diagrams) are beyond a perturbative treatment.

One possibility of including non-perturbative *confining and chiral-symmetry* aspects of QCD is to use some version the chiral quark model, an effective low-energy QCD model advocated by many authors [12-14]. To quote Weinberg [12], such a framework will introduce "fictitious elementary particles into the theory, in rough correspondence with the bound states" – pseudoscalar mesons among the degrees

[3] I.P. thanks M. Shifman for pointing out this fact.

of freedom of the constituent quark model. The *chiral-symmetry* of QCD and its Nambu-Goldstone realization is at work: instead of the "nearby" chiral symmetry, one observes (pseudo) Goldstone mesons. The chiral quark model starts with the ordinary QCD Lagrangian and adds a term $\mathcal{L}_\chi$ that takes care of chiral-symmetry breaking,

$$\mathcal{L}_\chi = -M(\bar{q}_R\, U q_L + \bar{q}_L\, U^\dagger q_R)\,, \tag{5}$$

where $\bar{q} = (\bar{u}, \bar{d}, \bar{s})$ and the 3×3 matrix $U \equiv \exp\left(2\mathrm{i}\Pi/f\right)$ contains the pseudoscalar octet mesons $\Pi = \sum_a \pi^a \lambda^a/2\ \ (a = 1,..,8)$, and f can be identified with the pion decay constant, $f = f_\pi = (92.4 \pm 0.2)$ MeV ($= f_K$, in the chiral limit). This term, proportional to the constituent quark mass $M \simeq 300$ MeV, includes the Goldstone meson octet in a chiral-invariant way, and provides a meson-quark coupling that makes it possible to calculate matrix elements of quark operators as loop diagrams. In this effective field theory it is of course no problem to handle off-shell quarks.

In our previous work [15], we found a substantial CP-violating amplitude for $\overline{K^0} \to \gamma\gamma$ from *irreducible* diagrams for $s \to d\gamma\gamma$. Owing to the Ward identities between $s \to d\gamma\gamma$ and $s \to d\gamma$ transitions, there is a cancellation between 1PI diagrams for $s \to d\gamma\gamma$ and reducible diagrams for the two-photon emission (where the 1PI transition $s \to d\gamma$ is a building block). However, one cannot expect this free-quark cancellation to persist in the real world: the hadronic matrix elements of the reducible graphs are of highly non-local character, whereas the matrix elements of the irreducible graphs are proportional to a quark current, having a well-known matrix element [15]. Therefore, let us take a closer look at the radiative flavour-changing transitions within the above-mentioned chiral quark model.

3. *Off-shellness in the $K_L \to \gamma\gamma$ amplitude and the anomaly link*

We were tempted to relate the off-shellness in the process $K_L \to \gamma\gamma$ to the well-known electromagnetic $\pi^0 \to \gamma\gamma$ decay governed by the axial anomaly.

Although the π^0 axial anomaly is not conventionally termed the off-shell effect, it can in fact be viewed in this way[4]

– either on account of being dominated by far off-shell triangle loop momenta ($q^2 \gg m_q^2$)

– or as represented by the term $\mathcal{A} = \dfrac{\alpha}{4\pi}\varepsilon_{\mu\nu\alpha\beta}F^{\mu\nu}F^{\alpha\beta}$,

[4] I.P. thanks D. Klabučar for turning his attention to a lecture by P. van Nieuwenhuizen, stressing this point.

which cannot be reproduced by classical equations of motion in the divergence of the axial current. This term is missing on the r.h.s. of

$$\partial^\mu(\bar{q}\gamma_\mu\gamma_5 q) = 2m_q\bar{q}\gamma_5 q \, .$$

This results from an inadequate application of EOM of the off-shell quark-field circulating in the triangle loop. It is reassuring that the quark triangle evaluation with the quark-meson coupling defined in Eq. (5) reproduces the required anomalous π^0 decay!

Let us now recall the appearance of the off-shellness [4,16] in $K_L \to \gamma\gamma$. The essential point is in overbridging the non-perturbative QCD ($\lesssim$ 1 GeV) scale and the electroweak ($\sim M_W$) scale at which the flavour change (FC) $s \to d$ takes place in the presence of external photons. The evaluation of the loop diagrams [15,17,18], without going to the mass shell, results in an effective Lagrangian [4]

$$\mathcal{L}(s \to d)_\gamma \; = \; B\,\epsilon^{\mu\nu\lambda\rho} F_{\mu\nu}\,(\bar{d}_L \; i \stackrel{\leftrightarrow}{D}_\lambda \gamma_\rho s_L) \; , \tag{6}$$

where $B \sim eG_F\lambda_{KM}$ depends on the loop integration, and quarks are interacting fields with respect to QCD. In order to follow the fate of the off-shell contribution, it is convenient to rewrite (6) in the form

$$\mathcal{L}(s \to d)_\gamma = \mathcal{L}_F + \mathcal{L}_\sigma \; , \tag{7}$$

where

$$\mathcal{L}_F = B_F\,\bar{d}[(i\gamma\cdot D - m_d)\,\sigma_{\mu\nu}F^{\mu\nu}L + \sigma_{\mu\nu}F^{\mu\nu}R(i\gamma\cdot D - m_s)]s \qquad , \tag{8}$$

and $\mathcal{L}_\sigma$ is the well-known magnetic-moment term,

$$\mathcal{L}_\sigma = B_\sigma\,\bar{d}\,(m_s\sigma_{\mu\nu}F^{\mu\nu}R + m_d\sigma_{\mu\nu}F^{\mu\nu}L)\,s \; . \tag{9}$$

Here we anticipate that the coefficients B_F and B_σ, being equal at the W-scale, evolve differently down to the scale $\simeq$ 1 GeV. It has been shown that $\mathcal{L}_F$ does *not* contribute to $s \to d\gamma\gamma$ when the external quarks are on-shell: The irreducible $s \to d\gamma\gamma$ part, with $iD_\mu \to e_{s(d)}A_\mu$, is exactly cancelled by reducible diagrams [17,18], i.e. with one photon on an external line of the $s \to d\gamma$ vertex, with $D_\mu \to \partial_\mu$. Thus, for on-shell quarks, the remaining contribution from $\mathcal{L}(s \to d)_\gamma$ to $s \to d\gamma\gamma$ is due to the reducible diagrams, where the effective flavour-changing vertex corresponds to $\mathcal{L}_\sigma$ alone. Moreover, this remaining contribution vanishes in the chiral limit $m_{s,d} \to 0$, as seen from (9). In the pure electroweak case, the CP-conserving part of the quantity B is very small, $\sim eG_F m_c^2/M_W^2$, owing to an effective GIM cancellation between u- and c-quarks, while the CP-violating part is substantial ($\sim eG_F$), owing to the heavy t-quark. In the CP-conserving case, a significant amplitude $\sim eG_F\alpha_s \log(m_c^2)$ is induced by perturbative QCD [19].

There has been considerable efforts [4,16,20] devoted to the study of the *direct* $K_L \to \gamma\gamma$ amplitude induced by the operators (6)–(9). By explicit calculation within the chiral quark model, we found a non-zero contribution to $\overline{K^0} \to \gamma\gamma$ from $\mathcal{L}_F$. Although formally suppressed by M^2/m_0^2 , $m_0 = 2\pi f_\pi \sqrt{6/N_c}$ being the chiral symmetry-breaking scale, its coefficient is sizeable, yielding a significant amplitude both in the CP-conserving [16] and CP-violating case [4,15]. Thus we disagree with some authors [18,21] who claim that this effect is unimportant.

We have shown that the quark off-shellness (lost in the beloved parton, free-quark picture) represents a piece of the electroweak $K_L \to \gamma\gamma$ amplitude, which has rather similar properties to the well-established $\pi^0 \to \gamma\gamma$ decay amplitude. It is suggestive that both of these processes are governed by a common effective interaction of the form

$$\mathcal{L}_X \;=\; \alpha C_X \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma} \Phi_X \;, \tag{10}$$

where $X = \pi^0$ or $X = K_2$. The $\pi^0 \to \gamma\gamma$ rate is reproduced by

$$C_{\pi^0} \;=\; \frac{N_c}{24\pi f_\pi} \;=\; 4.3 \times 10^{-4}\mathrm{MeV}^{-1} \,.$$

Similarly, the rate for $K_2 \to \gamma\gamma$ determines the phenomenological coupling

$$|C_{K_2}| \;=\; 5.9 \times 10^{-11}\mathrm{MeV}^{-1} \,.$$

As already mentioned, the adopted chiral quark model accounts for the full C_{π^0} amplitude, whereas the calculation in Ref. 16 shows that it accounts, combined with $\mathcal{L}_F$ in (8), for roughly a quarter of the $|C_{K_2}|$. All this refers to the "unrotated" (U) version of low-energy QCD [16].

The term $\mathcal{L}_\chi$ in (5) can be transformed into a pure mass term $-M\bar{\mathcal{Q}}\mathcal{Q}$ for rotated "constituent quark" fields $\mathcal{Q}_{L,R}$:

$$q_L \to \mathcal{Q}_L = \xi q_L \;; \quad q_R \to \mathcal{Q}_R = \xi^\dagger q_R \;; \;\; \xi \cdot \xi = U \;. \tag{11}$$

Then the meson-quark couplings in this "rotated" (R) picture are transformed into the kinetic (Dirac) part of the "constituent quark" Lagrangian. These interactions can be described in terms of vector and axial vector fields coupled to constituent quark fields $\mathcal{Q}$. In the rotated basis, where pions have derivative Goldstone couplings, the compensating Wess-Zumino-Witten (WZW) term ensures the anomaly matching. The unrotated-quark-triangle evaluation finds a counterpart in the anomalous WZW part of the chiral Lagrangian. The explicit diagrammatic evaluation, giving a zero result for $\pi^0 \to 2\gamma$ in the rotated picture, complies with the more general functional derivation of the WZW term [22], which is contained in a Jacobian of the quark field rotation in Eq. (11).

An important result in [16] is that the amplitude for $\overline{K^0} \to \gamma\gamma$ is also zero in the rotated basis. Thus, the results (non-zero in the U-basis, zero in the R-basis) in evaluating the $\overline{K^0} \to \gamma\gamma$ amplitude from quark-level diagrams within the chiral quark model, motivated us [16] to attribute a similar anomalous nature to this process. By employing the anomaly-matching principle, we argued for the existence of the related bosonic Lagrangian term corresponding to the WZW term. Our new anomalous term $\mathcal{L}_{\rm WZW}^{\Delta S=1}$ accounting for the transition $\overline{K^0} \to \gamma\gamma$ has the form [16]

$$\mathcal{L}_{\rm WZW}^{\Delta S=1}(4) \;\sim\; e\,B_F\,\frac{1}{f_\pi}\,\frac{M^2}{m_0^2}\,\epsilon_{\mu\nu\rho\sigma}A^\nu F^{\rho\sigma}\partial^\mu\overline{K^0}\,, \tag{12}$$

which can easily be brought to the form (10). Thus, besides the representative of the *direct* anomalous neutral kaon decay, which can be read off in Refs. 23, 24,

$$K_L \to \pi^+\pi^-\gamma\,,$$

we offer a new candidate. Our assertion is that our new WZW-extended $\Delta S = 1$ term adds a new process to the existing list of *direct* anomalous processes [23], namely the decay

$$K_L \to \gamma\gamma\,.$$

This result was recently confirmed by a bound-state calculation [20]. To evaluate the hadronic matrix elements in the bound-state approach, the variant of an effective meson bilocal theory [25] was used. With the model harmonic interaction, this calculation avoids both the divergences in quark-loop integrals and the divergences in the bound-state equations themselves, so that no regularizations or cut-offs are necessary. The price one pays are somewhat poor absolute values for the amplitudes. However, good chiral properties allow us again to decode the anomaly part in $K_L \to \gamma\gamma$ amplitude, using $\pi^0 \to \gamma\gamma$ as a "monitoring process".

It was checked that the pion-to-two-photons coupling (C_{π^0} in (10)), as extracted from the $\pi^0 \to \gamma\gamma$ evaluation in the bilocal bound-state approach [20], exhibits the independence from quark masses, which is characteristic of the anomaly contribution. For the corresponding direct $K_L \to \gamma\gamma$ amplitudes, the off-shell ($\mathcal{L}_F$) contributions turn out to be dominant (the model on-shell, $\mathcal{L}_\sigma$ amplitudes being at the 10-20% level). The bound-state calculation in essence confirms the previous chiral-quark results: Our off-shell contribution is an entirely new $\mathcal{O}(p^4)$ *direct-decay* piece, whereas the *reducible* pole contributions [26] are numerically uncertain, and the non-diagonal magnetic moment term belongs to the $\mathcal{O}(p^6)$ terms.

4. The quark-loop $B_s \to \gamma\gamma$ amplitude

As before [4,16], the flavour-changing radiative vertices have to be supplemented by the quark-meson vertex in order to perform the full quark-loop evaluation. In contradistinction to the analogous $K_L \to \gamma\gamma$ decay [4,16], the heavy B-meson cannot be treated as a Goldstone boson of the chiral quark model adopted earlier. However, the pseudoscalar character of B-mesons allows us to parametrize this vertex in a simple way, replacing the $\mathcal{L}_\chi$ term in (5) by

$$iG_B\bar{s}\gamma_5 b B_s\,. \tag{13}$$

This interaction may in general be non-local, i.e. G_B might be momentum-dependent [27]. Thereby, as usually done [4], we trade the meson-quark coupling G_B in favour of the meson-decay constant f_B. In calculating the contributions from $\mathcal{L}_F$ and $\mathcal{L}_\sigma$ in (8) and (9), respectively (with the obvious replacements $s \to b$ and $d \to s$), we obtained [28] an amplitude of the following form

$$M(B_s \to \gamma\gamma) \simeq e_D \, f_B[A_{(+)} \, F_{\mu\nu}F^{\mu\nu} + iA_{(-)} \, F_{\mu\nu}\tilde{F}^{\mu\nu}] \, , \tag{14}$$

$$A_{(\pm)} = \tau_F^{(\pm)} B_F + B_\sigma \tau_\sigma^{(\pm)} \, , \tag{15}$$

where the quantities $\tau_{F,\sigma}^{(\pm)}$ are dimensionless and depend on the bound-state dynamics. Numerically, they turn out to be of order one. The coefficients B_F of $\mathcal{L}_F$ and B_σ of $\mathcal{L}_\sigma$, now contain the KM factors relevant to the $b \to s$ transition and are renormalized at the scale $\mu = m_b$.

The QCD correction for B_σ is known [19,29]. Performing the same calculation, but with zero anomalous dimension for $\mathcal{L}_F$, we find at the $\mu = m_b$ scale that

$$\frac{B_F}{B_\sigma} \simeq \frac{4}{3} \, . \tag{16}$$

This result, obtained within a simplified calculation (with a truncated basis) is in agreement with the QCD corrections found in Ref. 15.

In the formal limit when the current quark masses $m_{b,s} \to 0$, the $F_{\mu\nu}\tilde{F}^{\mu\nu}$ term of (14) should reduce to the anomalous $\overline{K^0} \to 2\gamma$ amplitude described in the preceding section. However, in the real world $M_b \gg M_s \gtrsim M \simeq 300$ MeV, and the result for $\mathrm{Br}(B_s \to 2\gamma)$ will be rather different from $\overline{K^0} \to 2\gamma$. In order to estimate the (model-dependent) quantities $\tau_{F,\sigma}^{(\pm)}$, for illustrative purposes, we have considered two examples: 1) The limit of a constant G_B (local interaction), and 2) a form-factor damping of the light-quark momenta as in Ref. 27. In the case 1), we have considered the extreme limit when $M_b \gg M_s$ and we have kept only the leading terms that can be incorporated in f_B. In this simplified limit we obtain $\tau_\sigma^{(\pm)} = \tau_F^{(+)} = -\tau_F^{(-)} = 1$, giving a branching ratio

$$\mathrm{Br}(B_s \to 2\gamma) \simeq 2 \times 10^{-8}.$$

It is somewhat smaller than the free-quark estimate [30], which is proportional to the inverse of the light s-quark mass. Going beyond this simple limit, $\tau_\sigma^{(\pm)}$ are more important than $\tau_F^{(\pm)}$. For the case 2), the dominant terms $\tau_\sigma^{(\pm)}$ will be inversely proportional to the momentum-damping parameter Λ, which is somewhat larger than the constituent s-quark mass M_s. Then, qualitatively, the result is not far from that given in [30]. We find that $\tau_\sigma^{(\pm)} \simeq 2$ to 3, while the relative importance of $\tau_F^{(\pm)}$ is slightly reduced and formally suppressed by $1/M_b$ with respect to $\tau_\sigma^{(\pm)}$. In the case 2), $\mathrm{Br}(B_s \to 2\gamma)$ is increased with respect to 1). In general, we find that the genuine off-shell term $\mathcal{L}_F$ increases the rate by a factor of 1.5 to 3. We conclude that values of the order 10^{-8} to 10^{-7} are realistic. Our prediction is still two orders of magnitude above the LD estimates based on the vector-meson dominance [32].

In calculating the contribution from the off-shell operator $\mathcal{L}_F$, we have arrived at an important observation: The off-shellness of the light quark is characterized by the heavy-quark mass, whereas the off-shellness of the heavy quark is characterized by the light-quark mass. For the s- and b-quarks in play,

$$(\gamma \cdot p_b - M_b) \sim M_s \quad \text{and} \quad (\gamma \cdot p_s - M_s) \sim M_b \quad , \tag{17}$$

where p_q and M_q are the quark-momenta and quark-masses for $q = b, s$. However, if the light-quark momenta are damped as in Ref. 27, we have two competing effects, and the total effects will be less pronounced.

In a recent paper [31], it has been reported that there are off-shell bound state effects in the process $B \to K^*\gamma$. However, only the off-shellness of the b-quark was taken into account, whereas we find that for $B_s \to 2\gamma$ the off-shellness of the s-quark cannot be neglected. In our recent study of $B_s \to 2\gamma$ [28], we have shown that the contribution from the two-photon piece of $\mathcal{L}_F$ is exactly cancelled by parts of its contribution from the one-photon piece. The remaining contribution from the off-shell operator $\mathcal{L}_F$ corresponds to loop diagrams containing the effective $B_s\,\bar{b}b\gamma$ and $B_s\,\bar{s}s\gamma$ vertices. This result is equivalent to that presented in Ref. 4: $\mathcal{L}_F$ may be transformed into the wave function, but then it reappears in the bound-state dynamics.

5. *Conclusions*

We have demonstrated the quark off-shell effects in flavour-changing two-photon decays, such as $s \to d\gamma\gamma$ ($b \to s\gamma\gamma$) and its hadronic $\bar{K}^0 \to \gamma\gamma$ ($B_s \to \gamma\gamma$) counterparts. Thus, the same basic off-shell effect seems to take place in processes belonging to such different environments as the chiral perturbation theory and heavy-light bound-state calculation.

We have demonstrated that the naive use of the (perturbative) EOM (1) is not applicable in general. This should be no surprise because the low energy interactions, represented by (5) and (13), and the photon field appearing in the covariant derivative, might be interpreted as external fields. Thus the (perturbative) equations of motion (1) are changed.

The genuine off-shell effects are formally suppressed in a certain limit by $1/M_b$ for $B \to \gamma\gamma$ and by $(M/m^0)^2$ for $K \to \gamma\gamma$. Numerically, the suppression is not equally pronounced in these two cases. For $K \to \gamma\gamma$, the effect of $\mathcal{L}_F$ is even stronger than that of $\mathcal{L}_\sigma$. Indeed, the latter effect is chirally suppressed and of order $\mathcal{O}(p^6)$.

The quark off-shellness represents a link that brings close the electroweak $K_L \to \gamma\gamma$ decays and the electromagnetic $\pi_0 \to \gamma\gamma$ decay. However, the direct amplitude originating in the quark off-shellness in the kaon is only a fraction of the total $K_L \to$

$\gamma\gamma$ amplitude. The various LD aspects, including the reducible pole contributions, seem to play a dominant role in this case.

For $B_s \to \gamma\gamma$, we have also found a non-zero genuine off-shell contribution. Although the hadronic matrix element is model dependent, there are substantial off-shell contributions that increase the rate by a factor of $\simeq 1.5$ to 3. It is hoped that some of the uncertainties in calculating the effects of $\mathcal{L}_F$ could be resolved within some variant of a QCD sum rule [33] calculation.

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References

1) M.K. Gaillard and B.W. Lee, Phys. Rev. Lett. **33** (1974) 108;
G. Altarelli and L. Maiani, Phys. Lett. **B52** (1974) 351;

2) J. Collins, *Renormalization* (Cambridge U.P., Cambridge, 1984);

3) R.L. Kingsley, F. Wilczek and A. Zee, Phys. Lett. **61B** (1976) 259; A.I. Vainshtein, V.I. Zakharov and M.A. Shifman, Zh. Exper. Teor. Fiz. **72** (l977) 1275 [Sov. Phys. JETP **45** (1977) 670; F. Gilman and M.B. Wise, Phys. Rev. **D20** (1979) 2392; C.T. Hill and G.G. Ross, Nucl. Phys. **B171** (1980) 141;

4) J.O. Eeg and I. Picek, Phys. Lett. **B301**(1993) 423;

5) M. Cavicchi and A. Vairo, Bologna preprint DFUB-94-03, hep-ph/9302346;

6) S.P. Chia, Phys. Lett. **B147** (1984) 361;

7) E.P. Shabalin, Moscow preprint ITEP 86-112;

8) B. Guberina, R.D. Peccei and I. Picek, Phys. Lett. **B188**(1987) 258;

9) J.O. Eeg, Phys. Lett. **B196**(1987) 90;

10) J.F. Donoghue, Phys. Rev. **D19** (1979) 2772;

11) H. Politzer, Nucl. Phys. **B172** (1980) 349 H. Simma, Z. Phys. **C61** (1994) 67;

12) A. Manohar and H. Georgi, Nucl. Phys. **B234** (1984) 189; A. Manohar and G. Moore, Nucl. Phys. **B243** (1984) 55; S. Weinberg, Phys. Rev. Lett. **67** (1991) 3473;

13) J. Bijnens, H. Sonoda and M.B. Wise, Can. J. Phys. **64** (1986) 1; D.I. Diakonov, V.Yu. Petrov and P.V. Pobylitsa, Nucl. Phys. **B306** (1988) 809; D. Espriu, E. de Rafael and J. Taron, Nucl. Phys. **B345** (1990) 22; J. Bijnens, Nucl. Phys.**B367** (1991) 709;

14) A. Pich and E. de Rafael, Nucl. Phys. **B358** (1991) 311; A. Pich, CERN-TH 6368/92;

15) J.O. Eeg, B. Nižić and I. Picek, Phys. Lett. **B244** (1990) 513;
J.O. Eeg, B. Nižić and I. Picek, in Proceedings of the 25th International Conference on High Energy Physics, page 1259, Ed. K.K. Phua and Y. Yamaguchi, World Scientific, Singapore, 1991;
I. Picek, Nucl. Phys. B (Proc. Suppl.) **24A** (1991) 101;

16) J.O. Eeg and I. Picek, Phys.Lett.**B323**(1994) 193;

17) G.J. Lin, J. Liu and Y.-P. Yao, Phys. Rev. Lett. **61** (1990) 1498; H. Simma and D. Wyler, Nucl. Phys. **B344** (1990) 283;

18) S. Herrlich and J. Kalinowski, Nucl. Phys. **B381** (1992) 501;

19) M.A. Shifman, A.I. Vainshtein, and V.I. Zakharov, Phys. Rev. **D18** (1978) 2583; Ya.I. Kogan and M.A. Shifman, Yad. Fiz. **38** (1983) 1454 [Sov. J. Nucl. Phys. **38** (1983) 628;

20) D. Kekez, D. Klabučar, K. Kumerički and I. Picek, Bielefeld preprint BI-TP 94/21 and Zagreb preprint ZTF-94/05;

21) J. Liu, 1992, quoted in B. Winstein and L. Wolfenstein, Rev. Mod. Phys. **65** (1993) 1113;

22) J. Wess and B. Zumino, Phys. Lett. **B37** (1971) 95;
E. Witten, Nucl. Phys. **B233** (1983) 422;

23) G. Ecker, H. Neufeld and A. Pich, Phys. Lett **B278** (1992) 337;
G. Ecker, H. Neufeld and A. Pich, CERN-TH.6920/93, June 1993;

24) J. Bijnens, G. Ecker and A. Pich, Phys. Lett. **B286** (1992) 341;

25) Yu. L. Kalinovsky, L. Kaschluhn and V. N. Pervushin, Phys. Lett. **B231** (1989) 288. R. Horvat, D. Kekez, D. Klabučar and D. Palle, Phys. Rev. **D44** (1991) 1585. Yu.L. Kalinovsky and C. Weiss, Tübingen preprint UNITÜE-THEP-17/93; hep-ph/9402236;

26) R. Decker, P. Pavlopoulos and G. Zoupanos, Z. Phys. **C28** (1985) 117; L.-L. Chau and H.-Y. Cheng, Phys. Rev. Lett. **54** (1985) 1768, and Phys. Lett. **B195** (1987) 275; E.Golowich in *Flavour Mixing and CP-Violation*, Ed. J.Tran Thanh Van, Edition Frontiers, 1985; J.F.Donoghue, B.Holstein and G.Valencia, Int. J. Mod. Phys. **A2** (1987) 319; J.F.Donoghue, B.Holstein and Y.C.R.Lin Nucl. Phys. **B277** (1986) 651; J.L.Goity, Z.Phys. **C34** (1987) 341; G.Ecker, A.Pich and E.de Rafael, Nucl. Phys. **B303** (1988) 665;

27) B. Holdom and M. Sutherland, Phys. Rev. **D49** (1994) 2356;

28) J.O. Eeg and I. Picek, CERN-TH.7278/94 and ZTF - 94/6;

29) B. Grinstein, R. Springer and M. B. Wise, Nucl. Phys. **B339** (1990) 269;

30) G. Lin, J. Liu and Y. Yao, Phys. Rev. **D42** (1990) 2314; T.M. Aliev and G. Turan, Phys. Rev. **D48** (1994) 1176; P. Singer, Phys. Rev. **D49** (1994) R7;

31) C.E. Carlson and J. Milana, Maryland preprint UMD-PP-94-129, DOE/ER/40762-033, WM-94-104, hep-ph/9405344, May 1994;

32) J. Trampetić, Fizika **B2** (1994) 121;

33) S. Narison, Phys. Lett. **B327** (1994) 354;
A. Ali, V.M. Braun, and H. Simma, CERN-TH.7118/93, MPI-Ph/93-97, DESY93-193, hep-ph/9401277, submitted to Z. Phys.

DOPRINOSI KVARKOVA IZVAN MASENE LJUSKE RASPADIMA K, $B \to \gamma\gamma$

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Pokazana je važnost efekta kvarkova izvan masene ljuske na primjerima radijacijskih raspada pseudoskalarnih mezona, karakteriziranih najjednostavnijim hadronskim matričnim elementima. Istaknuto je da otprilike četvrtina opažene amplitude $K_L \to \gamma\gamma$ raspada ima izvorište u odstupanju od masene ljuske kvarkova u kaonu. Analogan efekt povećava vjerojatnost $B_S \to \gamma\gamma$ raspada za faktor 1.5 do 3.