ISSN 1330–0016
CODEN FIZBE7

SPECIAL RELATIVITY AND CAUSAL FASTER-THAN-LIGHT EFFECTS

MARIJAN RIBARIČ and LUKA ŠUŠTERŠIČ

Jožef Stefan Institute, University of Ljubljana, 61111 Ljubljana, Slovenia

Received 4 July 1994

UDC 530.12

PACS 03.30.+p, 03.65.Bz, 11.10.Kk, 05.60.+W

Interpretations of various quantum-mechanical, theoretical and experimental results suggest the existence of causal faster-than-light effects. To show how such effects can be reconciled with special relativity, we give an example of a classical causal system with local, covariant equations of motion. Its retarded solutions exhibit causal faster-than-light effects. Certain properties of these solutions propagate almost according to the Klein-Gordon equation and not faster than light.

1. *Introduction*

Many interpretations of quantum-mechanical theoretical results suggest the conjecture that there are changes in the state of a physical system that propagate with infinite speed from their sources, the so-called instantaneous effects [1-4]. Various experimentally observed violations of Bell's inequality [1,5] support this conjecture; they suggest the existence of causal effects that propagate faster than light (CFTLE).

According to special relativity, if a source at the time-space point $(ct_1, \vec{r}_1)$ causes CFTLE at $(ct_2, \vec{r}_2)$ so that $0 \leq t_2 - t_1 < |\vec{r}_2 - \vec{r}_1|/c$, then in an inertial reference frame $K^{'}$, moving with relative velocity $\vec{v}$ such that $t_2 - t_1 < c^{-2}\vec{v} \cdot (\vec{r}_2 - \vec{r}_1)$, the corresponding time-space points $(ct_1^{'}, \vec{r}_1^{\,'})$ and $(ct_2^{'}, \vec{r}_2^{\,'})$ are such that $t_2^{'} < t_1^{'}$, i.e., in $K^{'}$ the effect precedes its cause. This fact has two important consequences:

(i) It seems impossible to test directly by measurement whether CFTLE really exist. Namely, were it possible that the source at $(ct_1, \vec{r}_1)$ and CFTLE at $(ct_2, \vec{r}_2)$ each emits or scatters an electromagnetic wave, then, when observing these two electromagnetic signals from the inertial reference frame $K^{'}$, we would see consequences preceding their causes. So far *there have been no experimental indications that the time order of the cause and of the consequence depends on the motion of observer.* Thus, if we use time-varying sources that also emit or scatter electromagnetic waves, it follows that CFTLE, which they are possibly causing, cannot be directly observed and utilized for transmitting signals faster then light.

(ii) Suppose that we can conclude that CFTLE exist by interpreting certain quantum-mechanical phenomena of a given physical system. Then, from special relativity, we expect to conclude that there are some non-causal effects if we interprete the same phenomena when the system moves with certain uniform velocity. But this is not the case: *no interpretation of quantum mechanical phenomena or theoretical results suggests that there exist physical systems where changes of the state precede their causes* [1-4]. So, there are two possibilites: either CFTLE do not exist - they are just theoretical figments, or there is an open question how are we to apply, understand or possibly augment the principles of special relativity so as to encompass physical systems that exhibit CFTLE but no non-causal effects; e.g., do we have to introduce a preferred frame of reference to this end. The possibility that CTFLE suggested by quantum mechanics imply that all inertial reference frames are not equivalent has been pointed out by Eberhard [6]; and according to Hardy [7], CFTLE are "most naturally incorporated into a theory in which there is a special frame of reference." Our aim is to comment on this open question of theoretical physics and to point out an answer.

Relativistic field theories, such as classical electrodynamics and quantum field theories, study covariant relations between the effects at $(ct_2, \vec{r}_2)$ that are caused solely by sources at such time-space points $(ct_1, \vec{r}_1)$ that satisfy the Einstein causality condition $|\vec{r}_2 - \vec{r}_1|/c \leq t_2 - t_1$, the so-called locality condition. Therefore, such effects and their sources may be used to model observable effects and transmission of signals. The fields in relativistic field theories depend on the four independent time-space variables and satisfy covariant equations of motion that are local in time-space, i.e., they relate only the values of fields and of a finite number of their time and space derivatives computed an the same time-space point. Classical relativistic field theories do not exhibit CFTLE, but after quantization the interpretations of the results obtained suggest the existence of CFTLE.

If we take CFTLE as real and not just as theoretical artifacts, we have to presume that there are causal physical systems[1] whose states exhibit CFTLE, and the observable properties of which are quantitatively well described by quantum

[1]We call a physical system, its phenomena and its mathematical model causal if no changes of its state whatsoever, whether observable or non-observable, precede their causes. If real physical system were not causal, there would be no "free will," in experiments our choices of causes would be preordained by the preceding changes of their states.

field theories. The problem is how to construct mathematical models of such hypothetical, causal physical systems in such a way that two conditions are met in any inertial reference frame: (i) the equations of motion and initial and boundary conditions are such that states (i.e., their solutions) exhibit CFTLE implied by quantum-mechanical correlations but no non-causal effects, and (ii) the observable properties of state can be defined in such a way that they do not propagate faster than light and may be described with sufficient precision through fields determined by quantum field theories.

To accomplish such a construction, the de Broglie-Bohm causal interpretation of relativistic quantum fields [8,9] uses *non-covariant and non-local* partial differential equations of motion. The resulting theory exhibits CFTLE and yields Lorentz-covariant predictions of quantum field theories, but it ignores the first postulate of relativity [10] that forbids a preferred frame of reference implied by its non-covariant equations of motion. And it is not clear whether the de Broglie-Bohm theory can be reformulated so as to make it abide by the principle of locality of the basic equations of motion. So, this theory raises two basic theoretical questions: do CFTLE, suggested by interpretations of quantum-mechanical results and experimentally observed correlations, imply that: (i) there is a preferred frame of reference, and (ii) we have to abandon the principle that the basic equations of motion are local in time-space. According to Itzykson and Zuber [11], this principle of locality of basic equations is an important ingredient of modern field theories. A physical model, whose basic equations of motion relate directly the values of the state function at two distant time-space points, invites the question what goes on in between. Only an explanation local in time-space may be considered as complete. E.g., as Isaac Newton pointed out regarding gravity: "that one body may act upon another at a distance through a vacuum without the mediation of anything else, by and through which their action and may be conveyed from one to another, is to me so great an absurdity that I believe no man ... can ever fall into it" (see, e.g., Vigier [12]).

It is our purpose to point out that if a causal physical system that displays CFTLE existed, that would, in general, not necessarily imply a preferred frame of reference and/or that the basic equations of motion are not local in time-space. We will show that CFTLE are not incompatible with special relativity and the traditional belief that the basic physical phenomena are causal and local in time-space. To demonstrate this, we will put forward an example of explicitly solvable, linear, *local and covariant* equations of motion of a classical, causal system with a local Lagrangian density. The states of this system display CFTLE, though at each time-space point a certain property of them is defined in such a way that it does not propagate faster than light and closely satisfies the Klein-Gordon equation (the partial-differential equation satisfied by the Feynman propagator for a spin-0 boson and so governing the local propagation of free spin-0 bosons in quantum field theories [11]). Such a system is an analogue to the nonrelativistic rare gas that propagates changes of fluid-dynamics variables almost according to the linearized Euler partial differential equations with a finite speed of sound, though its states, which evolve according to the linearized Boltzmann equation, respond almost immediately everywhere to any local source due to the unlimited speeds of gas particles, (see, e.g., Ref. 13).

2. A classical model

2.1. Covariant transport process

Motivated by the kinetic theory of gases, we make the following assumptions: The state of physical system considered is given by a scalar, complex-valued function $\Psi(x,p)$ of the time-space variable $x = (ct, \vec{r}\,) \in \Re^{1,3}$ and of the four-momentum variable $p = (p^0, \vec{p}\,) \in \Re^{1,3}$. Its equation of motion is a local, covariant transport equation

$$p \cdot \nabla\Psi = \lambda^{-1} S_{-1}\Psi + S_0\Psi + Q, \tag{1}$$

where: (i) $p \cdot \nabla = c^{-1}p^0\partial/\partial t + \vec{p}\cdot\nabla$ is the covariant, substantial time derivative. (ii) S_{-1} and S_0 are two scattering operators that act on the state $\Psi(x,p)$ as if it were solely a function of the four-momentum variable p; they commute with the ortochronous Lorentz transformations to make transport equation (1) covariant. (iii) The positive parameter λ regulates the strength of scattering by S_{-1} and makes it predominant as λ tends toward zero, making thereby certain four-momentum averages of $\Psi(x,p)$ obey the Klein-Gordon partial differential equation in the limit $\lambda = 0$ [14]. (iv) $Q(x,p)$ are the independent sources.

Since we are going to study the causal dependence of state $\Psi(x,p)$ on the changes of independent sources $Q(x,p)$ and equation of motion (1) is linear, it is reasonable to assume: (A) The independent sources $Q(x,p)$ are localized within a sphere centered at the origin, of a radius $R_0 > 0$. They are absent before the time instant $t^{'}$ and after the time instant $t^{''} > t^{'}$. (B) The corresponding state $\Psi(x,p)$ satisfies the following initial and boundary conditions:

$$\Psi(x,p) = 0 \quad \text{if} \quad t < t^{'}, \tag{2}$$

and $\Psi(x,p) = 0$ if $|\vec{r}\,|$ is sufficiently large and either $p^0 \neq 0$ or $p^0 = 0$ and $\vec{p}\cdot\vec{r} < 0$. Thus, $\Psi(x,p) = 0$ if $|\vec{r}\,| \to \infty$ in such a way that $\vec{p}\cdot\vec{r} < 0$. Therefore, there are no incoming states at any time instant.

In what follows, we consider the explicit *retarded* solutions to a particulary simple local covariant transport equation (1) to *show the existence* of a classical causal physical system with covariant and local, linear equations of motion, which displays CFTLE and propagates certain property of the state $\Psi(x,p)$ (which we will refer to as the signal-field) not faster than light and almost like the retarded, Klein-Gordon Green's function in the limit $\lambda \to 0$. The specifications of this system are:

(i) The scattering operator

$$S_{-1}\Psi = f_0(p\cdot p)I_{p'}f_0^*(p'\cdot p')\Psi(x,p')$$

$$-f_1(p\cdot p)p\cdot I_{p'}f_1^*(p'\cdot p')p'\Psi(x,p') - \Psi(x,p), \tag{3}$$

where $p\cdot p = [p^0]^2 - |\vec{p}\,|^2$;

$$f_j(y) = A_j(\mu,\epsilon)\exp\left[-(y+4\mu^{-2})^2/2\epsilon^2\right], \quad j=0,1, \tag{4}$$

with $A_j(\mu,\epsilon)$ being positive normalization factors such that

$$\pi^2\int_0^\infty y|f_0(-y)|^2\mathrm{d}y = \frac{1}{4}\pi^2\int_0^\infty y^2|f_1(-y)|^2\mathrm{d}y = 1, \tag{5}$$

and μ and ϵ are two positive parameters; the integral over four-momentum,

$$I_pF(p) = \lim_{R\to\infty}\int_{-R}^{R}\mathrm{d}y\int_{|\vec{p}|^2\le R^2-y^2} F(\mathrm{i}y,\vec{p}\,)\mathrm{d}^3p, \tag{6}$$

is defined for any function $F(p) = F(p_0,\vec{p}\,)$ such that the limit (6) exists. The values of $\Psi(x,p)$ for complex values of the four-momentum variable p are defined by the analytic continuation of $\Psi(x,p)$ for $p\in\Re^{1,3}$. When Wick's rotation is permitted, the integral (6) of $F(p)$ over the four-momentum variable p is equivalent to the Lorentz-invariant, symmetric four-integral of $F(p)$ over p.

(ii) The scattering operator S_0 is the identity operator, i.e.,

$$S_0\Psi = \Psi(x,p). \tag{7}$$

(iii) The independent sources $Q(x,p)$ depend on the four-momentum p as follows:

$$Q(x,p) = f_0(p\cdot p)\varphi(x), \tag{8}$$

with $\varphi(x)$, $x\in\Re^{1,3}$, being a scalar, complex-valued signal-source localized within a sphere centered at the origin with radius $R_0>0$ and inactive before $t=t'$ and after $t=t''$. One can show that there is a local, covariant Lagrangian density corresponding to the equations of motion (1) – (8) [14].

2.2. Signal-field and its propagation

We assume that all observable and measurable properties of the state $\Psi(x,p)$ of the considered system are determined through its signal field

$$\varphi[x;\Psi] = I_p f_0^*(p\cdot p)\Psi(x,p), \quad x \in \Re^{1,3}, \tag{9}$$

an analogue of the fluid-dynamics variables in kinetic theory. To get an explicit expression for the signal-field $\varphi[x;\Psi]$ calculated from the equations of motion (1)-(8), we choose the parameter ϵ so small that it suffices to take into account only the zero-order terms in ϵ. We obtain the following covariant relation between the signal field $\varphi[x;\Psi]$ and its source $\varphi(x)$:

$$\varphi[x;\Psi] = -\varphi(x) + [C(\lambda)\mathcal{G}_{KG}(x;\mu_\lambda) - \mathcal{G}_\lambda(x)] * \nabla\cdot\nabla\varphi(x), \tag{10}$$

where $*$ denotes convolution with respect to the time-space variable x;

$$\mathcal{G}_{KG}(x;\mu) = \frac{\Theta(t)}{2\pi}\left[\delta(x\cdot x) + \frac{\mu J_1(\mu\sqrt{x\cdot x})}{2\sqrt{x\cdot x}}\Theta(x\cdot x)\right] \tag{11}$$

is the retarded, Klein-Gordon-Green's function, with $x\cdot x = c^2t^2 - |\vec{r}\,|^2$ and the unit step function $\Theta(t<0) = 0$ and $\Theta(t\geq 0) = 1$;

$$\mu_\lambda = \mu\sqrt{2}(1+\sqrt{1-4\lambda})^{-1/2}, \quad C(\lambda) = 1 + \lambda/\sqrt{1-4\lambda},$$

$$\mathcal{G}_\lambda(x) = \frac{2\lambda}{\pi(1-\lambda)}\int_1^\infty \frac{\sqrt{y-1}\mathcal{G}_{KG}(x,\frac{1-\lambda}{2\lambda}\mu\sqrt{y})}{(y-y_1)(y-y_2)}\mathrm{d}y, \tag{12}$$

$$y_{1,2} = \frac{2\lambda}{(1-\lambda)^2}[1\pm\sqrt{1-4\lambda}] \in [0,1),$$

provided $\lambda < \sqrt{5}-2$. From Eqs. (10) − (12) one can show that the value of the signal-source $\varphi(x)$ at $x = (ct_1, \vec{r}_1)$ may contribute to the value of the signal-field $\varphi[x;\Psi]$ carried by the state $\Psi(x,p)$ at $x = (ct_2, \vec{r}_2)$ only if the relativistic causality condition

$$|\vec{r}_2 - \vec{r}_1| \leq c(t_2 - t_1) \tag{13}$$

is satisfied. Hence the scalar, complex-valued state $\Psi(x,p)$, which is produced by the signal-source $\varphi(x)$, does not propagate faster than light the signal field $\varphi[x;\Psi]$; in particular, from (8) and (13) one obtains

$$\varphi[x;\Psi] = 0 \quad \text{if} \quad |\vec{r}\,| > R_0 + c(t - t^{'}). \tag{14}$$

In the limit $\lambda \to 0$, the behaviour of the Green's function $\mathcal{G}_{KG}(x;\mu)$ for large μ implies that $\mathcal{G}_\lambda(x) = O(\lambda^{1/2})$. Thus, in the limit $\lambda \to 0$, the signal field is given by

$$\varphi[x;\Psi] = -\mu^2 \mathcal{G}_{KG}(x;\mu) * \varphi(x), \tag{15}$$

and it satisfies *the Klein-Gordon equation*

$$\nabla \cdot \nabla\varphi[x;\Psi] + \mu^2\varphi[x;\Psi] = -\mu^2\varphi(x). \tag{16}$$

2.3. Causal faster-than-light effects

Taking account of Eqs. (1), (2), (8) and (14), we can express the state $\Psi(x,p)$ in terms of the signal-source $\varphi(x)$ and signal-field $\varphi[x;\Psi]$ as

$$\Psi(x,p) = \Theta(t - t^{'}) \int\limits_0^{\pm\infty} e^{-(\lambda^{-1}-1)y} q(x - yp, p)\mathrm{d}y, \tag{17}$$

with

$$q(x,p) = f_0(p\cdot p)\{\varphi(x) + \lambda^{-1}\varphi[x;\Psi]\}$$

$$-(\mu/\lambda) f_1(p\cdot p)[C(\lambda)\mathcal{G}_{KG}(x;\mu_\lambda) - \mathcal{G}_\lambda(x)] * p\cdot\nabla\varphi(x), \tag{18}$$

where the upper limit is $+\infty$ if $p^0 \geq 0$, and $-\infty$ if $p^0 < 0$ [13]. So the values of $q(x,p)$ and, therefore, the values of the signal-source $\varphi(x)$ at $x = (ct_1, \vec{r}_1)$ affect instantaneously the values of the state $\Psi(x,p)$ for certain four-momenta $p = (p^0 = 0, \vec{p}\,)$ at $x = (ct_2, \vec{r}_2)$, $t_2 = t_1$, regardless of the distance $|\vec{r}_2 - \vec{r}_1|$. Therefore, there exist *instantaneous effects* of the signal-source $\varphi(x)$ on the state $\Psi(x,p)$ of the causal physical system considered. In addition, the relation (17) shows that the value of the signal-source $\varphi(x)$ at $x = (ct_1, \vec{r}_1)$ affects the state $\Psi(x,p)$ for certain values of four-momentum p at $x = (ct_2, \vec{r}_2)$, $t_2 > t_1$, however small is the time interval $t_2 - t_1$ and/or however large is the distance $|\vec{r}_2 - \vec{r}_1|$. Therefore, the state $\Psi(x,p)$ displays CFTLE due to the signal-source $\varphi(x)$. For $t_2 > t_1$ and $p^0 < 0$ the effects of signal-source $\varphi(x)$ on the state $\Psi(x,p)$ do not decrease with distance $|\vec{r}_2 - \vec{r}_1|$; but from Eqs. (2), (14), (17) and (18),

$$\Psi(x,p) = 0 \quad \text{if} \quad |\vec{r}| > R_0 + \max(1, |\vec{p}/p^0|)c(t - t^{'}). \tag{19}$$

Thus, we have demonstrated that one can construct classical, linear, covariant equations of motion of a causal physical system with a local and covariant Lagrangian density, whose scalar, complex-valued state $\Psi(x,p)$ exhibits CFTLE, though the corresponding signal-field $\varphi[x;\Psi]$, which determines the observable properties of the state $\Psi(x,p)$, does not propagate faster than light and satisfies closely the Klein-Gordon equation as the parameter $\lambda \to 0$.

We obtained similar results also for analogous models of causal physical systems with local and covariant equations of motion whose states $\Psi(x,p)$ are spinor or four-vector, complex-valued fields of $x, p \in \Re^{1,3}$. These classical models with local, covariant Lagrangian densities display CFTLE but their solutions propagate certain properties of their states not faster than light and almost like the Dirac, Proca or Maxwell partial differential equations when $\lambda \to 0$.

3. Concluding remarks

We have shown that it is possible to reconcile classically causal faster-than-light effects, properties of state that do not propagate faster than light, and special relativity without abandoning our traditional belief that physical phenomena are causal[1] and basic equations of motion local in time-space and covariant. To this end we put forward, in effect, a new hypothesis: Physical phenomena are due to streaming and scattering of hypothetical pointlike entities, their speeds $c|\vec{p}/p^0|$ are not bounded and are causing CTFLE. The basic equations of motion are local and covariant transport equations such as Eq. (1), the partial differential equations of field theories providing only an averaged, macroscopic information that suffices to explain the experimental results obtained so far. That possibility was hinted at already by Feynman about thirty years ago [15].

The reason why the presented transport-theoretical model exhibits only CFTLE and never non-causal effects as expected by direct application of the Lorentz time-space transformations is as follows: In classical relativistic field theories, when studying how fields depend on their sources through certian linear, covariant equations of motion, it seems physically reasonable to make the following two covariant assumptions: (A) The sources are absent (i) before an initial time instant $t^{'}$, (ii) after certain time instant $t^{''} > t^{'}$, and (iii) outside a sphere with radius R_0 centered at a point $\vec{r}_0$. (B) There are no fields (i) before the time instant $t^{'}$, and (ii) outside the sphere with radius $R = R_0 + c(t - t^{'})$, $t \geq t^{'}$, and centered at the point $\vec{r}_0$. As a consequence, the relation between such fields and their sources is covariant, since equations of motion, and the initial and boundary conditions are all covariant. In our case, the signal-source $\varphi(x)$ and the signal-field $\varphi[x;\Psi]$ do satisfy such covariant assumptions, and dependence of the signal-field $\varphi[x;\Psi]$ on the signal-source $\varphi(x)$ is covariant. In contrast, the state $\Psi(x,p)$ satisfies the initial and boundary conditions (2) that are not covariant, though the assumed changes of independent sources $Q(x,p)$ defined by (8) are covariant; as a consequence, the relation between the state $\Psi(x,p)$ and its source $Q(x,p)$ is not covariant. In contrast to the de Broglie-Bohm theory [8,9] that uses non-covariant equations of motion to explain how a physical system can exhibit solely CFTLE and no non-causal effects, the proposed transport-theoretical model does not need a preferred inertial reference frame to explain such paradoxical phenomena. To this end it uses only non-covariant initial and boundary conditions for separating the part of the state due to the changes of independent sources from the part of the state due to the incoming states.

Two questions remain open: (A) Which transport-theoretical extension of classical field theories are relevant, i.e., which particular scattering operators S_{-1} and S_0 and independent sources Q correspond to real physical phenomena? (B) Are we to quantize, if possible, the proposed transport theory through path-integral method, or simply by replacing the Feynman propagators of quantum field theories with propagators of those transport-theoretical signal-fields that satisfy in the strong scattering limit ($\lambda \to 0$) the same basic field equation as the Feynman propagators do, namely the Klein-Gordon, Dirac or Proca equations [11].

Acknowledgement

The authors are grateful to their colleague M. Poljšak for many helpful remarks. This work was supported by the Slovenian Ministry of Science and Technology grants.

References

1) J. S. Bell, *Speakable and Unspeakable in Quantum Mechanics* (Cambridge University Press, Cambridge, 1987), Chs. 2 and 18;
2) A. Sudbery, *Quantum Mechanics and the Particles of Nature* (Cambridge University Press, Cambridge, 1988), Sects. 5.3−5.5;
3) B. d'Espagnat, in *Quantum Theory and Pictures of Reality*, edited by W. Schommers (Springer, Berlin, 1989), Ch. 3; P. Eberhard, *ibid*, Ch. 2; W. Schommers, *ibid*, Ch. 1;
4) G. C. Hegerfeldt, Phys. Rev. Lett. **72** (1994) 596;
5) J. F. Clauser, M. A. Horne, A. Shimony and R. A. Holt, Phys. Rev. Lett. **23** (1969) 880;
6) P. H. Eberhard, Nuovo Cimento **46**B (1978) 392, Sect. 4.2;
7) L. Hardy, Phys. Rev. Lett. **68** (1992) 2981;
8) D. Bohm, B. Hiley and P. Kaloyerou, Phys. Rep. **144** (1987) 321;
9) P. R. Holland, Phys. Rep. **224** (1993) 95, Eq. (3.16);
10) J. D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1975), 2nd ed., Sect. 11.1;
11) C. Itzykson and J. B. Zuber, *Quantum Field Theory* (McGraw-Hill, Singapore, 1987), Sect. 1-1-2 and Eqs. (1-178), (2-115), (2-111), (A-53b) and (3-134);
12) J. P. Vigier, in *Open Questions in Quantum Physics*, edited by G. Tarozzi and A. van der Merwe (Riedel, Dordrecht, 1985), p. 297;
13) M. M. R. Williams, *Mathematical Methods in Particle Transport Theory* (Butterworths, London, 1971), Sects. 2.3, 2.7 and 2.8;

14) M. Ribarič and L. Šušteršič, *Transport Theory and Statistical Physics*, in print (1995);
15) R. P. Feynman, R. B. Leighton and M. Sands, *The Feynman Lectures on Physics*, Vol. II (Addison-Wesley, Reading, Mass., 1965), Sect. 12-7.

SPECIJALNA TEORIJA RELATIVNOSTI I KAUZALNI BRŽI-OD-SVJETLOSTI UČINCI

MARIJAN RIBARIČ i LUKA ŠUŠTERŠIČ

Jožef Stefan Institute, University of Ljubljana, 61111 Ljubljana, Slovenia

UDK 530.12

PACS 03.30.+p, 03.65.Bz, 11.10.Kk, 05.60.+W

Tumačenje različitih kvantno-mehaničkih, teorijskih i eksperimentalnih rezultata ukazuje na postojanje kauzalnih bržih-od-svjetlosti učinaka. Da bi pokazali kako se ti učinci mogu uskladiti sa specijalnom teorijom relativnosti, navodimo primjer klasičnoga kauzalnog sustava s lokalnim, kovarijantnim jednadžbama gibanja, čija retardirana rješenja pokazuju kauzalne brže-od-svjetlosti učinke. Neka svojstva tih rješenja šire se gotovo prema Klein-Gordonovoj jednadžbi, ali ne brže od svjetlosti.