ISSN 1330–0016
CODEN FIZBE7

SPIN POLARIZATION PRECESSION AS A PROBE TO THE FUNDAMENTAL ORIGIN OF QUANTUM PHENOMENA

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Received 24 March 1994

UDC 530.145

PASC 03.65.Bz

By allowing a spin system to evolve according to discrete time dynamical processes, we calculate the value of the wave function at any time, given the initial value of the wave function. We also calculate the value of the x spin polarization at any discrete time and compare it with the value of the x spin polarization for the continuous-time case.

1. *Introduction*

For the ever searching student of quantum theory one persistent trademark of quantum processes stands out, namely, the discrete nature of quantum jumps and discrete nature of the eigenspectrum of a particle confined to a limited region of space and subject to quantum laws. It is only natural to ask if space and time themselves might be discrete in nature at some level and if the continuum is some low energy average when the space-time points are close together. To the mathematician, the subject of combinatorics and graph theory provide a natural setting to express a discrete space-time picture of the world [1,2]. Wheeler [3] quite long ago emphasized the discrete nature of space and time at some level and Finkelstein [4] has constructed an imaginative picture of a "quantum net" wherein the world is discrete and the continuum of space-time and quantum fields emerges after an averaging process. In a historical sense both Synder [5] and t'Hooft [6] suggested the use of a discrete space-time lattice to study the properties of QED and quantum gravity wherein a finite lenght renders these theories finite and calculable. In a quite another direction T.D. Lee [7] has advocated the calculation of path integrals in terms of a finite discrete time so as to eliminate the arbitrary nature of the measure in the path integral approach. In the above investigations a truly discrete space-time lattice structure was introduced that merges with the continuum when the separation between adjoining points goes to zero. Bombelli et al. [8] have discussed how a discrete causal set merges to the continuum of Minkowski space through the algebraic and topological relationship of points.

If space and time themselves do form a continuum, but a particle's position and time coordinate are uncertain due to a microscopic uncertainty principle that forbids a response of the wave function at the time and point of application of the Hamiltonian, then finite differences should replace derivates in the quantum equations of motion and the theory becomes a discrete space-time difference theory [9]. We have applied this idea to electron spin resonance [10], electron spin polarization [11], spectral shifts in hydrogen [12], as well as neutron interferometry [13]. Actually, at a submicroscopic level both of the above mentioned notions of discreteness might be operative. If this is the case, then we might ask if there is a fundamental discrete space-time dynamics, or is dynamics itself governed by a stochastic or Markov type process. A Markov process seems most fundamental since it does not in any way depend on the past history of the particle but is only sensitive to the transition from one point to the next. In what follows, we study the temporal evolution of a spin system in a z component magnetic field using three different dynamical schemes of discreteness. The first is a discrete time difference modified Schrödinger approach originally pioneered by Caldirola [14,15] and later studied by Santilli et al. as fitting into a Lie admissible structure [16,17]. We next study the time evolution of the spin system in a dynamical scheme that replaces the Schrödinger equation with a truly discrete time theory. In the third scheme we consider the spin system to evolve according to the usual continuous time theory with an environmental Markov influence with each jump representing a discrete time jump. In all these schemes we evaluate the expectation value of the x spin polarization and compare it with that calculated in the normal Schrödinger approach. At present, experimental studies find it difficult to probe for the discreteness. It is hoped that the following calculations will provide a motivation for the experimental community to look for observational consequences of discretness in physics through spin polarization phenomena.

2. *Spin polarization precession in discrete time physics*

We begin our analysis by considering a discrete time difference theory of the form suggested in Refs. 14-17. The Schrödinger equation is replaced by

$$H\Psi = \mathrm{i}\hbar \frac{[\Psi\,(t+\tau/2) - \Psi\,(t-\tau/2)]}{\tau}, \tag{2.1}$$

where τ is discrete time interval.

For the Hamiltonian of a spinning electron in a z component magnetic field we have

$$H = \frac{e\hbar}{2m}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} B. \tag{2.2}$$

The eigenstates are

$$\Psi = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} T(t).$$

Eq. (2.1) gives

$$\frac{e\hbar}{2m}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} B \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} T(t) = E \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} T(t) = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \mathrm{i}\hbar \frac{[T(t+\tau/2) - T(t-\tau/2)]}{\tau}. \tag{2.3}$$

The eigenvalues are

$$E_{\pm} = \pm \frac{e\hbar B}{2m},$$

with wave functions

$$\Psi_{+} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \exp\left[-\frac{2}{\tau} \sin^{-1}(\frac{E_{+}\tau}{2\hbar}) \mathrm{i}t \right] \tag{2.4}$$

$$\Psi_{-} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \exp\left[-\frac{2}{\tau} \sin^{-1}(\frac{E_{-}\tau}{2\hbar}) \mathrm{i}t \right].$$

For a state that was initially polarized in the x direction we have

$$\Psi = \begin{pmatrix} \frac{1}{\sqrt{2}} \exp\left[-\frac{2}{\tau} \sin^{-1}(\frac{E_{+}\tau}{2\hbar}) \mathrm{i}t \right] \\ \frac{1}{\sqrt{2}} \exp\left[-\frac{2}{\tau} \sin^{-1}(\frac{E_{-}\tau}{2\hbar}) \mathrm{i}t \right] \end{pmatrix}. \tag{2.5}$$

For the x spin polarization at time t we have

$$< S_x > = \Psi^{+} S_x \Psi = \Psi^{+} \frac{\hbar}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \Psi = \frac{\hbar}{2} \cos\left(\frac{4}{\tau} \sin^{-1} \frac{eB\tau}{4m} \right) t \tag{2.6}$$

where

$$\omega = \frac{4}{\tau} \sin^{-1}\left(\frac{eB\tau}{4m} \right) \simeq \frac{4}{\tau}\left[\frac{eB\tau}{4m} + \frac{1}{3!}\left(\frac{eB\tau}{4m} \right)^3 + ... \right]. \tag{2.7}$$

We see here, as in Ref. 11, that the spin polarization frequency has the usual continuous time value plus corrections due to the discrete time difference dynamics embodied in Eq. (2.1).

As a second approach to the discreteness we assume that the temporal component of the wave function advances in truly discrete time steps. Then Eq. (2.1) is replaced by

$$H\Psi(t_n) = \mathrm{i}\hbar \frac{[\Psi(t_n + \tau/2) - \Psi(t_n - \tau/2)]}{\tau} \tag{2.8}$$

where we assume the discrete time points to be given by

$$t_n = \frac{n\tau}{2}, \quad t_n + \frac{\tau}{2} = \frac{\tau}{2}(n+1), \quad t_n - \frac{\tau}{2} = \frac{\tau}{2}(n-1).$$

By choosing units that $\tau/2 = 1$, we have

$$H\Psi_n = \frac{\mathrm{i}\hbar}{2}\left[\Psi_{n+1} - \Psi_{n-1}\right]. \tag{2.9}$$

We assume

$$\Psi_n = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} U_n.$$

We still have for the spin component wave function for the eigenstates $(+, -)$

$$\Psi_+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad E_+ = \frac{e\hbar B}{2m}, \quad \Psi_- = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad E_- = -\frac{e\hbar B}{2m} \tag{2.10}$$

and for the discrete time difference component of the wave function we have for E_+

$$E_+ U_{+n} = \frac{\mathrm{i}\hbar[U_{+n+1} - U_{+n-1}]}{2}. \tag{2.11}$$

With the substitution

$$U_{+n} = cr^n$$

we obtain

$$E_+ = \frac{\mathrm{i}\hbar r}{2} - \frac{\mathrm{i}\hbar r^{-1}}{2}$$

$$r^2\left(\frac{\mathrm{i}\hbar}{2}\right) - rE_+ - \frac{\mathrm{i}\hbar}{2} = 0$$

$$r_\pm = \frac{E_+ \pm \sqrt{E_+^2 - \hbar^2}}{\mathrm{i}\hbar} = -\frac{\mathrm{i}E_+}{\hbar} \mp \frac{\mathrm{i}}{\hbar}\sqrt{E_+^2 - \hbar^2}.$$

Hence

$$r_+ = \left(\frac{E_+}{\hbar} + \sqrt{\frac{E_+^2}{\hbar^2} - 1}\right) \exp\left(\frac{\mathrm{i}3\pi}{2}\right)$$

$$r_- = \left(\frac{E_+}{\hbar} - \sqrt{\frac{E_+^2}{\hbar^2} - 1}\right) \exp\left(\frac{\mathrm{i}3\pi}{2}\right). \tag{2.12}$$

In the original units

$$r_+ = \left(\frac{E_+\tau}{2\hbar} + \left(\frac{E_+^2\tau^2}{4\hbar^2} - 1\right)^{1/2}\right) \exp\left(\frac{\mathrm{i}3\pi}{2}\right)$$

$$r_- = \left(\frac{E_+\tau}{2\hbar} - \left(\frac{E_+^2\tau^2}{4\hbar^2} - 1\right)^{1/2}\right) \exp\left(\frac{\mathrm{i}3\pi}{2}\right). \tag{2.13}$$

Since τ is expected to be small we have

$$\frac{E_+\tau}{2\hbar} \ll 1,$$

then

$$r_+ = \left(\frac{E_+\tau}{2\hbar} + \mathrm{i}\left(1 - \frac{E_+^2\tau^2}{4\hbar^2}\right)^{1/2}\right) \exp\left(\frac{\mathrm{i}3\pi}{2}\right) \approx -\frac{\mathrm{i}E_+\tau}{2\hbar} + \left(1 - \frac{E_+^2\tau^2}{4\hbar^2}\right)^{1/2} \approx 1 - \frac{\mathrm{i}E_+\tau}{2\hbar}. \tag{2.14}$$

Also

$$r_- \approx -\left(1 + \frac{\mathrm{i}E_+\tau}{2\hbar}\right). \tag{2.15}$$

We note that

$$(r_+)^n \approx \left(1 - \frac{\mathrm{i}E_+\tau}{2\hbar}\right)^n = \exp\left[n \ln_e\left(1 - \frac{\mathrm{i}E_+\tau}{2\hbar}\right)\right] \approx \exp\left(-\frac{\mathrm{i}E_+ n\tau}{2\hbar}\right) = \exp\left(-\frac{\mathrm{i}E_+ t_n}{\hbar}\right). \tag{2.16}$$

Since

$$\frac{n\tau}{2} = t_n, \quad \text{(for large } n).$$

This is the usual continuous time solution for large n. In the case of discrete time we have both solutions r_+, r_-, thus

$$U_{+n} = \frac{1}{\sqrt{2}}(1 - \epsilon_1) \exp\left(-\frac{\mathrm{i}E_+ n\tau}{2\hbar}\right) + \frac{\epsilon_1}{\sqrt{2}}(-1)^n \exp\left(\frac{\mathrm{i}E_+ n\tau}{2\hbar}\right) \tag{2.17}$$

$$\text{for large } n,\ n\tau/2 \to t_n$$

where ϵ_1 is a small variable parameter measuring the contribution of the solution $(r_-)^n$.

$$\left[-1\left(1+\frac{\mathrm{i}E_+\tau}{2\hbar}\right)\right]^n = (-1)^n \exp n \ln\left(1+\frac{\mathrm{i}E_+\tau}{2\hbar}\right) \approx (-1)^n \exp\left(\frac{\mathrm{i}E_+ n\tau}{2\hbar}\right)$$

$$\text{for large } n,\ n\tau/2 \to t_n.$$

In constructing the solution in Eq. (2.17) we have included both the solutions r_+ and r_- from Eq. (2.13). Since discrete time corrections to spin polarization precession frequencies and the sinusoidal dependence of the x spin polarization must be small from known measurements [18-20], contributions from the r_- solution must also be small since the known continuous results require only the solution r_+ in the continuous limit

$$n\tau/2 \to t_n \ (n \text{ large}).$$

We have first assumed that ϵ_1 is only measurable to first order and to ensure normalization at $n = 0$, we must have

$$U_{+n}(n=0) = \frac{1}{\sqrt{2}}.$$

For the first order effects in ϵ_1, the form of the solution in Eq. (2.17) is the only one allowable to ensure

$$U_{+n}(n=0) = \frac{1}{\sqrt{2}},$$

and similarly for $U_{-n}(n = 0)$, where normalization requires

$$(U_{+n}(n=0))^2 + (U_{-n}(n=0))^2 = 1.$$

Thus at n=0

$$U_{+n} = \frac{1}{\sqrt{2}}.$$

Also for E_- we have

$$U_{-n} = \frac{1}{\sqrt{2}}(1-\epsilon_2)\exp\left(-\frac{\mathrm{i}E_- n\tau}{2\hbar}\right) + \frac{\epsilon_2}{\sqrt{2}}(-1)^n \exp\left(\frac{\mathrm{i}E_- n\tau}{2\hbar}\right). \tag{2.18}$$

At n=0 we also obtain

$$U_{-n} = \frac{1}{\sqrt{2}}.$$

If we evaluate $< S_x >$ we have for an equal mixture of U_{+n} and U_{-n}

$$< S_x >_{n=0}= \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) \frac{\hbar}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \frac{\hbar}{2} \quad at \;\; n = 0.$$

Here we have assumed a linear superposition of Eq. (2.17) and Eq. (2.18) for the spin function to give $< S_x >_{n=0}= \hbar/2$. For any n we have

$$< S_x >= \frac{\left(U^*_{+n} U^*_{-n}\right) \frac{\hbar}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} U_{+n} \\ U_{-n} \end{pmatrix}}{\left(U^*_{+n} U^*_{-n}\right) \begin{pmatrix} U_{+n} \\ U_{-n} \end{pmatrix}} \tag{2.19}$$

$$< S_x >= \frac{\frac{\hbar}{2}(1 - \epsilon_1 - \epsilon_2) \cos\left(\frac{eB}{m}\frac{n\tau}{2}\right) + (\epsilon_1 + \epsilon_2)\frac{\hbar}{2}(-1)^n}{\left[1 - \epsilon_1 - \epsilon_2 + (-1)^n(\epsilon_1 + \epsilon_2) \cos\left(\frac{eB}{m}\frac{n\tau}{2}\right)\right]}. \tag{2.20}$$

Setting

$$\left(\frac{n\tau}{2} \to t\right)$$

we obtain

$$< S_x >= \frac{\frac{\hbar}{2}(1 - \epsilon_1 - \epsilon_2) \cos\left(\frac{eBt}{m}\right) + (\epsilon_1 + \epsilon_2)\frac{\hbar}{2}(-1)^n}{\left[1 - \epsilon_1 - \epsilon_2 + (-1)^n(\epsilon_1 + \epsilon_2) \cos\left(\frac{eBt}{m}\right)\right]}. \tag{2.21}$$

If we have $\epsilon_1 = \epsilon_2 = 0$, we have the usual continuous time value for $< S_x >$.

Eq. (2.21) represents a formula which has small chaotic variations of $< S_x >$ due to the term

$$(\epsilon_1 + \epsilon_2)\frac{\hbar}{2}(-1)^n$$

which arises in the truly discrete time formalism. Thus, any small unexplained chaotic fluctuations from the usual continuous value of

$$< S_x >= \frac{\hbar}{2} \cos\left(\frac{eB}{m}\right) t \tag{2.22}$$

might be evidence for an underlying discrete time dynamics. Eq. (2.21) represents a formula that can be used to ascertain both the qualitative and quantitative deviations from the continuous time formula for x spin polarization as a function of time.

In the third approach to a discrete time quantum dynamics we assume a Markov process operative between the up and down spin states. If p = probability of a jump from the down to the up state, q = probability of a jump from the up to the down state, and if the probabilities at t=0 are 1/2, 1/2 for the up and down states, we have after n steps

$$P(+)_n = \frac{p}{p+q} + (1-p-q)^n \left(\frac{1}{2} - \frac{p}{p+q} \right) \tag{2.23}$$

$$P(-)_n = \frac{q}{p+q} + (1-p-q)^n \left(\frac{1}{2} - \frac{q}{p+q} \right). \tag{2.24}$$

Here $P(+)_n$ = probability of the up state after n steps and $P(-)_n$ = probability of the down state after n steps. We also assume thet the Markov jump process is operative in addition to the usual quantum Schrödinger formalism and write the wave function as

$$\Psi(t) = \begin{pmatrix} \sqrt{P(+)_n} \exp\left(-\frac{\mathrm{i}E_+}{\hbar} t \right) \\ \sqrt{P(-)_n} \exp\left(-\frac{\mathrm{i}E_-}{\hbar} t \right) \end{pmatrix}. \tag{2.25}$$

Here $t = n\tau$, where τ is a fundamental interval of time.

For the x spin polarization we have

$$< S_x >= \Psi^+ \frac{\hbar}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \Psi = \hbar \sqrt{P(+)_n P(-)_n} \cos\left(\frac{eB}{m} \right) t. \tag{2.26}$$

We see that the effect of the Markov process is to alter the amplitude of the x spin polarization in a chaotic manner. The distinct signature of a Markov process would be an amplitude that fluctuates in a manner according to Eq. (2.26).

3. *Conclusions*

We have seen that Eq. (2.6), Eq. (2.20) and Eq. (2.26) give distinct formulae for the x spin polarization which have very specific signatures in each case. According to Eq. (2.6), the frequency of precession would be a nonlinear function of the external magnetic field, Eq. (2.20) would describe a behaviour for $< S_x >$ that is almost like the continuous time case except for the chaotic term varying as

$$(\epsilon_1 + \epsilon_2)(-1)^n \frac{\hbar}{2}$$

in the numerator and the term

$$(-1)^n (\epsilon_1 + \epsilon_2) \cos\left(\frac{eB}{m} \right) \frac{n\tau}{2}$$

in the denominator. The small parameters ϵ_1, ϵ_2 specify to what degree the second discrete time solution in Eq. (2.15) contributes. Eq. (2.26) would simply give a chaotic variation of the amplitude of the x component of spin polarization with the usual x component spin precession term providing a uniform background for the chaotic behaviour of the amplitude. Probably the best way to look for these discrete time effects would be to study the secondary effects generated by this spin precession. Such an effect might be the polarization induced by precessing particles in colliding $e^+ \ e^-$ pairs that generates a neutral current. The cross sections for various particle channels are sensitive to the polarization induced by the precession of particles (electrons, protons, etc.). With a continuous beam of initially unpolarized $e^+ \ e^-$ the products of the reaction would mimic the polarization at the time the reaction occured, i.e. the temporal behaviour of the spin precessing particles in the magnetic field. Another probe to these discrete time effects might be the study of secondary effects generated by a spin precessing particle in the atmosphere of a pulsar where B is high. In this setting we might have an electron spin resonance effect that could absorb outgoing gamma rays and leave missing lines in the emission spectrum. A careful spectral analysis of the gamma ray spectrum from the pulsar might thus be a probe of the discrete time effects. More concrete evidence for the discreteness might be found in experiments that probe for 4π rotation of a spinor to generate constructive interference in a neutron interferometer or electron interferometer [21-26]. The basic idea is outlined in Ref. 27 where two coherent beams of unpolarized particles are considered, allowing one to propagate through a magnetic field in a z direction and allowing the other to propagate without a magnetic field and then recombining them to interfere constructively. The beam in the magnetic field must undergo a 4π rotation of its phase. Since the parameters used in constructing the interferometer are known (magnetic field B, path line L) a comparison between the predicted phase change required for constructive interference and that which is experimentally observed can be made. In Ref. 27, a value of 716.8 $\pm$ (3.8) degrees was found experimentally assuming the usual continuous time precession frequency. The discrepancy could be attributed to discrete time effects and also possibly to a violation of the perfect SU(2) symmetry for spinors. It is hoped that the experimental community might suggest further tests for discrete time effects that lead to tests for the discrete time quantum dynamics.

Acknowledgement

The author would like to thank the Physics Department at Williams College and Harvard University for the use of their facilities.

References

1) M. Loebl, Discrete Mathematics (North Holland) **108** (1992) 333;

2) P. Pudlak and V. Rodl, Combinatorica **12(2)** (1992) 221;

3) J. Wheeler, Chapter on *Quantum Theory and Gravitation*, Proceedings of a symposium held at Loyala University, New Orleans, May 23-26, 1979 (Academic, New York, 1980);

4) D. Finkelstein, Int. J. of Theoretical Physics **27** (No. 4) (1985) 473;

5) H. S. Snyder, Phys. Rev. **71** (1947) 38;

6) G. t'Hooft, Phys. Rep. **104** (1984) 133;

7) T. D. Lee, Phys. Let. **122B** (1983) 217;

8) L. Bombelli, J. Lee, D. Meyer and R. Sorkin, Preprint IA SSNS-HEP-87/23, Inst. of Advanced Study, Princeton, NJ (1987);

9) E. Recami, personal communication at Fourth Workshop on Hadronic Mechanics and Non-Potential Interactions, 22-26 Aug. 1988, Skopje; (Nova Sci. Pub., N.Y.,1990);

10) C. Wolf, Phys. Let. **A123** (1987) 208;

11) C. Wolf, Il Nuovo Cimento Note Brevi **100B** (1987) 431;

12) C. Wolf, Eur. J. of Physics **10** (1989) 197;

13) C. Wolf, Foundations of Physics **20** (No. 1) (1990) 133;

14) P. Caldirola, Suppl. Nuovo Cimento **3** (1956) 297;

15) P. Caldirola, Lett. Nuovo Cimento **16** (1976) 151;

16) R. Mignani, H. C. Myung and R. M. Santilli, Hadronic J. **6** (1983) 1973;

17) A. Jannussis, C. Brodimas, V. Papatheou, C. Karayannis, P. Pangopoulos and W. Ioannidou, Hadronic J. **6** (1983) 1434;

18) P. B. Schwinberg, R. S. Van Dyck, Jr. and H. G. Dehmelt, Phys. Rev. Lett. **47** (1981) 1679;

19) R. S. Van Dyck, Jr., Bull. Am. Phys. Soc. **24** (1979) 758;

20) F. J. M. Farely and E. Picasso, Ann. Rev. Nuc. and Part. Sci. **29** (1979) 243;

21) Y. Aharanov and L. Susskind, Phys. Rev. **158** (1967) 1237;

22) H. J. Bernstein, Phys. Rev. Lett. **18** (1967) 1102;

23) G. C. Hegerfeld and K. Kras, Phys. Rev. **170** (1968) 1187;

24) G. T. Moore, Am. J. Phys. **38** (1970) 1177;

25) E. Drope, Z. Naturforschung **29a** (1974) 1117;

26) A. G. Klein and G. I. Opat, Phys. Rev. **D 11** (1975) 523;

27) H. Rauch, A. Wilfing, W. Bauspiess and U. Bonse, Z. Physik B **29** (1978) 281.

PRECESIJA SPINSKE POLARIZACIJE KAO PROBA OSNOVNOG PORIJEKLA KVANTNIH POJAVA

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UDK 530.145

PASC 03.65.Bz

Dozvoljavajući razvoj spinskog sistema u skladu s dinamičkim procesima diskretnog vremena, izračunali smo vrijednost valne funkcije u bilo koji trenutak uz danu početnu vrijednost. Također smo izračunali vrijednost polarizacije x spina za bilo koji diskretni trenutak i usporedili ga s vrijednošću za slučaj neprekinutog vremena.