ISSN 1330–0016
CODEN FIZBE7

ON THE ISOBARIC MULTIPLET WIDTH EQUATION[1]

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Received 3 August 1993

UDC 539.12

Original scientific paper

The isobaric multiplet width equation is verified on the lights of newer experimental information on masses and widths. The verification is carried out for the known level widths and the results came in support of the isobaric width equation removing all previous discrepancies. Multiplets with enough, but not complete, experimental information are also studied.

1. *Introduction*

In 1982 Awin and Shanley developed an isobaric multiplet level-width equation analogous to the isobaric mass formula. The results attained are that a quadratic equation in T_z (the z-component of the isospin) is obtained for the level widths if the relevant multiplet is hadronically unstable with respect to the nearest particle-decay channel; and a quartic equation in T_z is obtained if the masses are hadronically stable. Namely, if the isospin symmetry breaking term in the Hamiltonian is an isovector and an isotensor and if it can be treated perturbatively, then the level widths of an isobaric multiplet follow a quadratic or quartic equation in T_z. This

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depends on whether the hadronic decay conserves or violates the isospin. Accordingly, the purpose of the basic paper and the present update is to demonstrate this feature.

The hadronic multiplet mass m_0 is extracted using the available information on physical nuclear masses and the formula ($c = 1$)

$$m = m_0 + (m_n - m_p)T_z + E_c \tag{1}$$

through least squares fitting, and where the Coulomb energy E_c is estimated using Sengupta model; namely E_c is given by

$$E_c = e^2 \left[0.6z^2 - 0.46z^{4/3} - (1 - (-1)^z)\, 0.15\right] /r_0 A^{1/3} \tag{2}$$

r_0 is, also, a parameter of the fit.

The experimental information, available then, on widths has been analyzed to test the validity of the isobaric multiplet width equation (IMWE) and in general it was found that in most cases the quadratic-quartic behaviour of the widths is correlated with the hadronic instability (or stability) of the multiplet masses.

However, it is to be noted that one discrepancy occured which was owed either to the crudeness of the model used to estimate E_c or to Δm being small so as Coulomb effects may be strong and the perturbation treatment of the widths is not acceptable.

Now though the above and other inconclusiveness has been removed by introducing a simple model describing a quartet-doublet and a sextet-quartet situations, and where it was found that the quadratic-quartic nature of the widths was borne out in the model (Awin and Shanley 1982); newer information on masses (Wapstra and Audi 1985) and level widths (Ajzenberg-Selove 1985–1988) make it worthwhile to redo the calculations for the available data and check the IMWE.

Therefore in the next Section we check the available widths for being quadratic or quartic; and since no complete sextet is available, our conclusions about any particular multiplet widths are whether they are quadratic or not.

In Section 3 we extract the hadronic mass m_0 and the radius parameter r_0 corresponding to the multiplets of Section 3.

In Section 4, and taking the IMWE for granted, we study some of the level widths for multiplets with enough experimental information.

Finally, in Section 5 a concluding discussion is given.

2. *Experimental widths*

Here we analyze the multiplets with completely known experimental widths to extract their algebraic T_z-dependence; and where we assume that the radiative partial widths are negligible. Namely, the total widths, experimentally quoted, are taken to be particle-decay widths.

The isobaric quartets which have complete, or sufficient, information are those of mass numbers $A = 7, 9, 11, 13, 15, 17$ and 21. Their experimental widths are given in Table 1. Table 2 contains the available experimental widths for $A = 8, 12$ and 16 quintets.

Table 1. Experimental quartet widths (keV)

A	J^π	$\Gamma(3/2)$	$\Gamma(1/2)$	$\Gamma(-1/2)$	$\Gamma(-3/2)$
7^a	$3/2^-$	160 ± 30	260 ± 35	320 ± 30	$1\,400 \pm 200$
9^a	$3/2^-$	0	0.381 ± 0.033	0.395 ± 0.042	0
$11^{b,c}$	$1/2^+$	0	210 ± 20	270 ± 50	broad
$11^{b,d}$	$1/2^-$	0	155 ± 25	490 ± 40	740 ± 100
13^e	$3/2^-$	0	5.49 ± 0.25	0.86 ± 0.12	0
$15^{e,c}$	$1/2^+$	0	405 ± 6	–	$1\,000 \pm 200$
$15^{e,d}$	$5/2^+$	0	14 ± 5	135 ± 15	0.24 ± 0.03
17^f	$1/2^-$	0	2.4 ± 0.3	0.2 ± 0.04	0
21^h	$5/2^+$	0	2.8 ± 0.5	0.75 ± 0.15	0

a. F. Ajzenberg-Selove, Nucl. Phys. **A490** (1988) 1
b. F. Ajzenberg-Selove, Nucl. Phys. **A433** (1985) 1
c. The state is a ground one.
d. The state is the first excited one.
e. F. Ajzenberg-Selove, Nucl. Phys. **A449** (1986) 1
f. F. Ajzenberg-Selove, Nucl. Phys. **A460** (1986) 1
h. A. B. McDonald, H. B. Mak, H. C. Evans, G. T. Ewan and H. B. Trautvetter, Nucl. Phys. **A278** (1976) 477

Table 2. Experimental quintet widths (keV)

A	J^π	$\Gamma(2)$	$\Gamma(1)$	$\Gamma(0)$	$\Gamma(-1)$	$\Gamma(-2)$
8^a	0^+	0	< 12	5.5 ± 2	< 60	230 ± 50
12^b	0^+	0	85 ± 40	< 30	–	$\approx 400 \pm 250$
16^c	0^+	0	< 12	12.5 ± 2.5	–	122 ± 32

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c. F. Ajzenberg-Selove, Nucl. Phys. **A460** (1986) 1

We now discuss the various cases:

$A = 7, 13, 17$ and 21.

In these cases the widths are fitted to a quadratic function in T_z and large values of χ^2 are gotten implying that the widths are not quadratic in T_z.

$A = 9$

Here a one-parameter fit in T_z yields a value of χ^2 equal to 0.07. This case will be discussed more fully in the next Section.

$A = 1$ and 15 (first excited states)

The first excited states of $A = 11$ and 15 have fully known widths and a two-parameter fit in T_z will lead to $\chi^2 = 1.95$ and 0.29, respectively. Therefore the two systems are quadratic in T_z.

As to the ground state multiplets of $A = 11$ and 15, they will be discussed in Section 4.

$A = 8$ (quintets)

In this case we make exact, quadratic, fitting of the three known widths and predict the other two. The results obtained are $\Gamma(+1) = -24.63$ keV (unphysical) and $\Gamma(-1) = 90.38$ keV (exceeds bound). Therefore we conclude that the $A = 8$ quintet is not quadratic.

Again the $A = 12$ and 16 quintets will be discussed later.

Summarizing we see that the $A = 9, 11$ and 15 are well fitted by a quadratic expression in T_z.

3. *Hadronic multiplet masses*

In order to determine the hadronic masses of the multiplets we use equations (1) and (2) and make our fitting with m_0 and r_0 as the two parameters of the fit.

We make our analysis for the multiplets discussed in the previous Section as well as for any possible isospin-allowed decay products to determine which one lies lowest. The results are displayed in Table 3 and where we note that in several cases the magnitude of χ^2 is quite large, indicating shortcomings of Sengupta model. We should stress here that the purpose of fitting the multiplet masses is to determine which ones are hadronically stable and which ones are hadronically unstable and to test the IMWE accordingly.

Now to characterize the stability or the instability of a multiplet we define a mass difference Δm by the equation

$$\Delta m = m_0(\text{lowest-decay channel}) - m_0(\text{multiplet}) \tag{3}$$

and which represents the negative of the energy released in the absence of electromagnetism.

The values of Δm of the multiplets of interest are given in Table 4 and where the lowest isospin conserving decay channel in each case is indicated according to the scheme $A \to A' + A''$; A' and A'' are the mass numbers of the daugther nuclei.

Table 3. Hadronic multiplet masses

A	J^π	T	r_0(fm)	m_0(MeV)	χ^2
1	1/2	1/2	–	938.926	–
6	0^+	1	1.781	5603.739	231.3
7	$3/2^-$	3/2	0.937	6540.900	$\approx 10^6$
8	2^+	1	1.339	7467.947	3871.6
8	0^+	2	1.656	7479.504	11987.1
9	$3/2^-$	3/2	0.416	8403.314	10396.8
10	0^+	1	1.420	9321.164	11603.1
11	$1/2^+$	3/2	1.444	10259.716	1.0
11	$1/2^-$	3/2	1.597	10260.536	1.7
12	1^+	1	1.383	11182.631	1523.7
12	0^+	2	1.613	11196.088	106.7
13	$3/2^-$	3/2	1.322	12116.360	29361.8
14	0^+	1	1.337	13032.366	$\approx 10^6$
15	$1/2^+$	3/2	1.414	13970.541	2.6
15	$5/2^+$	3/2	1.426	13971.531	30627.7
16	2^-	1	1.399	14895.495	1404.9
16	0^+	2	1.435	14905.563	525.8
17	$1/2^-$	3/2	1.324	15827.938	152.3
20	2^+	1	1.363	18608.672	387.1
21	$5/2^+$	3/2	1.332	19539.387	253.3

Table 4. Multiplet decay energy (MeV)

A	J^π	A'	A''	Δm	Δm
				New An.	Old An.
7	$3/2^+$	1	6	1.765	−0.444
8	0^+	$1+1$	6	2.087	2.081
9	$3/2^-$	1	8	3.559	4.109
11	$1/2^+$	1	10	0.374	0.278
11	$1/2^-$	1	10	−0.446	−0.096
12	0^+	1	11	2.554	–
13	$3/2^-$	1	12	5.197	4.860
15	$1/2^+$	1	14	0.751	0.839
15	$5/2^+$	1	14	−0.239	0.250
16	0^+	1	15	3.904	–
17	$1/2^-$	1	16	6.483	6.136
21	$5/2^+$	1	20	8.211	8.202

$\Delta m > 0$ implies that we have a hadronically stable multiplet and this should be associated with quartic widths. For $A = 13, 17$ and 21, $\Delta m > 4.5$ MeV and this gives a margin large enough to conclude with certainty that they are hadronically stable and their widths to be non-quadratic.

As to the multiplet $A = 9$ we are to bear in mind that we can have $\Gamma(1/2) = = \Gamma(-1/2)$ and obtain quartic widths if the vector-tensor interference term is small for $\Delta T = 1$ particle decays. This was shown to be true in the simple quartet-sextet model developed by Awin and Shanley (1982).

For $A = 7$, the discrepancy in the previous work (Awin and Shanley 1982) has been removed and $\Delta m > 0$ implying a non-quadratic equation in T_z. $A = 8$ quintet, also, comply with our prediction ($\Delta m > 2\,\mathrm{MeV}$) even though, as was pointed out before, there is a channel instability problem with the $A = 6$ triplet.

The remaining completely available experimental multiplets are the first excited states with $A = 11$ and $A = 15$; for those $\Delta m < 0$ and this means that the multiplets are hadronically unstable and the widths are quadratic in T_z, as it is expected. It is to be noted that Δm is quite small here.

In summary we find that in all cases we got the quadratic or quartic behaviour for the widths and we found that it is correlated with the hadronic instability or stability of the corresponding multiplet. Moreover the recent experimental information on widths and masses removed all previous discrepancies.

4. *Other experimental multiplets*

In this Section we study a few other multiplets which are not available completely but there exist sufficient information on them to perform analysis and to make some predictions. We note that the missing state in the multiplet is interpolated and a large error in the mass is included.

$A = 11$ and 15 (ground state quartets)

From Table 4 the two multiplets are hadronically stable ($\Delta m > 0$ and small) and this implies that the widths are not quadratic. On the other hand if we assume them to be quadratic we predict a value of $\Gamma(-3/2) = 180$ keV for $A = 11$ and a value of $\Gamma(-1/2) = 738$ keV for $A = 15$, from which it is clear that the ground-state widths for $A = 11$ are not quadratic since $\Gamma(-3/2)$ is not at all broad as it is expected to be.

$A = 12$ and 16 quintets

Here the quintets are definitely not quadratic (hadronically stable, $\Delta m > > 2.5$ MeV). Moreover if we make an exact two-parameter fit and predict the other widths we either get an unphysical width or we get a value beyond the expected range, e.g. for $A = 12$, $\Gamma(0) = 180\,\mathrm{keV} > 30\,\mathrm{keV}$ (which is the bound). This is also the situation for $A = 16$ quintet where we have a value of $\Gamma(-1) = 90\,\mathrm{keV} > 60\,\mathrm{keV}$.

5. *Concluding discussion*

As we have seen the recent experimental information is in suport of the isobaric multiplet width equation which implies that the hadronically unstable multiplets have quadratic widths in T_z, while the stable ones have non-quadratic widths. To ascertain, however, the predicted quartic behaviour of the widths, experimentally, a complete sextet would be needed.

Moreover it should be added that when Δm is small, as in the case of $A = 11$ and 15, Coulomb effects may be so strong that their influence on the widths may not be amenable to perturbation treatment on which basis the quadratic-quartic behaviour of widths was predicted.

Acknowledgements

I like to thank Professor Abdus Salam, the International Atomic Energy Agency and UNESCO for the hospitality at the International Centre for Theoretical Physics, Trieste.

References

1) A. M. Awin and P. E. Shanley, Nucl. Phys. **A386** (1982) 101;

2) S. Sengupta, Nucl. Phys. **21** (1960) 542;

3) A. H. Wapstra and G. Audi, Nucl. Phys. **A432** (1985) 1.

O JEDNADŽBI ZA ŠIRENJE IZOBARNOG MULTIPLICITETA

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Originalni znanstveni rad

Novi eksperimentalni rezultati mjerenja širina čestično nestabilnih jezgara i pobuđenih stanja uzeti su kako bi se vidjelo da li je ranije predložena kvadratično-kvartična ovisnost o T_z (z-komponenti izospina) i dalje korektna. Zbog nedostatka eksperimentalnih podataka, samo kvadratična ovisnost je ispitana i nađeno je poboljšano slaganje.