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LETTER TO THE EDITOR

MULTIPLICITY DISTRIBUTIONS AND CORRELATIONS IN THE PRESENCE OF q-DEFORMED COHERENT STATES

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We study the effects of q-deformed coherent states on second-order correlations and multiplicity distributions.

In recent years there has been considerable interest in the study of quantum groups [1], in particular for their relation to certain models in field theory and statistical mechanics.

In order to find representations of quantum groups, a q-deformed harmonic-oscillator technique has been introduced, based on a deformed commutation relation containing q as a dimensionless parameter [2].

The physical nature of these q oscillators has been investigated by several authors [3], but with no clear answer. Most attempts have been on purely mathematical properties of q oscillators. It has also been observed that for q-coherent states [4] squeezing occurs for all finite q values not equal to unity (classical limit), in contrast to the usual case.

In processes where a large number of particles (usually pions) are produced, one faces the problem of identical particles, which is automatically taken into account by quantum statistics. Their multiplicity distribution and correlation functions depend on the statistics. For example, chaotic emission of pions leads to the geometric, or Bose-Einstein, multiplicity distribution, the statistics of which imply a noisy source. On the other hand, a fully coherent source is characterized by a Poisson multiplicity distribution. These sources are usually referred to as "classical", since they can be constructed from standard coherent states introduced by Glauber [5].

However, it has by now become well known that the standard coherent (Glauber) states are only a special case of a much wider class of coherent states [4,6]. Therefore, it is natural to ask what modifications should be expected to appear in multiplicity distributions and in correlation functions if these new, more general, coherent states are present in the source from which pions are emitted. In this Letter we present some of the results of our study of these modifications which are due to deformed coherent states.

Let us consider a conventional single-mode pion-oscillator algebra generated by the operators a, $a^\dagger$ and $N = a^\dagger a$ that satisfy $[a, a^\dagger] = 1$, $[a, N] = a$, $[a^\dagger, N] = -a^\dagger$.

Next we make a non-linear transformation of $(a, a^\dagger)$ into $(A, A^\dagger)$ defined by

$$\begin{aligned} A &= af(N) = f(N+1)a, \\ A^\dagger &= f(N)a^\dagger = a^\dagger f(N+1), \end{aligned} \tag{1}$$

where $f(N)$ is an arbitrary real function of N having a power series expansion.

The Fock space of the operator A, $A^\dagger$ is

$$\begin{aligned} &A^\dagger \left|0\right\rangle = \sqrt{1} f(1) \left|1\right\rangle , \\ &(A^\dagger)^2 \left|0\right\rangle = \sqrt{1} f(1) \sqrt{2} f(2) \left|2\right\rangle , \\ &\vdots \\ &(A^\dagger)^n \left|0\right\rangle = \left[\sqrt{n} f(n)\right]! \left|n\right\rangle \equiv \left([n]_f!\right)^{1/2} \left|n\right\rangle , \\ &\vdots \end{aligned} \tag{2}$$

where $[n]_f = nf^2(n)$ and $[n]_f ! [n-1]_f \dots [1]_f$, $[0]_f! = 1$.

We note the simple relation between the $|n\rangle$ and $(A^\dagger)^n |0\rangle$ states

$$\left|n\right\rangle = \frac{(a^\dagger)^n}{\sqrt{n!}} \left|0\right\rangle = \frac{(A^\dagger)^n}{\sqrt{[n]_f!}} \left|0\right\rangle . \tag{3}$$

This relation enables us to construct coherent states for the operator A in a simple way. Using (3) we find that the normalized coherent states $|\alpha\rangle$, $\alpha \in \mathbb{C}$ satisfying the relation

$$A \left|\alpha\right\rangle = \alpha \left|\alpha\right\rangle , \quad \langle\alpha|\alpha\rangle = 1 \tag{4}$$

are given by

$$|\alpha\rangle = C^{-1} \sum_{n=0}^{\infty} \frac{\alpha^n}{([n]_f!)^{1/2}} \, |n\rangle \,, \tag{5}$$

where

$$|C|^2 = \sum_{n=0}^{\infty} \frac{|\alpha|^{2n}}{[n]_f!} \equiv e_f(|\alpha|^2). \tag{6}$$

The idea of using deformed coherent states (Eq. (5)) to describe the emission of pions in hadron-hadron collisions was motivated by the possibility of measuring correlations between pairs of identical pions. We assume that the "vacuum" (source) from which the correlated pions are emitted is described by $|\alpha\rangle$. The corresponding multiplicity distributions are

$$\begin{aligned} P_n(\alpha) &= |\, \langle n|\alpha\rangle \,|^2 \\ &= \left[e_f(|\alpha|^2)\right]^{-1} \frac{|\alpha|^{2n}}{[n]_f!}. \end{aligned} \tag{7}$$

The quantity measuring the two-pion correlations seen in pion-interferometry experiments is the seconf-order correlation function given by

$$g^{(2)} \equiv \frac{\left\langle a^\dagger a a^\dagger a \right\rangle - \left\langle a^\dagger a \right\rangle}{(\langle a^\dagger a \rangle)^2}, \tag{8}$$

where $\left\langle a^\dagger a \right\rangle = \langle N\rangle = \left\langle \alpha | a^\dagger a | \alpha \right\rangle$.

If the source is classical, two extreme cases are possible: a completely coherent source with $g^{(2)} = 1$ and a completely noisy (thermal) source with $g^{(2)} = 2$. In general, for a classical source one can show that $1 \leq g^{(2)} \leq 2$. However, there are distributions that follow from squeezed coherent states [6] and/or deformed coherent states (Eq. (5)), which give $g^{(2)} < 1$ and $g^{(2)} > 2$. Emission is said to be antibunched if $g^{(2)} < 1$.

The correlation $g^{(2)}$ in our approach is given by

$$g^{(2)} = e_f(x) \frac{e_f''(x)}{[e_f'(x)]^2}, \quad x = |\alpha|^2. \tag{9}$$

We see that both shape of the multiplicity distribution $P_n(\alpha)$ and the second-order correlations function $g^{(2)}$ depend on the form of the function $f(n)$ entering $[n]_f$. The border line is $f(n) = 1$, which gives conventional coherent states, the Poisson distribution and $g^{(2)} = 1$.

The arbitrariness of the form of the function $f(n)$ may be reduced by the additional requirement that the operators $(A, A^\dagger)$ should satisfy the single-mode q--Heisenberg-Weyl algebra [2]

$$AA^\dagger - qA^\dagger A = 1. \tag{10}$$

In this case we find that

$$[n]_f = nf^2(n) = \frac{1-q^n}{1-q} \tag{11}$$

and the squeezing occurs for all values of $q \neq 1$.

References

1) P. P. Kulish and E. K. Sklyanin, J. Sov. Math. **19** (1982) 1596;
P. P. Kulish and N. Y. Reshetikhin, J. Sov. Math. **23** (1983) 2435;
E. K. Sklyanin, Funct. Anal. Appl. **16** (1982) 1262;
M. Jimbo, Lett. Math. Phys. **10** (1985) 63, **11** (1986) 247;

2) L. C. Biedenharn, J. Phys. **A22** (1989) L837;
A. J. Macferlane, J. Phys. **A22** (1989) 4581;
C. P. Sun and H. C. Fu, J. Phys. **A22** (1989) L983;

3) E. G. Floratos and R. N. Tomaras, Phys. Lett. **251B** (1990) 163;
D. Bonatas, C. Daskaloyannis and K. Kokkotas, J. Phys. A: Math. Gen. **24** (1991) L79;
D. Bonatos, P. Raychev and A. Faessler, Chem. Phys. Lett. **178** (1991) 221;
C. Daskaloyannis and K. Ypsilantis, J. Phys. A: Math. Gen. **25** (1992) 4157;
M. Chaichian, D. Ellinas and P. Kulish, Phys. Rev. Lett. **65** (1990) 980;
V. I. Man'ko, G. Marno, S. Solimeno and F. Zaccaria, Napoli preprint DSF-T-92/25 INFN-NA-IV-92/25 and DSF-T-93/06 INFN-NA.IV-93/6;

4) E. Celeghini, M. Rasetti and G. Vitiello, Phys. Rev. Lett. **66** (1991) 2056;
A. I. Solomon and J. Katriel, J. Phys. A: Math. Gen. **23** (1990) L1209;

5) J. Klauder and E. C. G. Sudarshan, *Quantum Optics* (Benjamin, New York, 1968);

6) D. F. Walls, Nature (London) **306** (1986) 141;
A. Vourdas and R. M. Weiner, Phys. Rev. **D38** (1988) 2209;
B. A. Bambah and M. V. Satyanarayana, Phys. Rev. **D38** (1988) 2202.

MULTIPLICITETNE RASPODJELE I KORELACIJE U PRISUTNOSTI q-DEFORMIRANIH KOHERENTNIH STANJA

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Proučavamo efekte q deformiranih koherentnih stanja na korelacije drugog reda i multiplicitetne raspodjele.