ISSN 1330–0016
CODEN FIZBE7

ANALYSIS OF ENERGY TRANSFER IN PARTICLE FIELD INTERACTION

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Received 21 January 1994

UDC 512.62

Original scientific paper

In this work we have analyzed dynamics of a particle in the one dimensional model of the plane electromagnetic wave. Our particular interest was to show that classical theory can describe this dynamics very well, and gives the correct momentum distribution of particle which depends on the frequency of the field as well as on its amplitude.

1. Introduction

Multiphoton excitation of particles is the field of great interest for fundamental research and for application [1,2]. Frequently, classical dynamics is used for analyzing such processes [3]. Among the classical methods, the one which is commonly used is the Monte Carlo method [4,5], in which randomization of initial position of the particle is assumed, while the momentum is taken from the law of energy conservation. The success of Monte Carlo simulation, in explaining energy transfer, is due to the fact that the momentum distribution of the particle is narrow. If the latter condition is not satisfied, the method fails to explain the correct energy transfer.

Recently, an improvement was made in classical dynamics, in order to take into account the momentum distribution [6,7]. The idea is to relate the position and the momentum distribution in accordance with the uncertainty relation

$$\Delta x \Delta p \geq \hbar. \tag{1}$$

The distributions which satisfy this condition are related by

$$P(x) = |\psi(x)|^2 \qquad Q(p) = |\phi(p)|^2 \tag{2}$$
$$\psi(x) = \sqrt{\frac{1}{2h\pi}} \int \mathrm{d}p \phi(p) e^{ipx\hbar}.$$

Based on this amendment, various examples were investigated including the relativistic dynamics of particles interacting with plane electromagnetic (EM) wave [6,7]. Excellent agreement with quantum theory was found for the position distribution, but the detailed analysis of momentum distribution was not made. In this article we will concentrate on the $Q(p,t)$ distribution in order to explain energy transfer between EM field and particle.

2. *Classical theory*

Classical time evolution of $P(r,t)$ and $Q(p,t)$, given the initial conditions (2), can be calculated in various ways. The simplest one is to calculate these distributions by taking random conditions for trajectories from the distributions $P_0(x)$ and $Q_0(p)$, and sample them conveniently after time t. Although the procedure gives satisfying results for dynamics of unbound particles, in general, the choice of the coordinate x depends on the choice of the momentum p. Therefore, one defines the distribution $\rho(x,p,t)$ in the phase space, with the property

$$P(x,t) = \int \mathrm{d}p\, \rho(x,p,t) \qquad Q(p,t) = \int \mathrm{d}x\, \rho(x,p,t) \tag{3}$$

and from this function one chooses the initial conditions.

The function $\rho(x,p,t)$ satisfies two important relationships. The first is

$$\rho\left[x(t), p(t), t\right] \delta x \delta p = \rho\left[x_0, p_0, t_0\right] \delta x_0 \delta p_0. \tag{4}$$

This requirement ensures that all the trajectories from the volume element $\delta x_0 \delta p_0$ are confined to the volume element $\delta x \delta p$ at any later time t.

The second relation is the Liouville's equation

$$\frac{\partial \rho}{\partial t} + \frac{p}{m}\frac{\partial \rho}{\partial x} + F\frac{\partial \rho}{\partial p} = 0 \tag{5}$$

from which one obtains the continuity equation for the probability and probability current. F is force acting on the particle with mass m.

It is essential now, if we try to analyze $Q(p,t)$, to find the connection between ρ and ψ. The Wigner's function is one possibility [8], but it is not the only one. This function is given by

$$\rho = \frac{1}{2\pi\hbar} \int\limits_{-\infty}^{\infty} \mathrm{d}y\, e^{-ipy} \psi^* \left(x - \frac{y}{2}\right) \psi \left(x + \frac{y}{2}\right) \tag{6}$$

which satisfies the conditions (3), and approximately the Liouville's equation [9]. From this function one is able to calculate the phase $\delta(x,t)$ of the wave function, defined by $\psi = \sqrt{P} e^{i\delta}$, which is given by

$$\delta(x,t) = \frac{m}{\hbar} \int\limits^{x} \mathrm{d}x' \, \frac{j(x',t)}{P(x',t)} \tag{7}$$

where $j(x,t)$ is the probability current. This connection is also used for obtaining $\psi(x,t)$ at any later time, what is essential for calculating the momentum distribution $Q(p,t)$ from

$$Q(x,t) = \frac{1}{2\pi\hbar} \left[\int \mathrm{d}x e^{-ixp/\hbar} \sqrt{P(x,t)} e^{i\delta(x,t)} \right]^2 \tag{8}$$

which is in accordance with the assumption (2).

In this article we present a 1D study of dynamics of a particle in the intense EM field. We approximate the probability distribution by a rectangular shape and the EM fields by an oscillating electric component. The interaction potential is of the form

$$V(x,t) = \frac{eE_0}{k} \cos(kx - \omega t) \tag{9}$$

where the wave number k and the frequency of the field ω are related by $kc = \omega$, where c is the speed of light. For this model the classical equation of motion is

$$m\ddot{x} = eE_0 \sin(kx - \omega t). \tag{10}$$

It is convenient to scale the equation by defining x as a product κx and time as κct, where $\kappa = mc/\hbar$ is the Compton's wave number. The classical equation of motion is now

$$\ddot{x} = \varepsilon \sin\left[k(x - t)\right] \tag{11}$$

where $\varepsilon = eE_0/(kmc^2)$, and where we also use k to represent k/κ. Solving this equation for a large number of initial conditions we obtain $P(x,t)$, the current $j(x,t)$ and from equation (8) the distribution $Q(p,t)$.

3. *Quantum theory*

In this section we will make a brief review of the quantum solution to the problem which was defined in the previous section. The aim is to compare the analysis of the energy transfer between a particle and the field based on the classical and quantum approaches. The basic quantum equation is

$$i\frac{\partial\psi}{\partial t} = -\frac{1}{2}\frac{\partial^2\psi}{\partial x^2} + \frac{\varepsilon}{k}\cos\left[k(x-t)\right]\psi \tag{12}$$

where the same scaled coordinates are used as in Eq. (11). For the initial ψ at $t = 0$, we assume a function confined to the region of space between $x = 0$ and $x = a$. In this paper we solve Eq. (12) numerically by the following procedure. It is assumed that the system is enclosed in a potential well, with infinitely high walls at the positions $x = -\delta$ and $x = a + \delta$, but otherwise being zero. We define functions

$$\phi_n(x) = \sqrt{\frac{2}{a+2\delta}}\sin\left[\frac{x+\delta}{a+2\delta}\pi n\right] \tag{13}$$

where a is the interval within which $P_0(x)$ is confined and the solution of Eq. (12) is written as

$$\psi(x,t) = \sum_n c_n(t)\phi_n(x). \tag{14}$$

A set of the first order ordinary differential equations in the time variable is obtained for the coefficients $c_n(t)$ if Eq. (14) is replaced in Eq. (12), and they are then solved numerically.

4. *Examples*

Before any analysis of typical examples, it is worth mentioning numerical values of the parameters involved in our study. We will be dealing with electron as the particle in which case, if the intensity of the incident radiation is 1 W/cm^2, and its wavelength is 10^{-7} m, then the coupling coefficient is $\varepsilon = 2.07 \cdot 10^{-15}$, and the ratio k/κ, or k in the notation of the previous sections, is $2.43 \cdot 10^{-5}$. For these numbers it is difficult to do any numerical study, and so the parameters which will be used have larger values. In our study, in order to obtain the nicest features for energy transfer, we will consider broad probability distributions in the coordinate x. A broad distribution means that the width of the distribution is an order of magnitude larger than the wave length of the EM wave.

First, we have investigated the weak coupling examples, and by that we mean the cases when $\varepsilon/k^2 \ll 1$. In this limit, however, the classical calculations are numerically very demanding and so this parameter should not be too small. The example which we could treat with a reasonable effort is for $\varepsilon/k^2 = 0.054$, and the results are shown in Fig. 1. The wave number of the EM field is $k = 0.135$

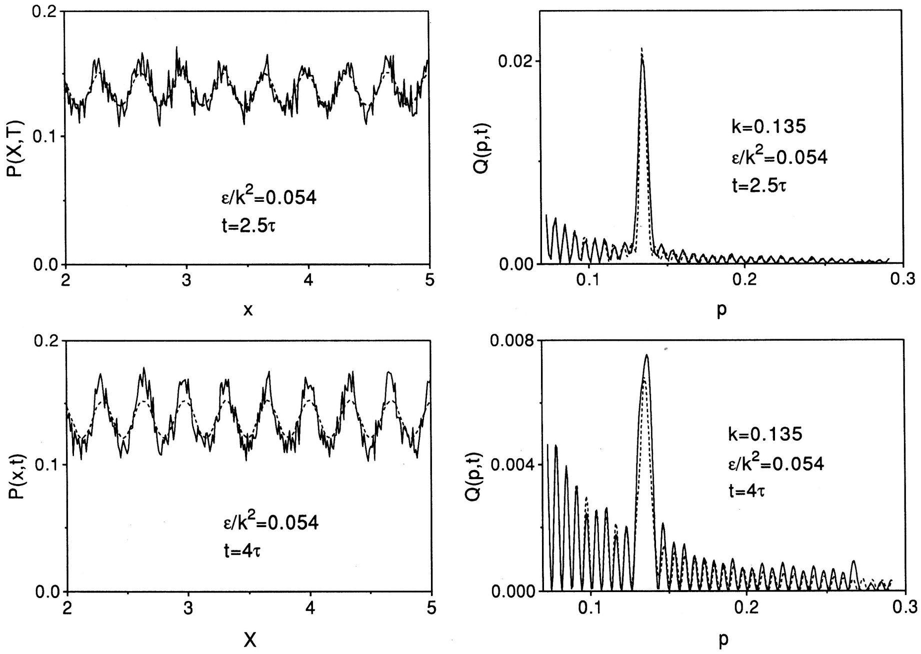

Fig. 1. Classical (solid line) and quantum (broken line) distribution $P(x,t)$ and $Q(x,t)$ for the weak coupling between the EM plane wave and the particle, given at two time intervals. Only the central segment of a much wider distribution $P(x,t)$ is shown for better comparison.

and the time is measured in the units of the period of its oscillations ($\tau = 2\pi/\omega$). The blowup part near the central segment of $P(x,t)$ is shown ($a = 986$) for better comparison. Both the quantum (broken line) and classical (solid line) results show very good agreement for $P(x,t)$, sligth deviations being more obvious for longer time.

The distribution of moments is also shown. The central peak at $p = 0$ is not shown because it has much greater amplitude. Also, the distribution is symmetric with respect to the change in sign of p, and so this is also not shown. The agreement between the classical (solid line) and the quantum (broken line) calculations is very good, and the distribution peak is exactly at $p = k$. In the language of the QED this peak corresponds to the "one photon energy transfer".

In the next example we consider the intermediate coupling when $\varepsilon/k^2 = 0.54$ and $k = 0.135$. The width of the distribution is the same as in the previous example. Figure 2 shows again two typical results for $P(x,t)$ and $Q(p,t)$ for the short and longer time. For the short time the agreement between the quantum (broken line) and the classical (solid line) calculations is very good, both for $P(x,t)$ and the momentum distribution $Q(p,t)$. Again, in the latter case we do not show the central peak and the distribution for the negative p. We notice the single peak at $p = k$

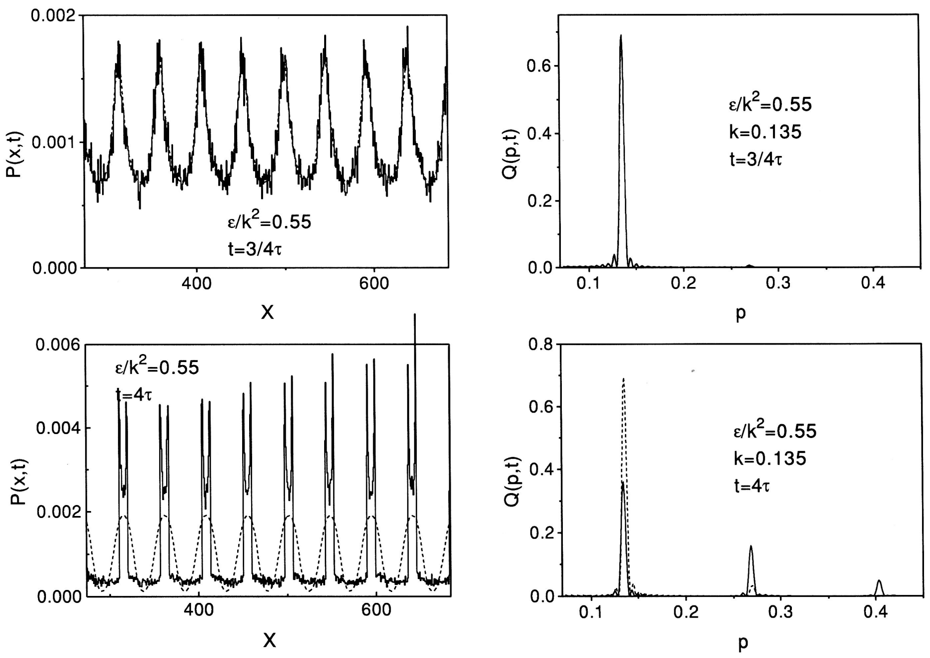

Fig. 2. The same as in Fig. 1, except for the intermediate coupling.

which corresponds to the "one photon energy transfer". However, for longer time the deviation between the classical and the quantum $P(x,t)$ is obvious. The classical $P(x,t)$ appears to be getting singularities whilst the quantum $P(x,t)$ is a smooth function, although the general oscillation pattern is reproduced very well. The momentum distributions are similar, showing also peaks for "two photon" and "three photon" energy transfer, but the intensities are not the same.

The singularities in $P(x,t)$ are more pronounced for longer time, and their explanation is found using Eqs. (3) and (4). The distribution $P(x,t)$ is

$$P(x,t) = \int \mathrm{d}\rho\, \rho(x,p,t) = \int \mathrm{d}x_0 \frac{\rho_0\left[x_0(p_0,x,t),p_0\right]}{\dfrac{\mathrm{d}x}{\mathrm{d}x_0}} \tag{15}$$

and the denominator may be zero for a particular value of the initial x_0. This can be verified by solving sligthly modified set of equations (11). The set is divided by δx_0, in which case its solution represents the derivatives $\delta x/\delta x_0$ and $\delta p/\delta x_0$ if the initial conditions are $\delta x/\delta x_0 = 1$ and $\delta p/\delta x_0 = 0$. For the other initial conditions it was assumed that the initial velocity of the particle is zero while its initial position is taken from the interval defined by $P_0(x)$. The results are shown in Fig. 3, for the same parameters as in Fig. 2, except that $t = 5\tau$. The broken line represents the derivative $\mathrm{d}x/\mathrm{d}x_i$, and whenever its value goes through zero, a singularity in $P(x,t)$ appears. Therefore, the deviations between the classical

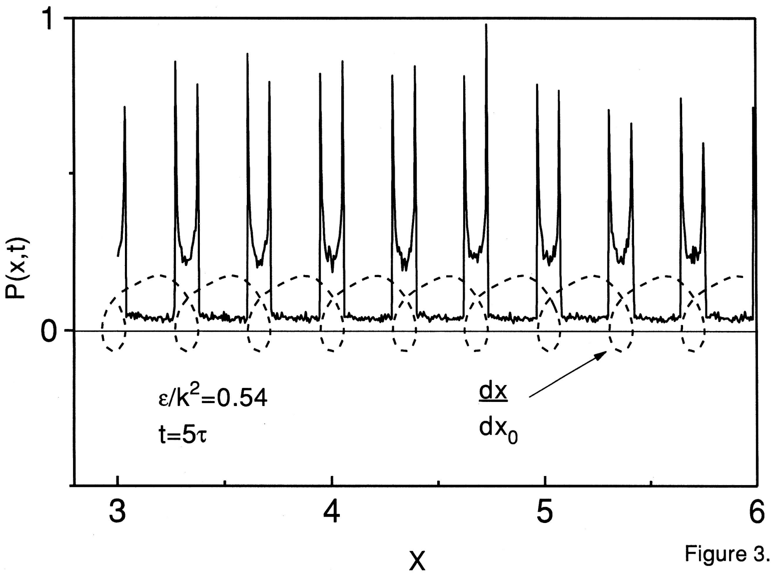

Fig. 3. Explaining the singularities of the classical $P(x,t)$ in Fig. 2. Whenever the denominator in Eq. (15) is zero a singularity appears in $P(x,t)$.

and the quantum calculations are not fundamental but they are of the sort which are found in the description of the classical and quantum rainbow. For greater ε/k^2 ratio the quantum calculations become more demanding because of the poor convergence of the series (15). On the other hand, the classical calculations are very easy to do without numerical instabilities.

5. *Discussion*

In this work we have analyzed dynamics of a particle in the one dimensional model of the plane EM wave. Our particular interest was to show that classical theory can describe this dynamics very well, and gives the correct momentum distribution of the particle which depends on the frequency of the field as well as on its amplitude. It was shown that in the weak coupling limit classical theory gives the same results as the quantum analysis, whilst in the strong coupling limit the probability distributions get singularities which are the source of discrepancies. The analysis is not complete without taking into account the radiation field produced by the particle. This work is, therefore, a preliminary study which tests to what degree classical theory is able to explain some of the fundamental processes in the EM field.

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ANALIZA PRIJENOSA ENERGIJE U MEĐUDJELOVANJU ČESTICE I POLJA

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Originalni znanstveni rad

U ovom radu smo analizirali interakciju čestice i ravnog elektromagnetskog vala. Koristili smo jednodimenzionalnu aproksimaciju. Pokazali smo da u takvoj aproksimaciji, klasična teorija u potpunosti opisuje prijenos energije, te daje respodjelu impulsa koja ovisi prvenstveno o frekvenciji polja.