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ELECTROMAGNETIC-AXION DISPERSIVE EFFECTS IN TWO ALTERNATIVE THEORIES OF ELECTROMAGNETISM

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By comparing the dispersive effects generated in the usual axion electromagnetic theory with that of the two-potential theory of Cabibbo and Ferrari we arrive at criteria sufficient to distinguish these two theories using astrophysical observations.

1. *Introduction*

With the vast avalanche of theories invented to accommodate particle phenomenology it seems worthwhile to search for signatures outside of accelerator physics to distinguish these theories. In particle accelerator experiments the search for a fundamental scalar field has to date produced no positive results [1]. The foundations of particle theory depend on the existence of a fundamental scalar doublet to generate the quark masses and gauge boson masses [2], also the existence of a fundamental scalar might have a more generic basis originating in a technicolour scenario or supergravity theory [3,4]. The whole program of supersymmetry is built around a desire to keep scalars light by generating cancellations in loop diagrams that otherwise would lead to a divergent scalar mass [4]. With the advent of topological considerations or QCD the unwanted topological term could be made to vanish by introducing the $U(1)_{\mathrm{PQ}}$ global symmetry upon whose breaking the pseudo-scalar axion appears [6]. In somewhat the same spirit both the pseudo-scalar familon and majoron appear after the relevant global symmetries are broken that were invented to explain quark mass generation and majorana neutrino mass generation [7,8]. Scalars also appear in gravitational theory in the guise of the Brans-Dicke scalar [9], Barber's theory of creation [10] and Zee's spontaneously generated theory of gravity [11]. In supergravity theory, both the scalar dilation and graviscalar appear with the graviscalar producing small deviations from the equivalence principle [12,13,14]. Lastly, the higher dimensional Kaluza-Klein theories produce a scalar that couples to electromagnetism when the theory is compactified [15].

In what follows we seek to study the interaction of the axion with electromagnetism. Through a quark loop the axion has well defined couplings with electromagnetism that may produce measurable dispersive effects as well as birefringence in pseudo-scalar electromagnetic propagation [16]. With regard to the pure electromagnetic Lagrangian, Cabibbo and Ferrari have studied a two-potential theory of electromagnetism which has the beauty that it can describe electromagnetic phenomena in the presence of magnetic charge without Dirac strings [17]. As pointed out by Vinciarelli [18], the dual symmetry of Maxwell's equations is unique to four space-time dimensions which may suggest that monopoles might be condensations from higher dimensions. To study the predictions that a two-potential theory generates, we study the propagation of electromagnetic pseudo-scalar waves in both the usual one-potential theory and also in the two-potential theory. We find that the two-potential theory produces two dispersive components, while the one-potential theory produces three dispersive components. A fine resolution of radiation from extra galactic sources may thus provide experimental tests for the two-potential theory. Also, Calligari et al. [19] have calculated the corrections to the earth's magnetic field produced by the two-potential theory and in a separate note we have studied the effects that two photons with small rest masses would have on pure electromagnetic wave propagation [20]. Though conventional electromagnetic phenomena might not distinguish between these two theories, astrophysical signatures may provide us with a means to distinguish these two theories and thus provide us with evidence for or against a consistent theory of electromagnetism admitting magnetic charge.

2. *Pseudo-scalar electromagnetic propagation in two alternative theories of electromagnetism*

We begin our analysis by writing the Lagrangian of electromagnetism coupled to the pseudo-scalar axion [16].

$$\mathcal{L} = -\frac{1}{16\pi} F_{\mu\nu} F^{\mu\nu} \sqrt{-g} + \frac{\partial_\mu \Phi \partial^\mu \Phi}{2} - \frac{m_a^2 c^2 \Phi^2}{2\hbar^2} + \alpha \left(\frac{\varepsilon^{\mu\nu\alpha\beta} F_{\alpha\beta} F_{\mu\nu}}{\sqrt{-g}} \right) \Phi \sqrt{-g}. \quad (2.1)$$

Here

$F_{\mu\nu}$ = electromagnetic field tensor, Φ = pseudo-scalar axion field,
α = axion coupling constant, m_a = axion mass.

Varying Eq. (2.1) with respect to A_μ and Φ gives

$$\frac{\partial}{\partial x^\nu}\left(\frac{\sqrt{-g}F^{\mu\nu}}{4\pi}\right) - 4\alpha\frac{\partial}{\partial x^\nu}\left(\varepsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}\right) = 0 \tag{2.2}$$

$$-\Box\Phi - \frac{m_a^2c^2}{\hbar^2}\Phi + \alpha\left(\frac{\varepsilon^{\mu\nu\alpha\beta}F_{\alpha\nu}F_{\mu\nu}}{\sqrt{-g}}\right) = 0. \tag{2.3}$$

We now consider propagation in the x direction, with

$$F_{13} = -B_0 - B_y, \quad F_{12} = B_z, \quad F_{24} = E_y, \quad F_{34} = E_z\ .$$

Here B_0 = constant is static magnetic field in y direction: B_y, B_z, E_y, E_z are fluctuating fields propagating in x direction. Eq. (2.2) reads upon setting $u = 2, 3$

$$-\frac{1}{4\pi}\frac{\partial B_z}{\partial x} - \frac{1}{4\pi c}\frac{\partial E_y}{\partial t} + \frac{8\alpha B_0}{c}\frac{\partial\Phi}{\partial t} = 0 \tag{2.4}$$

$$\frac{1}{4\pi}\frac{\partial B_y}{\partial x} - \frac{1}{4\pi c}\frac{\partial E_z}{\partial t} = 0. \tag{2.5}$$

Eq. (2.3) gives

$$-\frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} + \frac{\partial^2\Phi}{\partial x^2} - \frac{m_a^2c^2}{\hbar^2}\Phi - 8\alpha E_y B_0 = 0. \tag{2.6}$$

Upon differentiating Eq. (2.4) with respect to t and Eq. (2.5) with respect to t and using

$$\frac{\partial E_y}{\partial x} = -\frac{1}{c}\frac{\partial B_z}{\partial t}; \qquad \frac{\partial E_z}{\partial x} = \frac{1}{c}\frac{\partial B_y}{\partial t}$$

or

$$\frac{\partial^2 E_y}{\partial x^2} = -\frac{1}{c}\frac{\partial^2 B_z}{\partial t\partial x}; \qquad \frac{\partial^2 E_z}{\partial x^2} = \frac{1}{c}\frac{\partial^2 B_y}{\partial t\partial x}$$

we have

$$\frac{\partial^2 E_y}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 E_y}{\partial t^2} + \frac{32\pi\alpha B_0}{c^2}\frac{\partial^2\Phi}{\partial t^2} = 0 \tag{2.7}$$

$$\frac{\partial^2 E_z}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 E_z}{\partial t^2} = 0 \tag{2.8}$$

and from Eq. (2.6)

$$-\frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} + \frac{\partial^2\Phi}{\partial x^2} - \frac{m_a^2c^2\Phi}{\hbar^2} - 8\alpha E_y B_0 = 0. \tag{2.9}$$

For plane wave propagation we have

$$\begin{aligned} E_y &= \overline{E}_{0y} e^{i(kx-\omega t)} \\ E_z &= \overline{E}_{0z} e^{i(kx-\omega t)} \\ \Phi &= \overline{\Phi}_0 e^{i(kx-\omega t)}. \end{aligned} \tag{2.10}$$

Substituting Eq. (2.10) into Eq. (2.7), Eq. (2.8) and Eq. (2.9) we have

$$\begin{aligned} \overline{E}_{0y}\left(\frac{\omega^2}{c^2}-k^2\right)+\overline{\Phi}_0\left(-\frac{\omega^2}{c^2}(32\pi\alpha B_0)\right)&=0 \\ \overline{E}_{0z}\left(\frac{\omega^2}{c^2}-k^2\right)&=0 \\ \overline{E}_{0y}(-8\alpha B_0)+\overline{\Phi}\left(\frac{\omega^2}{c^2}-k^2-\frac{m_a^2c^2}{\hbar^2}\right)&=0. \end{aligned} \tag{2.11}$$

For the above system to have a solution for $\overline{E}_{0y}$, $\overline{E}_{0z}$ and Φ_0 we have

$$\begin{aligned} \omega_0 &= kc, \\ \omega_+ &= \Bigg[c^2k^2+\frac{m_a^2c^4}{2\hbar^2}+128\pi\alpha^2B_0^2c^2+ \\ &\quad +\frac{1}{2}\sqrt{\left(2c^2k^2+\frac{m_a^2c^4}{\hbar^2}+256\pi\alpha^2B_0^2c^2\right)^2-4\left(k^4c^4+\frac{k^2m_a^2c^6}{\hbar^2}\right)}\,\Bigg]^{1/2} \\ \omega_- &= \Bigg[c^2k^2+\frac{m_a^2c^4}{2\hbar^2}+128\pi\alpha^2B_0^2c^2- \\ &\quad -\frac{1}{2}\sqrt{\left(2c^2k^2+\frac{m_a^2c^4}{\hbar^2}+256\pi\alpha^2B_0^2c^2\right)^2-4\left(k^4c^4+\frac{k^2m_a^2c^6}{\hbar^2}\right)}\,\Bigg]^{1/2}. \end{aligned} \tag{2.12}$$

The above system gives three dispersive branches. To estimate the contribution from the axion coupling, we know that the dimensions of α are $\sqrt{\frac{L}{E}}$ or according to the mass scale that breaks the P.Q. symmetry it would be proportional to $\sqrt{\frac{\hbar}{m^2c^3}}$; for microwaves ($k \simeq 1\,\mathrm{cm}^{-1}$), we have for $\frac{(\alpha B_0)^2}{k^2} = 1$, ($B_0 = 10^{12}$ gauss, i.e. $B_0 = 10^8$ T, found in field of pulsars), $m \approx 3\times10^{-21}$ kg, $m \simeq 10^6$ GeV.

Thus for the external magnetic field to generate measurable dispersive effects, the mass scale of the P.Q. symmetry breaking must be about 10^6 GeV. Also for the window on the axion mass, we have the experimental limits ($m_a < 10^{-3}$ eV, $m_a > 10^{-5}$ eV) [21] thus for the term containing $\left(\frac{m_a c^2}{\hbar}\right)^2$ for $m_a \simeq 10^{-5}$ eV we have $\left(\frac{m_1 c}{\hbar}\right) \simeq 10^{-1}$ which is small compared to k for microwaves.

We now consider the same Lagrangian as in Eq. (2.1) for the two-potential theory of Cabibbo & Ferrari [17] where

$$F_{\mu\nu} = \frac{\partial A_\mu}{\partial x^\nu} - \frac{\partial A_\nu}{\partial x^\mu} - \frac{\varepsilon^{\alpha\beta}_{\mu\nu}}{2\sqrt{-g}}\left(\frac{\partial B_\alpha}{\partial x^\beta} - \frac{\partial B_\beta}{\partial x^\alpha}\right) \tag{2.13}$$

here
A_μ = electric-like four-vector potential
B_μ = magnetic-like four-pseudovector potential.

Using Eq. (2.13) and Eq. (2.1) and varying Eq. (2.1) with respect to A_μ, B_μ we find

$$\frac{\partial}{\partial x^\nu}\left(\frac{\sqrt{-g}F^{\mu\nu}}{4\pi}\right) - 4\alpha\frac{\partial}{\partial x^\nu}\left(\varepsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}\Phi\right) = 0 \tag{2.14}$$

$$\frac{\partial}{\partial x^\nu}\left(\frac{1}{4\pi}\widetilde{F}^{\mu\nu}\right) - 4\alpha\frac{\partial}{\partial x^\nu}\left(\frac{\varepsilon^{\mu\nu\alpha\beta}\widetilde{F}_{\alpha\beta}\Phi}{\sqrt{-g}}\right) = 0 \tag{2.15}$$

$$\left(\widetilde{F}^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}\right).$$

Again, varying Eq. (2.1) with respect to Φ gives

$$-\Box\Phi - \frac{m_a^2c^2}{\hbar^2}\Phi + \alpha\left(\frac{\varepsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}F_{\mu\nu}}{\sqrt{-g}}\right) = 0\,. \tag{2.16}$$

Again, using $F_{12} = B_z$, $F_{13} = B_0 - B_y$, $F_{24} = E_y$, $F_{34} = E_z$ and setting $\mu = 2{,}3$ in Eqs. (2.14) and (2.15) we have

$$\begin{aligned}
-\frac{1}{4\pi}\frac{\partial B_z}{\partial x} - \frac{1}{4\pi c}\frac{\partial E_y}{\partial t} + \frac{8\alpha B_0}{c}\frac{\partial\Phi}{\partial t} &= 0\\
\frac{1}{4\pi}\frac{\partial B_y}{\partial x} - \frac{1}{4\pi c}\frac{\partial E_z}{\partial t} &= 0\\
\frac{1}{4\pi}\frac{\partial E_z}{\partial x} - \frac{1}{4\pi c}\frac{\partial B_y}{\partial t} &= 0\\
-\frac{1}{4\pi}\frac{\partial E_y}{\partial x} - \frac{1}{4\pi c}\frac{\partial B_z}{\partial t} + 8\alpha B_0\frac{\partial\Phi}{\partial t} &= 0
\end{aligned} \tag{2.17}$$

and (from Eq. (2.16))

$$-\frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} + \frac{\partial^2\Phi}{\partial x^2} - \frac{m_a^2c^2}{\hbar^2}\Phi - 8\alpha B_0 E_y = 0.$$

Upon differentiating and substituting the above five equations give

$$\frac{\partial^2 E_y}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 E_y}{\partial t^2} + \frac{32\pi\alpha B_0}{c^2}\frac{\partial^2 \Phi}{\partial t^2} - 32\pi\alpha B_0 \frac{\partial^2 \Phi}{\partial x^2} = 0$$
$$\frac{\partial^2 E_z}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 E_z}{\partial t^2} = 0 \tag{2.18}$$
$$-\frac{1}{c^2}\frac{\partial^2 \Phi}{\partial t^2} + \frac{\partial^2 \Phi}{\partial x^2} - \frac{m_a^2 c^2}{\hbar^2}\Phi - 8\alpha B_0 E_y = 0.$$

We now substitute $\Phi = \overline{\Phi}_0 e^{i(kx-\omega t)}$, $E_y = \overline{E}_{0y} e^{i(kx-\omega t)}$, $E_z = \overline{E}_{0z} e^{i(kx-\omega t)}$ in Eq. (2.18) to obtain

$$\overline{E}_{0y}\left(\frac{\omega^2}{c^2} - k^2\right) + \overline{\Phi}_0 \left(32\pi\alpha B_0 \left(k^2 - \frac{\omega^2}{c^2}\right)\right) = 0$$
$$\overline{E}_{oz}\left(\frac{\omega^2}{c^2} - k^2\right) = 0 \tag{2.19}$$
$$\overline{E}_{0y}(-8B_0\alpha) + \overline{\Phi}_0 \left(\frac{\omega^2}{c^2} - k^2 - \frac{m_a^2 c^2}{\hbar^2}\right) = 0$$

giving for a solution

$$\left(\frac{\omega^2}{c^2} - k^2\right)\left[\left(\frac{\omega^2}{c^2} - k^2\right)\left(\frac{\omega^2}{c^2} - k^2 - \frac{m_a^2 c^2}{\hbar}\right) - 256\pi\left(B_0^2\alpha^2\right)\left(\frac{\omega^2}{c^2} - k^2\right)\right] = 0$$
$$\left(\frac{\omega^2}{c^2} - k^2\right)^2 = 0 \tag{2.20}$$
$$\frac{\omega^2}{c^2} = k^2 + \frac{m_a^2 c^2}{\hbar^2} + 256\pi(B_0\alpha)^2.$$

Thus for the two-potential we have two dispersive branches given by Eq. (2.20).

3. *Conclusion*

The fact that one-potential theory yields three dispersive branches and a two--potential theory yields two dispersive branches provides us with a definite way of testing for the two-potential theory in an astrophysical setting. If microwave bursts are identified wherein the microwaves pass through an intense magnetic field (as that found in a pulsar atmosphere) then each dispersive branch will propagate at a different speed, thus within a burst we would find repetitive signals since there would be a time delay between each dispersive branch. Thus, the number of repetitive signals within a primary signal would provide us with a test for the above theories. The waves would also have to pass through an axion cloud and the above analysis would also provide us with a way to look for the axion since the dispersive frequencies in Eq. (2.12) and Eq. (2.20) are sensitive to the axion mass. For scalar, pseudo-scalar, electromagnetic propagation we have previously discussed the above type of analysis and have given realistic possibilities for radiation emitted by a certain class of RSCV n stars [22]. With that type of analysis applied to dispersive branches in this note, we have a definite way to discriminate between the above two theories of electromagnetism if repetitive signals within microwave bursts are found.

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DISPERZIVNI EFEKTI U INTERAKCIJI AXIONA I ELEKTROMAGNETIZMA U DVIJE TEORIJE ELEKTROMAGNETIZMA

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Uspoređujući disperzivne efekte generirane u uobičajenoj axionskoj elektromagnetskoj teoriji s onima u Cabbibo-Ferrarijevoj dvo-potencionalnoj teoriji, našli smo kriterije dovoljne za njihovo razlikovanje korištenjem astrofizičkih promatranja.