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PION FIELDS AS GAUSSIAN RANDOM VARIABLES AND THE KOBA–NIELSEN–OLESEN SCALING

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We discuss pion distributions resulting from a Gaussian random nucleon source. Connection with a chaotic–coherent mixture, the negative–binomial distribution, and the Koba–Nielsen–Olesen scaling is derived.

1. *Introduction*

The central question in soft hadronic processes (low-p_T hadrons) and at the same time a long-standing one is to explain why the charged-particle multiplicity distributions $P_n(s)$ in pp collisions exhibit the Koba–Nielsen–Olesen (KNO) scaling [1] only in the ISR energy range ($\sqrt{s} = 20$ - 65 GeV).

A number of other quantities measured in pp collisions, such as [2] $\langle p_T \rangle$, $\sigma_{\rm el}/\sigma_{\rm tot}$ and $B/\sigma_{\rm tot}$, where B is the slope of the diffraction peak, also show approximate energy independence only in the ISR energy range. The constant $\sigma_{\rm el}/\sigma_{\rm tot}$ is usually referred to as geometrical scaling (GS) [3]. Without a satisfactory answer to these question it is not possible to fully understand the observed violation [4] of the KNO scaling and the increase of $\sigma_{\rm el}/\sigma_{\rm tot}$ with energies in the range $\sqrt{s} > 100$ GeV.

In the framework of the eikonal formalism [5] it is usually assumed that the multiplicity distributions $P_n(b,s)$ at fixed b satisfy the KNO scaling. Then the KNO scaling of $P_n(s)$ follows under the assumption of the geometrical scaling, namely that $\sigma_{\rm inel}(b,s)$ and $n(b,s)/\overline{n}(s)$ depend only on the scaled impact parameter $b(s) = b/R(s)$.

Recently, the geometrical branching model [6] (GBM) in the b space has been suggested as a model for multiparticle production which combines the GS and Furry branching [7] and leads to the KNO scaling for $\sqrt{s} < 100$ GeV.

In this paper we emphasize a statistical approach to pp collisions in the ISR energy range similar to that developed in quantum optics [8]. We study under which circumstances the multiplicity distributions of pions emitted from a classical random source satisfy the KNO scaling.

It is assumed that the fields describing pions are gaussian random variables. The incident leading protons which can neither be created nor destroyed are treated as a classical random source for pions; their presence is parametrized by the rapidity difference $Y = \ln(s/m^2)$ and the relative impact parameter b.

The unitary S matrix following from such a classical random source is $\widehat{S}(\vec{b},s)$ [9]. It is an operator in the space of pions. The initial-state vector for the pion field is $\widehat{S}(\vec{b},s)|0\rangle$. The vacuum state $|0\rangle$ is a state with no pions but with two leading protons present. In practice, we rarely have any information about the initial state $\widehat{S}(\vec{b},s)|0\rangle$. This means that physical quantities should be averaged over the initial-state ensemble. In quantum statistics, the ensemble average is usually performed using the density operator, which, in our case, is of the form

$$\varrho(\vec{b},s) = \{\widehat{S}(\vec{b},s)|0\rangle\langle 0|\widehat{S}^{\dagger}(\vec{b},s)\}_{\rm av} \qquad (1)$$

and normalized to unity: $\mathrm{Tr}\{\varrho\} = 1$.

2. *High-energy approximation and the n-particle scattering amplitude*

Many results in hadron-hadron collisions can be understood in terms of a simple picture that the outgoing particles have three origins: a) beam fragmentation, b) target fragmentation and c) central production. At high energies most of the particles are produced in the central region. To isolate the central production, we adopt high-energy longitudinally-dominated kinematics, with two leading particles retaining a large fraction of their incident momenta. The essential restrictions are as follows [9]:

$$\begin{aligned} &\text{(i)} \quad \text{the leading-particle effect;} \\ &\text{(ii)} \quad |y_i| \leq \frac{1}{2}(1-\varepsilon)Y, \quad Y = \ln(s/m^2), \quad \varepsilon > 0, \\ &\text{(iii)} \quad \text{small} \quad |\vec{q}_{iT}|, \\ &\text{(iv)} \quad \overline{n}(s) \ll (s/m^2)^{\varepsilon/2}, \quad \overline{n}(s) = \text{average multiplicity.} \end{aligned} \tag{2}$$

Here, y_i and $\vec{q}_{iT}$ denote the rapidity and the transverse momenta, respectively, of the pions. At high energies restriction (ii) in (2) forces the outgoing nucleons to have energies of the order $\sqrt{s}/2$ and equal but opposite longitudinal momenta. As long as the average multiplicity satisfies restriction (iv) in (2), we can omit the pion variables q_{iL} and E_i from the argument of the energy momentum-conservation δ-function which appears in the phase-space volume element for n pions and two nucleons.

Using the following set of $3n+2$ independent variables s, $\vec{\Delta}$, $\{\vec{q}_{iT}, y_i\} \equiv q_i$, $i = 1, 2, \dots, n$, the n-particle contribution to the s-channel unitarity becomes

$$A_n(s, \vec{\Delta}) = \frac{1}{4s} \int \mathrm{d}^2 b \mathrm{e}^{\mathrm{i}\vec{\Delta}\vec{b}} \prod_{i=1}^{n} \mathrm{d}q_i |T_n(\vec{b}, s; 1 \dots n)|^2, \tag{3}$$

where $\vec{\Delta} \equiv \frac{1}{2}(\vec{p}_a{}' - \vec{p}_a) - \frac{1}{2}(\vec{p}_b{}' - \vec{p}_b)$ and $\mathrm{d}q = \mathrm{d}^2 q_T \mathrm{d}y / 2(2\pi)^3$. The nucleon momenta are labelled by p_a and p_b. The normalization is such that

$$\begin{aligned} A_n(s, 0) &= s\sigma_n(s), \\ \sigma_{\mathrm{inel}}(s) &= \sum_{n=1}^{\infty} \sigma_n(s). \end{aligned} \tag{4}$$

3. *Pion field as a random variable*

As we have seen in the preceding section, the leading-particle effect is crucial for an approximate treatment of the multiparticle s-channel unitarity integral, Eq. (3), which enables us to consider the colliding nucleons as a classical source for pions [10]. The basic equation for the pion field is

$$(\Box + \mu^2)\, \pi(\vec{b}, s; x) = j(\vec{b}, s; x), \tag{5}$$

where j is a classical random source. The reference to the nucleon states is in the diagonal variables $\vec{b}$ and s.

Eq. (5) has the standard solution [11] in terms of in- and out-fields:

$$\pi_{\mathrm{in}}(\vec{b}, s; x) = \pi_{\mathrm{out}}(\vec{b}, s; x) + \int \mathrm{d}^4 x' \, \Delta(x - x', \mu)\, j(\vec{b}, s; x'), \tag{6}$$

which are connected by means of the unitary $\widehat{S}$ matrix $\widehat{S}(\vec{b},s)$ as follows:

$$\pi_{\text{out}} = \widehat{S}^{\dagger}\pi_{\text{in}}\widehat{S} = \pi_{\text{in}} + \pi_{\text{classical}}. \tag{7}$$

If $\widehat{S}(\vec{b},s)$ is written as

$$\widehat{S}(\vec{b},s) = \exp[\mathrm{i}\chi(\vec{b},s)]; \quad \chi = \chi^{\dagger}, \tag{8}$$

then it is easy to see that the solution of Eq. (7) is obtained if $\chi(\vec{b},s)$ has the following form:

$$\chi(\vec{b},s) = \int \mathrm{d}^4x\, j(\vec{b},s;x)\,\pi_{\text{in}}(\vec{b},s;x). \tag{9}$$

It is convenient to rewrite the $\widehat{S}$ matrix in the normal-order product form

$$\widehat{S}(\vec{b},s) = \mathrm{e}^{-\frac{1}{2}A(\vec{b},s)} \;:\; \mathrm{e}^{\mathrm{i}\chi(\vec{b},s)} \;:, \tag{10}$$

where

$$A(\vec{b},s) = \int \mathrm{d}q\, |J(\vec{b},s;q)|^2 \tag{11}$$

and

$$J(\vec{b},s;q) = \int \mathrm{d}^4x\, \mathrm{e}^{\mathrm{i}qx} j(\vec{b},s;x). \tag{12}$$

If j is treated as a fixed classical source, Eq. (8) becomes the AASB model [9].

Because of the random character of j, the transition probability

$$|\langle 1,2,\dots n\,\text{pions}|\widehat{S}(\vec{b},s)|0\rangle|^2 \tag{13}$$

is something that is defined only in a statistical sense. This means that we must average (13) over the random source $j(\vec{b},s;x)$ to obtain the measurable transition probability

$$|\langle 1,2,\dots,n\,\text{pions}|\widehat{S}(\vec{b},s)|0\rangle|^2_{\text{av}} = \mathrm{Tr}\{\varrho(\vec{b},s)|\text{ pions}\rangle\langle\text{ pions}|\} = \frac{1}{4s^2}|T_n(\vec{b},s;1\dots n)|^2. \tag{14}$$

The density operator ϱ is defined by Eq. (1).

In terms of the pion-number operator

$$N = \int \mathrm{d}q\, a^{\dagger}(q)\, a(q), \tag{15}$$

the S matrix with no pions emitted can be written as

$$|\langle 0|\widehat{S}(\vec{b},s)|0\rangle|^2_{\rm av} = {\rm Tr}\{\varrho(\vec{b},s) : {\rm e}^{-N} :\} = \langle \exp\{-A(\vec{b},s)\}\rangle, \tag{16}$$

where the angular brackets $\langle\ldots\rangle$ denote an average in the ensemble of classical random source $j(\vec{b},s;x)$. The connection with the inelastic cross section and the exclusive cross section for the production of n pions is

$$\begin{aligned}\sigma_{\rm inel}(\vec{b},s) &= 1 - \langle \exp[-A(\vec{b},s)]\rangle \\ &= 1 - \exp\{-\Omega(\vec{b},s)\},\end{aligned} \tag{17}$$

and

$$\sigma_n(\vec{b},s) = \left\langle \frac{[A(\vec{b},s)]^n}{n!} \exp[-A(\vec{b},s)]\right\rangle, \quad n \geq 1, \tag{17$'$}$$

where $\Omega(\vec{b},s) = -\ln\langle\exp[-A(\vec{b},s)]\rangle$ is the usual eikonal function (or the opacity function) of the geometrical model [12]. Note that $\langle A(\vec{b},s)\rangle$ is related to the average number of emitted pions at fixed $\vec{b}$:

$$n(\vec{b},s)\sigma_{\rm inel}(\vec{b},s) = \langle A(\vec{b},s)\rangle. \tag{18}$$

The pion-multiplicity distribution $P_n(\vec{b},s) = \sigma_n(\vec{b},s)/\sigma_{\rm inel}(\vec{b},s)$ in the impact--parameter space $\vec{b}$ is most easily studied using the pion-generating function [13]

$$\begin{aligned}Q(\vec{b},s;\lambda) &= {\rm Tr}\{\varrho(\vec{b},s) : {\rm e}^{-\lambda N} :\} \\ &= \sum_{n=0}^{\infty}(1-\lambda)^n \sigma_n(\vec{b},s) \\ &= \langle \exp[-\lambda A(\vec{b},s)]\rangle.\end{aligned} \tag{19}$$

Note that

$$\begin{aligned}\sigma_{\rm inel}(\vec{b},s) &= 1 - Q(\vec{b},s;1), \\ \sigma_n(\vec{b},s) &= (-1)^n \frac{1}{n!}\,\partial^n_\lambda Q(\vec{b},s,1).\end{aligned} \tag{20}$$

4. *Chaotic-coherent mixture and the KNO scaling*

In proton-proton collisions for energies $\sqrt{s} < 100$ GeV the transverse momenta of pions are sharply limited $\langle q_T \rangle = \text{const.}$ As a result, correlations in the transverse momenta are not expected to play too strong a role in determining the energy dependence of the S matrix. In this spirit, the source function J may be written in the separable form

$$J(\vec{b}, s; q) = g(\vec{q}_T) f(\vec{b}, s; y). \tag{21}$$

Since we are interested only in the rapidity interval $|y| \leq \frac{1}{2}(1-\varepsilon)Y$, we can expand $f(\vec{b}, s; y)$ in a Fourier series over the rapidity interval Y:

$$f(\vec{b}, s; y) = \sum_{l=0}^{\infty} g_l(\vec{b}, s) u_l(y), \tag{22}$$

where

$$u_l(y) = Y^{-1/2} \exp[-2\pi \mathrm{i} \frac{l}{Y} y] \tag{23}$$

and

$$\int\limits_{-Y/2}^{Y/2} u_l(y)\, u^*_{l'}(y)\, \mathrm{d}y = \delta_{ll'}.$$

Let us define

$$Y G(y) = \Big\langle \int \mathrm{d}y'\, f^*(y')\, f(y'+y) \Big\rangle. \tag{24}$$

For simplicity, we have used the abbreviations

$$f(\vec{b}, s; y) \equiv f(y) \quad \text{and} \quad G(\vec{b}, s; y) \equiv G(y).$$

The averaging procedure is now performed over the random Fourier coefficients $g_l \equiv g_l(\vec{b}, s)$.

For the independent Gaussian random coefficients g_l we find

$$G(y) = \frac{1}{Y} \sum_l \mathrm{e}^{-2\pi \mathrm{i} \frac{l}{Y} y} G_l, \tag{25}$$

where

$$G_l = \langle g_l^* g_l \rangle = \frac{1}{\pi} \int \mathrm{d}^2 g_l\, |g_l|^2 \frac{1}{G_l} \exp[-\frac{|g_l|^2}{G_l}]. \tag{26}$$

The generating function $Q(\lambda) \equiv Q(\vec{b}, s; \lambda)$ is

$$Q(\lambda) = \prod_l \frac{1}{1 + \lambda g^2 G_l} = \exp[-\sum_l \ln(1 + \lambda g^2 G_l)], \tag{27}$$

where

$$g^2 = \frac{1}{4\pi} \int \frac{\mathrm{d}^2 q_T}{(2\pi)^2} |g(\vec{q}_T)|^2. \tag{28}$$

The convergence criterion reads

$$G(0) = \frac{1}{Y} \sum_l G_l < \infty. \tag{29}$$

The quantities G_l are obtained from $G(y)$ as

$$G_l = \int_{-Y/2}^{Y/2} G(y) \exp[2\pi \mathrm{i} \frac{l}{Y} y] \mathrm{d}y. \tag{30}$$

Since the mass of the emitted pion is fixed, $f(y)$ should fall when y becomes very large. This means that $G(y)$ will also fall off when y becomes larger than some "correlation length" ξ. More precisely, we define

$$\gamma(y) = \frac{G(y)}{G(0)}; \quad |\gamma(y)| \leq 1, \tag{31}$$

then the quantity

$$\xi = 2 \int_0^{\infty} |\gamma(y)|^2 \, \mathrm{d}y \tag{32}$$

is a possible measure of the "correlation length" if, for example,

$$G(y) \sim \mathrm{e}^{-|y|/\xi}.$$

In the limit when Y is small compared with ξ, we may replace $G(y)$ by $G(0) \exp(-2\pi \mathrm{i} \frac{l_0}{Y} y)$ where l_0/Y plays the role of the centre of rapidity frequency. Hence (30) shows that $G_{l_0} = Y G(0)$ with all other G_l vanishing. This leads to the generating function Q of the form

$$Q_{\mathrm{BE}}(\lambda) = \frac{1}{1 + \lambda g^2 Y G(0)}, \tag{33}$$

which is recognized as the generating function for a geometric or Bose-Einstein distribution.

In the next approximation for Y which is not too large ($Y \sim \xi$), we can account for a slow variation of $G(y)$ if we set

$$G(y) = G_0(y) \exp(-2\pi \mathrm{i} \frac{l_0}{Y} y) = G(0) \gamma_0(y) \exp(-2\pi \mathrm{i} \frac{l_0}{Y} y), \tag{34}$$

where $G_0(y) \simeq G(0)$ in the interval $|y| \leq \frac{1}{2} Y$.

The largest term is obviously

$$G_{l_0} = \int_{-Y/2}^{Y/2} G_0(y) \, \mathrm{d}y = G(0) \int_{-Y/2}^{Y/2} \gamma_0(y) \, \mathrm{d}y, \tag{35}$$

while the remaining terms

$$G_{l_0+l} = \int_{-Y/2}^{Y/2} G_0(y) \exp[2\pi \mathrm{i} \frac{l}{Y} y] \, \mathrm{d}y \tag{36}$$

are assumed to be small for all $l \neq 0$.

For the remaining terms in (36) we expand the logarithm in (27) to the first order so as to obtain

$$Q(\lambda) = \frac{1}{1 + \lambda g^2 G_{l_0}} \exp[-\lambda g^2 \sum{}' G_{l_0+l}], \tag{37}$$

where the prime signifies that the term $l = 0$ has been omitted from the sum. Eq. (37) is recognized as the product of two generating functions [14] – one for a Bose-Einstein distribution and the other for a Poisson distribution. It follows that the resultant distribution $\sigma_n(\vec{b}, s)$ is given as a convolution:

$$\sigma_n(\vec{b}, s) = \sum_{m=0} \sigma_{\mathrm{BE}}(n - m; \vec{b}, s) \sigma_{\mathrm{Poisson}}(m; \vec{b}, s). \tag{38}$$

It is also easy to check that

$$\sum_n n \int \sigma_n(\vec{b}, s) \, \mathrm{d}^2 b = \sigma_{\mathrm{inel}}(s) \overline{n}(s) = \sigma_{\mathrm{BE}}(s) \overline{n}_{\mathrm{BE}}(s) + \sigma_{\mathrm{P}}(s) \overline{n}_{\mathrm{P}}(s). \tag{39}$$

Note that $\sigma_{\mathrm{BE}} + \sigma_{\mathrm{P}} \neq \sigma_{\mathrm{inel}}$.

In the limiting form at energies, $Y \gg \xi$; one sees that all G_l for wich $l \ll Y/\xi$ are equal and that G_l become negligible for $l \gg Y/\xi$. We regard G_l as a slowly varying function of l. In this case it is possible to use a continuum approximation and replace $\sum_l$ by $Y \int \mathrm{d}\nu$:

$$Q(\lambda) = \exp\{-Y \int_0^\infty \mathrm{d}\nu \, \ln(1 + \lambda g^2 \widetilde{G}(\nu))\}, \tag{40}$$

where we have sensibly chosen

$$\widetilde{G}(\nu) = \int\limits_{-\infty}^{+\infty} G(y) \exp[2\pi \mathrm{i}\nu y] \,\mathrm{d}y. \tag{41}$$

If we set $\lambda = 1 - \mathrm{e}^{\mathrm{i}x}$, then

$$\sigma_n(\vec{b}, s) = \frac{1}{2\pi} \int\limits_{-\pi}^{\pi} \mathrm{d}x \exp\{-\mathrm{i}nx - Y \int\limits_{0}^{\infty} \mathrm{d}\nu \ln[1 + (1 - \mathrm{e}^{\mathrm{i}x}) g^2 \widetilde{G}(\nu)]\}. \tag{42}$$

This expression is valid for large Y and a relatively broad spectrum $\widetilde{G}(\nu)$, both of which make

$$\overline{n}(s) = \sum_n n \int \sigma_n(\vec{b}, s) \mathrm{d}^2 b \gg 1. \tag{43}$$

It is then reasonable to assume that $P_n(s) = [\sigma_{\mathrm{inel}}(s)]^{-1} \int \sigma_n(\vec{b}, s)\, \mathrm{d}^2 b$ is a slowly varying function of n and that we can approximate $P_n(s)$ by a smooth density function $\psi(z, s)$ interpolated on the basis of the points

$$\psi(z_n, s) \approx \overline{n}(s) P_n(s), \tag{44}$$

where

$$z_n = n/\overline{n}(s). \tag{45}$$

The normalization is such that

$$\int\limits_{0}^{\infty} \psi(z, s) \mathrm{d}z = \sum_n \psi(z_n, s) \frac{1}{\overline{n}(s)} = 1. \tag{46}$$

The smooth approximation is obtained by replacing the integration limits for $x \to$ $\to x\overline{n}$ by $\pm\infty$, and retaining only the first term in an expansion of x in the logarithm (42), that is

$$\overline{n}(s) \sigma_n(\vec{b}, s) \to (2\pi)^{-1} \int\limits_{-\infty}^{+\infty} \mathrm{d}x \exp\{-\mathrm{i}zx - Y \int\limits_{0}^{\infty} \mathrm{d}\nu \ln[1 - \mathrm{i}\frac{x}{\overline{n}(s)} g^2 \widetilde{G}(\nu)]\}. \tag{47}$$

This expression may be viewed as the limit of (44) as $\overline{n} \to \infty$, $n \to \infty$, $\widetilde{G}(\nu) \to \infty$ and $z_n \to z$ and $\widetilde{G}(\nu)/\overline{n} \sim$ finite.

For all practical purposes, when $n, \overline{n} \gg 1$, we may use the distribution

$$P_n(s) = \overline{n}^{\,-1} \psi\left(\frac{n}{\overline{n}}, s\right), \tag{48}$$

where $\psi(z,s)$ is defined by (47). The probability distribution P_n will exhibit the KNO scaling property if $\widetilde{G}(\nu) \equiv \widetilde{G}(\vec{b},s;\nu)$ depends only on the scaled impact parameter $b(s) = b/R(s)$.

5. Negative–binomal distributions

To illustrate the effectiveness of the procedure proposed in the preceding section, let us consider a simple model of the square spectrum of $\widetilde{G}(\nu)$, which nevertheless gives all the main features of pp collisions in the ISR energy region.

We assume that $\widetilde{G}(\nu)$ is of the form

$$\widetilde{G}(\nu) = \frac{Y}{k}G(0) \quad \text{for} \quad 0 < \nu_1 \le \nu \le \nu_1 + \frac{k}{Y},$$
$$\widetilde{G}(\nu) = 0 \quad \text{elsewhere}, \tag{49}$$

where k is not necessarily an integer. The corresponding pion-field correlation function (25) is

$$G(y) = \int_0^\infty \mathrm{d}\nu\, \widetilde{G}(\nu)\mathrm{e}^{-2\pi\mathrm{i}\nu y}$$
$$= \frac{Y}{k}G(0)\frac{\sin(\pi\frac{k}{Y}y)}{\pi y}\exp\{-2\pi\mathrm{i}(\nu_1 + \frac{k}{2Y})y\}. \tag{50}$$

From (32) we find that the correlation length is

$$\xi = \frac{Y}{k}. \tag{51}$$

The pion generating function obtained using (40) is now

$$Q(\lambda) = \left(1 + \lambda\frac{\langle A\rangle}{k}\right)^{-k}, \tag{52}$$

where $\langle A\rangle \equiv \langle A(\vec{b},s)\rangle = Yg^2G(0)$.

We recognize that (52) represents the generating function of the negative binomial (NB) distribution:

$$\sigma_n^{NB}(\vec{b},s) = \frac{\Gamma(n+k)}{n!\Gamma(k)}\left(\frac{\langle A\rangle}{k}\right)^n\left(1 + \frac{\langle A\rangle}{k}\right)^{-n-k}. \tag{53}$$

The average number of pions is

$$\overline{n}(s) = \frac{1}{\sigma_{\mathrm{inel}}}\int \mathrm{d}^2b\langle A\rangle, \tag{54}$$

where

$$\sigma_{\mathrm{inel}} = \int \mathrm{d}^2 b [1 - (1 + \frac{\langle A \rangle}{k})^{-k}]. \tag{55}$$

If n and $\langle A \rangle$ are both large, we find that $\langle A \rangle \sigma_n(\vec{b}, s)$ depends only on the single variable $n/\langle A \rangle$:

$$\langle A \rangle \sigma_n(\vec{b}, s) = \frac{k^k}{\Gamma(k)} \left(\frac{n}{\langle A \rangle} \right)^{k-1} \mathrm{e}^{-\frac{n}{\langle A \rangle} k}. \tag{56}$$

Of course, there is no quarantee that the simple scaling function (56) will be preserved after integration over b.

The geometrical properties of pp collisions in the ISR energy range 20 - 65 GeV may be related to the geometrical scaling of the pion-field correlation function $G(0) \equiv G(\vec{b}, s; 0)$ i.e. that it depends only on the dimensionless scaling variable $b(s) = b/R(s)$. For simplicity, we assume that $G(0)$ has a form of a step function $\theta(R - b)$, so that

$$\langle A \rangle = A \theta(R - b), \quad A = Y g^2. \tag{57}$$

Other functional forms are also possible (e.g. $\exp(-b^2/R^2)$ etc.). However, they do not essentially change our conclusions concerning the KNO scaling of $P_n(s) = \sigma_n(s)/\sigma_{\mathrm{inel}}(s)$.

In the model given by (57)

$$\overline{n}(s) = A \Big(1 + \frac{A}{k} \Big)^k / \Big[\Big(1 + \frac{A}{k} \Big)^k - 1 \Big]$$
$$\sim A \;\text{ if }\; A \;\text{ is very large,} \tag{58}$$

$$\sigma_{\mathrm{inel}}(s) = \pi R^2 \frac{(1 + \frac{A}{k})^k - 1}{(1 + \frac{A}{k})^k} \sim \pi R^2 \;\text{ if }\; A \;\text{ is large} \tag{59}$$

and

$$P_n(s) = \frac{\Gamma(n+k)}{n! \Gamma(k)} \Big(\frac{A}{k} \Big)^n \Big(1 + \frac{A}{k} \Big)^{-n} / \Big[\Big(1 + \frac{A}{k} \Big)^k - 1 \Big]$$
$$\sim \frac{\Gamma(n+k)}{n! \Gamma(k)} \Big(\frac{A}{k} \Big)^n \Big(1 + \frac{A}{k} \Big)^{-n-k} \;\text{ if }\; A \;\text{ is very large.} \tag{60}$$

The KNO-scaling form is

$$A P_n(s) = \frac{k^k}{\Gamma(k)} \Big(\frac{n}{A} \Big)^{k-1} \exp \Big[- \frac{n}{A} k \Big], \tag{61}$$

where A is related to $\overline{n}(s)$ via Eq. (58).

The interpretation of NB distributions in terms of a stochastic model has been proposed earlier [15]. It assumes stimulated or partially stimulated emission of identical bosons. In stimulated emission, k is an integer, while in partially stimulated emission, k is a continuous parameter. It has been found experimentally [16] that pp collisions at the ISR energies are well fitted by the NB distribution provided that k takes values between 8 and 11.

6. Conclusion

In this paper we have investigated the origin of the KNO scaling in pp collisions at ISR energies. Our approach to pion production is based on the leading-particle approximation in which the colliding protons are considered as a classical random (Gaussian) source of pions. By analyzing the pion-pair correlation function, we have shown that, depending on the ratio of the correlation function length ξ and the rapidity interval $Y = \ln(s/m^2)$, we obtain

a) chaotic (or Bose-Einstein) distribution of pions if $\xi \gg Y$,

b) mixture of chaotic and Poisson (coherent) distribution of pions if $\xi \sim Y$,

c) negative binomial distribution and the KNO scaling if $Y \gg \xi$ and the pion-pair correlation function satisfies the geometrical scaling.

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PIONSKA POLJA KAO GAUSSOVE SLUČAJNE VARIJABLE I KOBA–NIELSEN–OLESENOVO SKALIRANJE

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Diskutirane su pionske raspodjele slučajnog Gaussovog nukleonskog izvora. Dobivena je veza s kaos-koherentnom mješavinom, Koba-Nielsen-Olesen skaliranjem te s negativnom binomnom raspodjelom.