Printed ISSN 1330–0016
Online ISSN 1333–9133
CD ISSN 1333–8390
CODEN FIZBE7

STRING COSMOLOGICAL MODEL WITH STRANGE QUARK MATTER IN KANTOWSKI-SACHS SPACE-TIME

S. D. KATORE[a], K. S. ADHAV[a], R. S. RANE[b] and V. B. KURKURE[a]

[a]*PG Department of Mathematics, SGB Amravati University, Amravati. 444602 India*

[b]*Department of Mathematics, Y. C. Science and Arts College, Mangrulpir, 444403 India*
E-mail addresses: katore777@gmail.com, ati_ksadhav@yahoo.co.in, rsrane53@rediffmail.com, vb_kurkure@rediffmail.com

Received 1 March 2010; Accepted 2 February 2011
Online 26 May 2011

We study the string cloud cosmological model in the context of Kantowski-Sachs space-time. For this purpose, the Einstein's field equations are solved for Kantowski-Sachs space-time with strange quark matter coupled to the string cloud by using the relation between metric potentials. The physical and kinematical parameters of the models are studied.

PACS numbers: 11.27.+d, 98.80.-k UDC 524.83
Keywords: string cosmology, quark matter, Kantowski-Sachs space-time

1. *Introduction*

In recent years, attempts to unify gravity with other fundamental forces in nature are an active field of research at the interface between particle physics and cosmology. According to particle physicists, one of the most important problems of the early universe is the grand unification of the strong and electroweak forces of nature. It was realized that this grand symmetry present in the universe at the very high energy scale had to be broken in order to account for the electroweak and nuclear forces as different fields in the present day low-energy universe. One of the immediate consequences of this symmetry breaking is the formation of topological defects such as monopoles, cosmic strings or domain walls.

Among these topological defects, cosmic strings are believed to be one-dimensional topologically stable objects, which might have been formed during a phase transition in the early universe (Kibble [1]), among others like monopoles and domain walls. The present day configurations of the universe are not contradicted by a large-scale network of the strings in the early universe. They may be one of the sources of density perturbations that are required for the formation of galaxies (Vilenkin [2], Gott [3], Stachel [4], Krori et al. [5], Banerjee et al. [6], Tikekar and Patel [7], Bhattacharjee and Baruah [8]). All that work has been confined to the relativistic string models in the context of general relativity.

One of the interesting consequences of phase transition in the early universe is the formation of strange quark matter. Itoch [9], Bodmar [10] and Witten [11] proposed two ways of formation of quark matter, namely, the quark-hadron phase transition in the early universe and conversion of neutron stars into strange at ultrahigh densities. In this respect, Alcock et al. [12], Haensel [13], Cheng et al. [14], Yavuz et al. [15], Yilmaz [16, 17] studied the quark matter attached to the topological defects in general relativity.

In the present paper, we solve the Einstein's field equations for Kantowski-Sachs space-time with strange quark matter coupled to the string cloud by using the relation between metric potentials. The physical and kinematical parameters of the model are studied. Lastly some discussions are given.

2. *String with quark matter solutions*

We consider the Kantowski-Sachs space-time metric in the form

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + A^2\mathrm{d}r^2 + B^2(\mathrm{d}\psi^2 + \sin^2\psi\mathrm{d}\phi^2)\,, \tag{1}$$

where A and B are functions of t alone. Here (x_1, x_2, x_3, x_4) corresponds to (r, ψ, ϕ, t). The energy momentum tensor for string cloud (Letelier [18]) is taken as

$$T_{ij} = \rho U_i U_j - \lambda x_i x_j\,, \tag{2}$$

where ρ is the rest energy for the cloud of strings with particles attached to them and λ is the string tension density. They are related by

$$\rho = \rho_p + \lambda, \quad \text{or} \quad \rho_p = \rho - \lambda\,, \tag{3}$$

where ρ_p is the particle energy density. The string is free to vibrate and different vibration modes of the string represent the different particle types, since different modes are seen as different masses or spins. So we will take quarks instead of particles in the string cloud. From (3), we obtain in this case

$$\rho = \rho_q + \lambda + B_c, \quad \text{or} \quad \rho_q + B_c = \rho_p = \rho - \lambda\,, \tag{4}$$

where ρ_q is the quark energy density and B_c is a bag constant. Using (4) into equation (2), we have the following energy-momentum tensor for strange quark matter coupled to the string cloud (Yavuz et al. [15])

$$T_{ij} = (\rho_q + \lambda + B_c)U_i U_j - \lambda x_i x_j\,, \tag{5}$$

together with $U_\alpha U^\alpha = -1$, $x_\alpha x^\alpha = 1$ and $U_\alpha x^\alpha = 0$, where U_α is the four velocity $U^\alpha = \delta_4^\alpha$, and x^α is the unit space-like vector in the radial direction $x^\alpha = \delta_1^\alpha$ which represent the strings directions in the cloud, i.e. the anisotropic direction. Using co-moving coordinates systems, we have

$$U^\alpha = (0,0,0,-1) \quad \text{and} \quad x^\alpha = (\frac{1}{A},0,0,0)\,.$$

The Einstein's field equations (with gravitational units $8\pi G = c = 1$)) read

$$R_{ij} - \frac{1}{2}Rg_{ij} = -T_{ij}\,. \tag{6}$$

Using Kantowski-Sachs line element (1), the field equations (6) with (2) take the form

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{1}{B^2} = \lambda\,, \tag{7}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = 0 \tag{8}$$

and

$$\frac{\dot{B}^2}{B^2} + 2\frac{\dot{A}\dot{B}}{AB} + \frac{1}{B^2} = \rho\,, \tag{9}$$

where the dot denotes differentiation with respect to t. The expansion scalar θ and shear scalar σ have the following expressions

$$\theta = u^\alpha_{;\alpha} = \frac{\dot{A}}{A} + 2\frac{\dot{B}}{B} \tag{10}$$

and

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{2}{3}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)^2. \tag{11}$$

Equations (7) – (9) are three independent equations in four unknown quantities A, B, ρ and λ. Therefore, we need one more relation to obtain exact solutions connecting the unknown quantities. We assume that the scalar of expansion (θ) in the model is proportional to the eigenvalue σ_1^1 of the shear tensor σ_i^i. This condition leads to the following relation between the metric coefficients,

$$A = \mu B^n, \tag{12}$$

where μ and n are non-zero constants. If we substitute Eq. (12) into Eq. (8), we have

$$\frac{\ddot{B}}{B} + \frac{n^2}{n+1}\frac{\dot{B}}{B} = 0\,,$$

which on integration gives

$$B = (at + b)^{\gamma} , \tag{13}$$

where a and b are constants of integration. From Eqs. (12) and (13), we get

$$A = \mu(at + b)^{n\gamma}, \tag{14}$$

where $\gamma = \dfrac{n+1}{n^2+n+1}$ is a constant. Hence the geometry of the universe for Kantowski-Sachs string cloud cosmological model can be written as

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + \mu^2(at+b)^{2n\gamma}\mathrm{d}r^2 + (at+b)^{2\gamma}(\mathrm{d}\psi^2 + \sin^2\psi \mathrm{d}\phi^2) . \tag{15}$$

3. Physical and kinematical parameters

The model (15) represents the cosmological model in time with strange quark matter attached to the string cosmology. This model has no initial singularity.

Substituting equations (13) and (14) into equations (7), (9), (10) and (11), we obtain the expressions for the physical and kinematical quantities, the rest energy density (ρ), the string tension density (λ), the particle energy density (ρ_p), the scalar of expansion (θ) and the shear scalar (σ) for the model (15):

$$\rho = \frac{a^2\gamma(2n^2+3n+1)}{(n^2+n+1)(at+b)^2} + \frac{1}{(at+b)^{2\gamma}} ,$$

$$\lambda = \frac{a^2\gamma(-2n^2+n+1)}{(n^2+n+1)(at+b)^2} + \frac{1}{(at+b)^{2\gamma}} ,$$

$$\rho_q + B_c = \rho_p = \rho - \lambda = \frac{2na^2(2n^2+3n+1)}{(n^2+n+1)^2(at+b)^2} ,$$

$$\theta = \frac{a(n+1)(n+2)}{(n^2+n+1)(at+b)} ,$$

and

$$\sigma^2 = \frac{2a^2(n^2-1)^2}{3(n^2+n+1)^2(at+b)^2} .$$

4. Discussion and conclusion

We considered the solutions of the Einstein's field equations in Kantowski-Sachs space-time when the strange quark matter is attached to the string. For the model

(15), we observe that initially for $t = -b/a$, the quantities like the scalar of expansion (θ) and shear scalar (σ^2) diverge. As t increases, θ and σ^2 decrease and finally vanish at large t. This agrees with the results obtained by Back et al. [19], Adams et al. [20], Adcox et al. [21] and Yilmaz [17]. Further, we observe that the string tension density (λ) and the rest energy density (ρ) have initial singularities. However as t increases, the parameters ρ and λ decrease, and finally vanish when $t \to \infty$. But for $t \to -b/a$, ρ and λ are infinite.

For $n = -1/2$, the matter disappears, and $\rho_p = 0$ and $\rho = \lambda = (at + b)^{-4/3}$, which represents the geometric string solution. In this case, $\rho_p = -B_c$ (negative quark energy density). Hence in the geometric string (i.e. $\rho = \lambda$), we observe that the negative quark energy density is proportional to the bag constant B_c of string cosmology. So the bag constant B_c may have negative quark energy density because the bag model is repulsive.

Acknowledgements

The authors are thankful to acknowledge the UGC, New Delhi for sanctioning research project and financial support. They are also thankful to IUCAA, Pune and TIFR, Mumbai for sending preprints and papers.

References

[1] T. W. B. Kibble, J. Phys. A **9** (1976) 1387.
[2] A. Vilenkin, Phys. Rev. D **23** (1981) 853.
[3] G. R. Gott, Appl. Phys. J. **288** (1985) 422.
[4] J. Stachel, Phys. Rev. D **21** (1980) 2171.
[5] K. D. Krori, T. Choudhary, C. R. Mahanta, Gen. Rel. Grav. **22** (1990) 123.
[6] S. Banerjee, B. Bhui, Mon. Not. R. Astron. Soc. **247** (1990) 57.
[7] R.Tikekar, L. K.Patel, Gen. Rel. Grav. **26** (1994) 647.
[8] R. Bhattacharjee, K. K. Baruah, Pure Appl. Math. **32** (2001) 47.
[9] N. Itoch, Prog. Theor. Phys. **44** (1970) 291.
[10] A. R. Bodmar, Phys. Rev. D **4** (1971) 1601.
[11] E. Witten, Phys. Rev. D **30** (1984) 272.
[12] C. Alcock, E. Farthi, A. Olinto, Astrophys. J. **310** (1986) 261.
[13] P. Haensel, J. L. Zdunik, R. Sehaeffer, Astron. Astrophys. **160** (1986) 121.
[14] K. S. Cheng, Z. G. Dai, T. Lu, Int. J. Mod. Phys. D **7** (1998) 139.
[15] I. Yavuz, I. Yilmaz, H. Baysal, Int. J. Mod. Phys. D **14** (2005) 1365.
[16] I. Yilmaz, Phys. Rev. D **71** (2005) 103503.
[17] I. Yilmaz, Gen. Rel. Grav. **38** (2006) 1397.
[18] P. S. Letelier, Phys. Rev. D **28** (1983) 2414.
[19] B. B. Back et al., PHOBOS Collab, Nucl. Phys. A **757** (2005) 28.
[20] J. Adams et al., STAR Collab, Nucl Phys. A **757** (2005) 102.

[21] K. Adcox et al., PHENIX Collab, Nucl Phys. A **757** (2005) 184.

KOZMOLOŠKI MODEL SA STRUNAMA I STRANOM KVARKOVSKOM TVARI U KANTOWSKI-SACHSOVOM PROSTORU-VREMENU

Proučavamo kozmološki model s oblakom struna u Kantowski-Sachsovom prostoru-vremenu. S tim su ciljem riješene Einsteinove jednadžbe polja za Kantowski-Sachsov prostor-vrijeme, sa stranom kvarkovskom tvari vezanom s oblakom struna, primjenom relacija među metričkim potencijalima. Razmatraju se fizički i kinematički parametri modela.