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# CYLINDRICALLY SYMMETRIC MAGNETIZED INHOMOGENEOUS STRING COSMOLOGICAL MODELS OF PERFECT FLUID DISTRIBUTION WITH VARIABLE MAGNETIC PERMEABILITY

RAVI PRAKASH SINGH[a,1] and LALLAN YADAV[b]

[a]*Department of Physics, St., Andrew's College, Gorakhpur - 273 001, India*
*E-mail address: rpsingh_standrewsgkp@yahoo.in*

[b]*Department of Physics, D. D. U. Gorakhpur University, Gorakhpur-273 009, India*
*E-mail address: lallan_yadav07@yahoo.co.in*

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A new class of cylindrically-symmetric magnetized inhomogeneous perfect-fluid string cosmological models with variable magnetic permeability is investigated. We assume that $F_{12}$ is the only non-vanishing component of the electromagnetic field tensor $F_{ij}$. The Maxwell's equations show that $F_{12}$ is the function of $x$ alone, whereas the magnetic permeability $\bar{\mu}$ may be the function of both $x$ and $t$. To get the deterministic solution, it has been assumed that the metric coefficients are separable in the form as $A = f(x)\ell(t)$, $B = g(x)k(t)$, $C = g(x)\nu(t)$. Also, the Einstein field equations have been solved with string source in which magnetic field is absent. Some physical and geometric aspects of the models in the presence and absence of magnetic field are discussed.

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## 1. Introduction

Cosmic strings play an important role in the study of the early universe. These strings arise during the phase transition after the big-bang explosion as the temperature goes down below some critical temperature as predicted by grand unified theories [1] – [5]. It is believed that cosmic strings give rise to density perturbations

[1]Corresponding author

which lead to formation of galaxies [6]. These cosmic strings have stress energy and couple to the gravitational field. Therefore, it is interesting to study the gravitational effect which arises from the strings. The general treatment of strings was initiated by Letelier [7, 8] and Stachel [9]. The occurrence of magnetic fields on galactic scale is a well-established fact today, and their importance for a variety of astrophysical phenomena is generally acknowledged as pointed out by Zel'dovich [10]. Also Harrison [11] has suggested that magnetic field could have a cosmological origin. As a natural consequence, we should include magnetic fields in the energy-momentum tensor of the early universe. The choice of anisotropic cosmological models in Einstein system of field equations leads to the cosmological models more general than the Robertson-Walker model [12]. The presence of primordial magnetic fields in the early stages of evolution of the universe has been discussed by several authors (Misner, Thorne and Wheeler [13]; Asseo and Sol [14]; Pudritz and Silk [15]; Kim, Tribble, and Kronberg [16]; Perley and Taylor [17]; Kronberg, Perry and Zukowski [18]; Wolfe, Lanzetta and Oren [19]; Kulsrud, Cen, Ostriker and Ryu [20]; Barrow [21]). Melvin [22], in his cosmological solution for dust and electromagnetic field, suggested that during the evolution of the universe, the matter was in a highly ionized state and was smoothly coupled with the field, subsequently forming neutral matter as a result of the universe expansion. Hence the presence of magnetic field in the string dust universe is not unrealistic.

Benerjee et al. [23] investigated an axially-symmetric Bianchi type I string dust cosmological model in the presence and absence of magnetic field. The string cosmological models with a magnetic field are also discussed by Chakraborty [24], Tikekar and Patel [25, 26]. Patel and Maharaj [27] investigated stationary rotating world model with magnetic field. Ram and Singh [28] obtained some new exact solutions of string cosmology with and without a source-free magnetic field for Bianchi type I space-time, considered in the different basic form by Carminati and McIntosh [29]. Singh and Singh [30] investigated string cosmological models with magnetic field in the context of space-time with $G_3$ symmetry. Singh [31] studied string cosmology with electromagnetic fields in Bianchi type-II, -VIII and -IX space-times. Lidsey, Wands and Copeland [32] reviewed aspects of super-string cosmology with the emphasis on the cosmological implications of duality symmetries in the theory. Bali et al. [33, 34, 35] investigated Bianchi type-I magnetized string cosmological models.

Cylindrically-symmetric space-time plays an important role in the study of the universe on a scale in which anisotropy and inhomogeneity are not ignored. Inhomogeneous cylindrically-symmetric cosmological models have significant contribution in the understanding of some essential features of the universe such as the formation of galaxies during the early stages of their evolution. Bali and Tyagi [36], Pradhan et al. [37] – [40], Kilinc [41] investigated cylindrically-symmetric inhomogeneous cosmological models in the presence of electromagnetic field. Barrow and Kunze [42, 43] found a wide class of exact cylindrically-symmetric flat and open inhomogeneous string universes. In their solutions, all physical quantities depend on at most one space coordinate and the time. The case of cylindrical symmetry is natural because of the mathematical simplicity of the field equations whenever there exists a direction in which the pressure is equal to energy density. In recent time, cylindrically-symmetric inhomogeneous string cosmological models in the presence of magnetic field have been studied by several authors [44] – [49] in various contexts.

Maxwell considered the magnetic permeability ($\bar{\mu}$) to be a constant for a given material. Maxwell considered the spatial gradient of the magnetic field intensity in the steady state to be exclusively determined by a variation in the velocity of the molecular vortices within the magnetic lines of force. To this day, it is assumed that the magnetic permeability is a constant for a given material. Many authors have investigated string cosmological models with constant/unknown magnetic permeability. But from 'The Double Helix Theory of the Magnetic Field' [50], we must look to a variable magnetic permeability in order to account for variations in magnetic flux density in the steady state, and if we look at the solenoidal magnetic field pattern around a bar magnet, this is not very difficult to visualize. The magnetic field lines are clearly more concentrated at the poles of the magnet than elsewhere. It should be quite obvious that the density of the vortex sea, as denoted by the quantity $\bar{\mu}$, is a variable quantity and that this density visibly varies according to how tightly the magnetic lines of force are packed together [51].

Recently Bali [52] obtained Bianchi type-V magnetized string dust universe with variable magnetic permeability. Kilinc and Yavuz [53] investigated some string cosmological models with magnetic field in cylindrically-symmetric space-time. Motivated by the situation discussed above, in this paper, we revisit these solutions [53] by assuming metric coefficient to be separable in a new form.

This paper is organized as follows. The metric and the field equations are presented in Section 2. In Section 3 we deal with the solution of the field equations by revisiting solutions obtained by Kilinc and Yavuz [53]. Section 4 describes solutions in the absence of magnetic field. Finally, in Section 5 concluding remarks are given.

## 2. The metric and field equations

We consider the metric in the form

$$\mathrm{d}s^2 = A^2(\mathrm{d}x^2 - \mathrm{d}t^2) + B^2\mathrm{d}y^2 + C^2\mathrm{d}z^2, \tag{1}$$

where $A$, $B$ and $C$ are functions of $x$ and $t$. The energy momentum tensor for the string with electromagnetic field has the form

$$T_i^j = (\rho + p)u_i u^j + p g_i^j - \lambda x_i x^j + E_i^j, \tag{2}$$

where $u_i$ and $x_i$ satisfy conditions

$$u^i u_i = -x^i x_i = -1, \tag{3}$$

and

$$u^i x_i = 0. \tag{4}$$

Here $\rho$ is the rest energy density of the system of strings, $p$ is the isotropic pressure, $\lambda$ the tension density of the strings, $x^i$ is a unit space-like vector representing the direction of strings so that $x^1 = 0 = x^2 = x^4$ and $x^3 \neq 0$, and $u^i$ is the four-velocity-vector satisfying the following conditions

$$g_{ij}u^i u^j = -1. \tag{5}$$

In Eq. (2), $E_i^j$ is the electromagnetic field given by Lichnerowicz [54]

$$E_i^j = \bar{\mu}\left[h_l h^l \left(u_i u^j + \frac{1}{2}g_i^j\right) - h_i h^j\right], \tag{6}$$

where $\bar{\mu}$ is the magnetic permeability and $h_i$ the magnetic flux vector defined by

$$h_i = \frac{1}{\bar{\mu}}\, {}^*F_{ji}u^j, \tag{7}$$

where the dual electromagnetic field tensor ${}^*F_{ij}$ is defined by Synge [55]

$${}^*F_{ij} = \frac{\sqrt{-g}}{2}\epsilon_{ijkl}F^{kl}. \tag{8}$$

Here $F_{ij}$ is the electromagnetic field tensor and $\epsilon_{ijkl}$ is the Levi-Civita tensor density.

In the present scenario, the comoving coordinates are taken as

$$u^i = \left(0, 0, 0, \frac{1}{A}\right). \tag{9}$$

We choose the direction of string parallel to the $x$-axis so that

$$x^i = \left(\frac{1}{A}, 0, 0, 0\right). \tag{10}$$

We consider the magnetic field as flowing along the $z$-axis so that $F_{12}$ is the only non-vanishing component of $F_{ij}$. Maxwell's equations

$$F_{ij;k} + F_{jk;i} + F_{ki;j} = 0\,, \tag{11}$$

$$\left[\frac{1}{\bar{\mu}}F^{ij}\right]_{;j} = J^i, \tag{12}$$

require that $F_{12}$ is the function of $x$-alone and the magnetic permeability is the functions of both $x$ and $t$. The semicolon represents a covariant differentiation.

The Einstein's field equations (with $\frac{8\pi G}{c^4} = 1$)

$$R_i^j - \frac{1}{2} R g_i^j = -T_i^j, \tag{13}$$

for the line-element (1) lead to the following system of equations:

$$-\frac{A_1}{A}\left(\frac{B_1}{B} + \frac{C_1}{C}\right) - \frac{A_4}{A}\left(\frac{B_4}{B} + \frac{C_4}{C}\right) - \frac{B_1 C_1}{BC} + \frac{B_4 C_4}{BC} + \frac{C_{44}}{C} + \frac{B_{44}}{B}$$

$$= -pA^2 + \lambda A^2 - \frac{F_{12}^2}{2\bar{\mu} B^2}, \tag{14}$$

$$\frac{A_1}{A}\left(\frac{B_4}{B} + \frac{C_4}{C}\right) + \frac{A_4}{A}\left(\frac{B_1}{B} + \frac{C_1}{C}\right) - \frac{B_{14}}{B} - \frac{C_{14}}{C} = 0, \tag{15}$$

$$\frac{A_{11}}{A} - \frac{A_1^2}{A^2} - \frac{A_{44}}{A} + \frac{A_4^2}{A^2} + \frac{C_{11}}{C} - \frac{C_{44}}{C} = pA^2 + \frac{F_{12}^2}{2\bar{\mu} B^2}, \tag{16}$$

$$\frac{A_{11}}{A} - \frac{A_1^2}{A^2} - \frac{A_{44}}{A} + \frac{A_4^2}{A^2} + \frac{B_{11}}{B} - \frac{B_{44}}{B} = pA^2 - \frac{F_{12}^2}{2\bar{\mu} B^2}, \tag{17}$$

$$\frac{A_1}{A}\left(\frac{B_1}{B} + \frac{C_1}{C}\right) + \frac{A_4}{A}\left(\frac{B_4}{B} + \frac{C_4}{C}\right) - \frac{B_1 C_1}{BC} + \frac{B_4 C_4}{BC} - \frac{C_{11}}{C} - \frac{B_{11}}{B}$$

$$= \rho A^2 + \frac{F_{12}^2}{2\bar{\mu} B^2}, \tag{18}$$

where the sub-indices 1 and 4 in $A$, $B$, $C$ and elsewhere denote ordinary differentiation with respect to $x$ and $t$, respectively.

The velocity field $u^i$ is irrotational. The scalar expansion $\theta$, shear scalar $\sigma^2$, acceleration vector $\dot{u}_i$ and proper volume $V^3$ are, respectively, found to have the following expressions:

$$\theta = u^i_{;i} = \frac{1}{A}\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right), \tag{19}$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{3}\theta^2 - \frac{1}{A^2}\left(\frac{A_4 B_4}{AB} + \frac{B_4 C_4}{BC} + \frac{C_4 A_4}{CA}\right), \tag{20}$$

$$\dot{u}_i = u_{i;j}u^j = \left(\frac{A_1}{A}, 0, 0, 0\right), \tag{21}$$

$$V^3 = \sqrt{-g} = A^2BC, \tag{22}$$

where $g$ is the determinant of the metric (1). Using the field equations and the relations (19) and (20) one obtains the Raychaudhuri's equation as

$$\dot{\theta} = \dot{u}^i_{;i} - \frac{1}{3}\theta^2 - 2\sigma^2 - \frac{1}{2}\rho_p\,, \tag{23}$$

where the dot denotes differentiation with respect to $t$ and

$$R_{ij}u^iu^j = \frac{1}{2}\rho_p\,. \tag{24}$$

With the help of Eqs. (1) - (4), (9) and (10), the Bianchi identity $\left(T^{ij}_{;j}\right)$ is reduced to two equations:

$$\rho_4 - \frac{A_4}{A}\lambda + \left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right)\rho = 0 \tag{25}$$

and

$$\lambda_1 - \frac{A_1}{A}\rho + \left(\frac{A_1}{A} + \frac{B_1}{B} + \frac{C_1}{C}\right)\lambda = 0. \tag{26}$$

Thus, due to all three (strong, weak and dominant) energy conditions, one finds $\rho \geq 0$ and $\rho_p \geq 0$, together with the fact that the sign of $\lambda$ is unrestricted, i.e., it may take positive or negative values, or zero as well.

## 3. *Solution of the field equations*

We revisit the solutions obtained by Kilinc and Yavuz [41]. Equations (16) and (17) lead to

$$\frac{F_{12}^2}{\bar{\mu}B^2} = \frac{B_{44}}{B} - \frac{C_{44}}{C} + \frac{C_{11}}{C} - \frac{B_{11}}{B}\,, \tag{27}$$

and

$$\frac{2A_{11}}{A} - \frac{2A_1^2}{A^2} - \frac{2A_{44}}{A} + \frac{2A_4^2}{A^2} + \frac{C_{11}}{C} + \frac{B_{11}}{B} - \frac{C_{44}}{C} - \frac{B_{44}}{B} = 0. \tag{28}$$

The field equations (14) - (18) constitute a system of five equations with six unknowns parameters $A$, $B$, $C$, $\lambda$, $\rho$ and $F_{12}$. Therefore, some additional constraints relating these parameters are required to obtain explicit solutions of the system of equations. Assuming that the metric coefficients are separable in the following way

$$A = f(x)\ell(t), \tag{29}$$

$$B = g(x)k(t), \tag{30}$$

$$C = g(x)\nu(t), \tag{31}$$

and

$$\frac{\ell_4}{\ell} = m \text{ (constant)}. \tag{32}$$

From Eq. (15) we get

$$\frac{\dfrac{k_4}{k} + \dfrac{\nu_4}{\nu}}{\dfrac{k_4}{k} + \dfrac{\nu_4}{\nu} - \dfrac{2\ell_4}{\ell}} = \frac{\dfrac{g_1}{g}}{\dfrac{f_1}{f}} = n\text{(constant)}. \tag{33}$$

Eq. (33) leads to

$$\frac{g_1}{g} = n\frac{f_1}{f}, \tag{34}$$

which after integration gives

$$g = \alpha f^n, \tag{35}$$

where $\alpha$ ($\neq 0$) is a constant of integration. Eqs. (32) and (33) reduce to

$$\frac{k_4}{k} = -a - \frac{\nu_4}{\nu}, \tag{36}$$

where $a = \dfrac{2mn}{1-n}$ is constant. From Eq. (32) we get

$$\ell = e^{mt}. \tag{37}$$

Using Eqs. (29) - (31) in (28), we get

$$2\left(\frac{f_{11}}{f} - \frac{f_1^2}{f^2}\right) + 2\frac{g_{11}}{g} = \frac{k_{44}}{k} + \frac{\nu_{44}}{\nu} = s \text{ (constant)}. \tag{38}$$

From the right-hand side of Eq. (38), we have

$$\frac{k_{44}}{k} + \frac{\nu_{44}}{\nu} = s. \tag{39}$$

Equations (36) and (39) lead to

$$\nu_4^2 + a\nu\nu_4 + \left(\frac{a^2 - s}{2}\right)\nu^2 = 0. \tag{40}$$

The differential equation (40) has two solutions

$$\nu = e^{-\frac{1}{2}(a \pm \sqrt{2s - a^2})t}. \tag{41}$$

Now we consider the following two cases.

### *3.1. Model I*

Taking negative sign, Eq. (41) leads to

$$\nu = e^{\frac{1}{2}(-a+\sqrt{2s-a^2})t}. \tag{42}$$

From Eqs. (36) and (42), we get

$$k = e^{-\frac{1}{2}(a+\sqrt{2s-a^2})t}. \tag{43}$$

From the left-hand side of Eq. (38), we have

$$2\frac{f_{11}}{f} - 2\frac{f_1^2}{f^2} + 2\frac{g_{11}}{g} = s. \tag{44}$$

From Eqs. (34) and (44), we have

$$\frac{f_{11}}{f} + \frac{2(n^2-n-1)}{(2n+1)}\frac{f_1^2}{f^2} = s, \tag{45}$$

which leads to

$$f = \left(c_1 e^{bx} + c_2 e^{-bx}\right)^{\frac{2n+1}{2n^2-1}}, \tag{46}$$

where $b = \sqrt{\dfrac{(2n^2-1)s}{(2n+1)}}$ and $c_1$, $c_2$ are constants of integration. Hence Eq. (35) gives

$$g = \alpha\left(c_1 e^{bx} + c_2 e^{-bx}\right)^{\frac{n(2n+1)}{2n^2-1}}. \tag{47}$$

Therefore, we have

$$A = \left(c_1 e^{bx} + c_2 e^{-bx}\right)^{\frac{2n+1}{2n^2-1}} e^{mt}, \tag{48}$$

$$B = \alpha\left(c_1 e^{bx} + c_2 e^{-bx}\right)^{\frac{n(2n+1)}{2n^2-1}} e^{-\frac{1}{2}(a+\sqrt{2s-a^2})t}, \tag{49}$$

$$C = \alpha\left(c_1 e^{bx} + c_2 e^{-bx}\right)^{\frac{n(2n+1)}{2n^2-1}} e^{\frac{1}{2}(-a+\sqrt{2s-a^2})t}. \tag{50}$$

Hence, the metric (1) reduces to

$$\begin{aligned}
\mathrm{d}s^2 &= \left(c_1 e^{bx} + c_2 e^{-bx}\right)^{\frac{2(2n+1)}{2n^2-1}} e^{2mt}(\mathrm{d}x^2 - \mathrm{d}t^2) \\
&+ \alpha^2\left(c_1 e^{bx} + c_2 e^{-bx}\right)^{\frac{2n(2n+1)}{2n^2-1}} e^{-(a+\sqrt{2s-a^2})t}\mathrm{d}y^2 \\
&+ \alpha^2\left(c_1 e^{bx} + c_2 e^{-bx}\right)^{\frac{2n(2n+1)}{2n^2-1}} e^{(-a+\sqrt{2s-a^2})t}\mathrm{d}z^2.
\end{aligned} \tag{51}$$

Since the magnetic permeability is a variable quantity, we have assumed it as

$$\bar{\mu} = e^{(a+\sqrt{2s-a^2})t}. \tag{52}$$

Thus $\bar{\mu} \to 0$ as $t \to \infty$ and $\bar{\mu} = 1$ when $t \to 0$. Zel'dovich [56] has explained that $\rho_s/\rho_c \sim 2.5 \times 10^{-3}$, where $\rho_s$ is the mass density and $\rho_c$ the critical density then the string frozen in plasma would change their density like $R^{-2}$ i.e. like $t^{-1}$ in the radiation dominated universe where $R$ is the radius of the universe.

### Some physical and geometric properties of the model

The pressure ($p$), the string tension density ($\lambda$), the energy density ($\rho$) and the particle density ($\rho_p$) for the model (51) are given by

$$p = \frac{1}{(c_1 e^{bx} + c_2 e^{-bx})^{\frac{2(2n+1)}{2n^2-1}} e^{2mt}} \left[ -\frac{1}{2}s + \frac{(2n+1)(n+1)b^2}{(2n^2-1)} \right.$$

$$\left. + \frac{(2n+1)(1+n-n^2)b^2}{(2n^2-1)^2} \frac{(c_1 e^{bx} - c_2 e^{-bx})^2}{(c_1 e^{bx} + c_2 e^{-bx})^2} \right], \tag{53}$$

$$\lambda = \frac{1}{(c_1 e^{bx} + c_2 e^{-bx})^{\frac{2(2n+1)}{2n^2-1}} e^{2mt}} \left[ ma + \frac{1}{2}(s + a^2 + a\sqrt{2s-a^2}) \right.$$

$$\left. - \frac{n(n+2)(2n+1)^2 b^2}{(2n^2-1)^2} \frac{(c_1 e^{bx} - c_2 e^{-bx})^2}{(c_1 e^{bx} + c_2 e^{-bx})^2} \right], \tag{54}$$

$$\rho = \frac{1}{(c_1 e^{bx} + c_2 e^{-bx})^{\frac{2(2n+1)}{2n^2-1}} e^{2mt}} \left[ -ma - \frac{1}{2}(s - a^2 + a\sqrt{2s-a^2}) \right.$$

$$\left. - \frac{2n(2n+1)b^2}{(2n^2-1)} - \frac{n^2(4n^2-1)b^2}{(2n^2-1)^2} \frac{(c_1 e^{bx} - c_2 e^{-bx})^2}{(c_1 e^{bx} + c_2 e^{-bx})^2} \right], \tag{55}$$

$$\rho_p = \frac{1}{(c_1 e^{bx} + c_2 e^{-bx})^{\frac{2(2n+1)}{2n^2-1}} e^{2mt}} \left[ -s - a(2m + \sqrt{2s-a^2}) \right.$$

$$\left. - \frac{2n(2n+1)b^2}{(2n^2-1)} + \frac{2n(3n+1)(2n+1)b^2}{(2n^2-1)^2} \frac{(c_1 e^{bx} - c_2 e^{-bx})^2}{(c_1 e^{bx} + c_2 e^{-bx})^2} \right]. \tag{56}$$

The non-vanishing component $F_{12}$ of electromagnetic field tensor for model (51) is given by

$$F_{12}^2 = \alpha^2 a\sqrt{2s-a^2}\left(c_1 e^{bx} + c_2 e^{-bx}\right)^{\frac{2n(2n+1)}{2n^2-1}}, \tag{57}$$

which is function of $x$ alone as it is required by Maxwell's equations (11) and (12). The scalar of expansion ($\theta$), shear tensor ($\sigma$), acceleration vector ($\dot{u}_i$) and the proper volume ($V^3$) for the model (51) are given by

$$\theta = \frac{(m-a)}{\left(c_1 e^{bx} + c_2 e^{-bx}\right)^{\frac{2n+1}{2n^2-1}} e^{mt}}, \tag{58}$$

$$\sigma^2 = \frac{(2m^2 + 2ma + 3s - a^2)}{6\left(c_1 e^{bx} + c_2 e^{-bx}\right)^{\frac{2(2n+1)}{2n^2-1}} e^{2mt}}, \tag{59}$$

$$\dot{u}^i = \left(\frac{(2n+1)b}{(2n^2-1)}\frac{(c_1 e^{bx} - c_2 e^{-bx})^2}{(c_1 e^{bx} + c_2 e^{-bx})^2}, 0, 0, 0\right), \tag{60}$$

$$V^3 = \alpha^2 (c_1 e^{bx} + c_2 e^{-bx})^{\frac{2(2n^2+3n+1)}{2n^2-1}} e^{(2m-a)t}. \tag{61}$$

From Eqs. (58) and (59), we have

$$\frac{\sigma^2}{\theta^2} = \frac{(2m^2 + 2ma + 3s - a^2)}{6(m-a)^2} = \text{constant}. \tag{62}$$

The model (51) is expanding (for $m < 0$ only), shearing, accelerating, non-rotating and singularity-free model. For $m > 0$, $\theta$ = constant when $t = 0$ and $\theta \to 0$ when $t \to \infty$. For $m < 0$, $\theta$ = constant when $t = 0$ and $\theta \to \infty$ when $t \to \infty$. From Eq. (60), we observe that for $b = 0$ or $n = -\frac{1}{2}$, $\dot{u}^i$ vanishes. In this case the pressure $p$, the energy density $\rho$, the string tension density $\lambda$ and particle density $\rho_p$ tend to a constant value as $t \to 0$ and $x \to 0$. At a later stage $p$, $\rho$, $\lambda$ and $\rho_p$ approach zero when $t \to \infty$ and $x \to \infty$, as expected. For suitable values of the constants, our solution satisfies the energy conditions $\rho > 0$, $\rho_p \geq 0$. The electromagnetic field tensor $F_{12}$ does not vanish if $s \neq \dfrac{a^2}{2}$. The proper volume increases as time increases. Since $\dfrac{\sigma}{\theta}$ = constant, the model does not approach isotropy.

### 3.2. *Model II*

Taking positive sign, Eq. (41) leads to

$$\nu = e^{-\frac{1}{2}(a+\sqrt{2s-a^2})t}. \tag{63}$$

From Eqs. (36) and (63), we get

$$k = e^{\frac{1}{2}(-a+\sqrt{2s-a^2})t}. \tag{64}$$

Therefore, we have

$$A = \left(c_1 e^{bx} + c_2 e^{-bx}\right)^{\frac{2n+1}{2n^2-1}} e^{mt}, \tag{65}$$

$$B = \alpha \left(c_1 e^{bx} + c_2 e^{-bx}\right)^{\frac{n(2n+1)}{2n^2-1}} e^{\frac{1}{2}(-a+\sqrt{2s-a^2})t}, \tag{66}$$

$$C = \alpha \left(c_1 e^{bx} + c_2 e^{-bx}\right)^{\frac{n(2n+1)}{2n^2-1}} e^{-\frac{1}{2}(a+\sqrt{2s-a^2})t}. \tag{67}$$

Hence, the metric (1) reduces to

$$\begin{aligned} \mathrm{d}s^2 &= \left(c_1 e^{bx} + c_2 e^{-bx}\right)^{\frac{2(2n+1)}{2n^2-1}} e^{2mt}(\mathrm{d}x^2 - \mathrm{d}t^2) \\ &+ \alpha^2 \left(c_1 e^{bx} + c_2 e^{-bx}\right)^{\frac{2n(2n+1)}{2n^2-1}} e^{(-a+\sqrt{2s-a^2})t}\mathrm{d}y^2 \\ &+ \alpha^2 \left(c_1 e^{bx} + c_2 e^{-bx}\right)^{\frac{2n(2n+1)}{2n^2-1}} e^{-(a+\sqrt{2s-a^2})t}\mathrm{d}z^2. \end{aligned} \tag{68}$$

Since the magnetic permeability is a variable quantity, we have assumed it as

$$\bar{\mu} = e^{-(-a+\sqrt{2s-a^2})t}. \tag{69}$$

Thus we again observe that $\bar{\mu} \to 0$ as $t \to \infty$ and $\bar{\mu} = 1$ when $t \to 0$.

## Some physical and geometric properties of the model

The pressure ($p$), the string tension density ($\lambda$), the energy density ($\rho$) and the particle density ($\rho_p$) for the model (68) are given by

$$\begin{aligned} p = \frac{1}{\left(c_1 e^{bx} + c_2 e^{-bx}\right)^{\frac{2(2n+1)}{2n^2-1}} e^{2mt}} \Bigg[ -\frac{1}{2}s + \frac{(2n+1)(n+1)b^2}{(2n^2-1)} \\ + \frac{(2n+1)(1+n-n^2)b^2}{(2n^2-1)^2} \frac{(c_1 e^{bx} - c_2 e^{-bx})^2}{(c_1 e^{bx} + c_2 e^{-bx})^2} \Bigg], \end{aligned} \tag{70}$$

$$\lambda = \frac{1}{\left(c_1 e^{bx} + c_2 e^{-bx}\right)^{\frac{2(2n+1)}{2n^2-1}} e^{2mt}} \left[ ma + \frac{1}{2}(s + a^2 - a\sqrt{2s - a^2}) \right.$$

$$\left. - \frac{n(n+2)(2n+1)^2 b^2}{(2n^2-1)^2} \frac{(c_1 e^{bx} - c_2 e^{-bx})^2}{(c_1 e^{bx} + c_2 e^{-bx})^2} \right], \tag{71}$$

$$\rho = \frac{1}{\left(c_1 e^{bx} + c_2 e^{-bx}\right)^{\frac{2(2n+1)}{2n^2-1}} e^{2mt}} \left[ -ma + \frac{1}{2}(a^2 - s + a\sqrt{2s - a^2}) \right.$$

$$\left. - \frac{2n(2n+1)b^2}{(2n^2-1)} - \frac{n^2(4n^2-1)b^2}{(2n^2-1)^2} \frac{(c_1 e^{bx} - c_2 e^{-bx})^2}{(c_1 e^{bx} + c_2 e^{-bx})^2} \right], \tag{72}$$

$$\rho_p = \frac{1}{\left(c_1 e^{bx} + c_2 e^{-bx}\right)^{\frac{2(2n+1)}{2n^2-1}} e^{2mt}} \left[ -2ma - s + a\sqrt{2s - a^2}) \right.$$

$$\left. - \frac{2n(2n+1)b^2}{(2n^2-1)} + \frac{2n(3n+1)(2n+1)b^2}{(2n^2-1)^2} \frac{(c_1 e^{bx} - c_2 e^{-bx})^2}{(c_1 e^{bx} + c_2 e^{-bx})^2} \right]. \tag{73}$$

The non-vanishing component $F_{12}$ of electromagnetic field tensor for model (67) is given by

$$F_{12}^2 = \frac{mn\alpha^2}{(n-1)} \sqrt{2s - a^2} \left(c_1 e^{bx} + c_2 e^{-bx}\right)^{\frac{2n(2n+1)}{2n^2-1}}, \tag{74}$$

which is the function of $x$ alone. So it is consistent with the demand of Maxwell's equations (11) and (12) that $F_{12}$ be function of $x$ alone.

The kinematics parameters $\theta$, $\sigma$, $\dot{u}^i$ for the model (68) are the same as in the case I. The model (68) possesses the same behaviour as the previous model (51).

## 4. *Solution in the absence of magnetic field*

In the absence of magnetic field, if we use the same assumption as with the presence of magnetic field, we obtain the following field equations

$$-\frac{A_1}{A}\left(\frac{B_1}{B} + \frac{C_1}{C}\right) - \frac{A_4}{A}\left(\frac{B_4}{B} + \frac{C_4}{C}\right) - \frac{B_1 C_1}{BC} + \frac{B_4 C_4}{BC} + \frac{C_{44}}{C} + \frac{B_{44}}{B}$$

$$= -pA^2 + \lambda A^2, \tag{75}$$

$$\frac{A_1}{A}\left(\frac{B_4}{B}+\frac{C_4}{C}\right)+\frac{A_4}{A}\left(\frac{B_1}{B}+\frac{C_1}{C}\right)-\frac{B_{14}}{B}-\frac{C_{14}}{C}=0, \tag{76}$$

$$\frac{A_{11}}{A}-\frac{A_1^2}{A^2}-\frac{A_{44}}{A}+\frac{A_4^2}{A^2}+\frac{C_{11}}{C}-\frac{C_{44}}{C}=pA^2, \tag{77}$$

$$\frac{A_{11}}{A}-\frac{A_1^2}{A^2}-\frac{A_{44}}{A}+\frac{A_4^2}{A^2}+\frac{B_{11}}{B}-\frac{B_{44}}{B}=pA^2, \tag{78}$$

$$\frac{A_1}{A}\left(\frac{B_1}{B}+\frac{C_1}{C}\right)+\frac{A_4}{A}\left(\frac{B_4}{B}+\frac{C_4}{C}\right)-\frac{B_1C_1}{BC}+\frac{B_4C_4}{BC}-\frac{C_{11}}{C}-\frac{B_{11}}{B}=\rho A^2. \tag{79}$$

Following the same techniques as with the presence of magnetic field, in this case also, we find equations (33) - (40). From Eqs. (77) and (78), we obtain

$$\frac{k_{44}}{k}=\frac{\nu_{44}}{\nu}. \tag{80}$$

Eqs. (39) and (80) leads to

$$\frac{k_{44}}{k}=\frac{s}{2}. \tag{81}$$

Therefore, our solutions will be equivalent to (41). But Eq. (80) will be satisfied if $2s=a^2$. In this case our solutions (41) are reduced only to one equation

$$\nu=e^{-\frac{1}{2}at}. \tag{82}$$

Accordingly, Eq. (36) reduces to

$$k=e^{-\frac{1}{2}at}. \tag{83}$$

But in the absence of magnetic field, the expressions for $f$ and $g$ are the same as with the presence of magnetic field, i.e., given by Eqs. (46) and (47), respectively.

Hence, in this case, we have

$$A=\left(c_1e^{bx}+c_2e^{-bx}\right)^{\frac{2n+1}{2n^2-1}}e^{mt}, \tag{84}$$

$$B=C=\alpha\left(c_1e^{bx}+c_2e^{-bx}\right)^{\frac{n(2n+1)}{2n^2-1}}e^{-\frac{1}{2}at}. \tag{85}$$

Therefore, the geometry of the universe (1) reduces to

$$\mathrm{d}s^2=\left(c_1e^{bx}+c_2e^{-bx}\right)^{\frac{2(2n+1)}{2n^2-1}}e^{2mt}(\mathrm{d}x^2-\mathrm{d}t^2)+$$

$$\alpha^2\left(c_1e^{bx}+c_2e^{-bx}\right)^{\frac{2n(2n+1)}{2n^2-1}}e^{-at}(\mathrm{d}y^2+\mathrm{d}z^2). \tag{86}$$

### Some physical and geometric properties of the model

The pressure $(p)$, the string tension density $(\lambda)$, the energy density $(\rho)$, the particle density $(\rho_p)$ and the kinematic parameters $\theta$, $\sigma$, $\dot{u}_i$ and $V^3$ for the model (86) are given by

$$p = \frac{1}{\left(c_1 e^{bx} + c_2 e^{-bx}\right)^{\frac{2(2n+1)}{2n^2-1}} e^{2mt}} \left[ -\frac{a^2}{4} + \frac{(2n+1)(n+1)b^2}{(2n^2-1)} \right.$$

$$\left. + \frac{(2n+1)(1+n-n^2)b^2}{(2n^2-1)^2} \frac{(c_1 e^{bx} - c_2 e^{-bx})^2}{(c_1 e^{bx} + c_2 e^{-bx})^2} \right], \tag{87}$$

$$\lambda = \frac{1}{\left(c_1 e^{bx} + c_2 e^{-bx}\right)^{\frac{2(2n+1)}{2n^2-1}} e^{2mt}} \left[ a^2 \left(m + \frac{1}{2}\right) + \frac{(2n+1)(n+1)b^2}{(2n^2-1)} \right.$$

$$\left. - \frac{(2n+1)(2n^3+6n^2+n-1)b^2}{(2n^2-1)^2} \frac{(c_1 e^{bx} - c_2 e^{-bx})^2}{(c_1 e^{bx} + c_2 e^{-bx})^2} \right], \tag{88}$$

$$\rho = \frac{1}{\left(c_1 e^{bx} + c_2 e^{-bx}\right)^{\frac{2(2n+1)}{2n^2-1}} e^{2mt}} \left[ -a^2 \left(m - \frac{1}{4}\right) - \frac{2n(2n+1)b^2}{(2n^2-1)} \right.$$

$$\left. - \frac{n^2(4n^2-1)b^2}{(2n^2-1)^2} \frac{(c_1 e^{bx} - c_2 e^{-bx})^2}{(c_1 e^{bx} + c_2 e^{-bx})^2} \right], \tag{89}$$

$$\rho_p = \frac{1}{\left(c_1 e^{bx} + c_2 e^{-bx}\right)^{\frac{2(2n+1)}{2n^2-1}} e^{2mt}} \left[ -a^2 \left(2m + \frac{1}{4}\right) - \frac{(2n+1)(3n+1)b^2}{(2n^2-1)} \right.$$

$$\left. - \frac{(2n+1)(4n^3+5n^2+n-1)b^2}{(2n^2-1)^2} \frac{(c_1 e^{bx} - c_2 e^{-bx})^2}{(c_1 e^{bx} + c_2 e^{-bx})^2} \right], \tag{90}$$

$$\theta = \frac{(m-a)}{\left(c_1 e^{bx} + c_2 e^{-bx}\right)^{\frac{2n+1}{2n^2-1}} e^{mt}}, \tag{91}$$

$$\sigma^2 = \frac{\frac{1}{3}(m-a)^2 + a\left(m - \frac{a}{4}\right)}{\left(c_1 e^{bx} + c_2 e^{-bx}\right)^{\frac{2(2n+1)}{2n^2-1}} e^{2mt}}, \tag{92}$$

$$V^3 = \alpha^2 (c_1 e^{bx} + c_2 e^{-bx})^{\frac{4(2n+1)}{2n^2-1}} e^{4mt}. \tag{93}$$

From Eqs. (91) and (92), we have

$$\frac{\sigma^2}{\theta^2} = \frac{1}{3} + \frac{a(4m-a)}{4(m-a)^2} = \text{constant}. \tag{94}$$

## 5. *Concluding remarks*

In this paper, we have investigated new exact solutions of Einstein's field equations for the cylindrically-symmetric space-times with string source in the presence and absence of magnetic field. In these solutions, we take magnetic field and string together as the source of gravitational field. It is known that magnetic fields are anisotropic stress sources. In general, the expressions for physical and kinematic quantities depend on at most one space coordinate and the time. Our solutions are new and different from the other author's solutions.

If we choose suitable values for the constants, we see that our solutions in the presence and in the absence of magnetic field satisfy energy conditions (strong, weak and dominant). We also observe that our solutions also satisfy the Raychaudhuri's equation. In the presence and absence of magnetic field, the pressure $p$ the energy density $\rho$, the string tension density $\lambda$ and the particle density $\rho_p$ tend to constant values as $t \to 0$ and $x \to 0$. At a later stage, $p$, $\rho$, $\lambda$ and $\rho_p$ approach zero when $t \to \infty$ and $x \to \infty$, as expected. In all these cases, the proper volumes also increase as time increases. For $m > 0$, $\theta =$ constant when $t = 0$ and $\theta \to 0$ when $t \to \infty$. For $m < 0$, $\theta =$ constant when $t = 0$ and $\theta \to \infty$ when $t \to \infty$.

In both cases 3.1 and 3.2, the electromagnetic field tensors given in equations (57) and (74) are function of $x$ alone and hence they match with the requirment of Maxwell's equations (11) and (12). The electromagnetic field tensor does not vanish if $s \neq \dfrac{a^2}{2}$. Either in the presence of magnetic field or in its absence, we observe that all kinematic quantities $\theta$, $\sigma$, $\dot{u}^i$ and proper volume $V^3$ tend to constant values as $t \to 0$ and $x \to 0$. Our solutions are free from singularity. Since $\dfrac{\sigma}{\theta} =$ constant, the models (51), (68) and (86) do not approach to isotropy at any time. For $b = 0$ or $n = -\frac{1}{2}$, the acceleration in the models in the presence of magnetic field vanishes.

Either in the presence of a magnetic field or in its absence, if we put $m = 0$, our solutions become a function of $x$ only. Thus, these solutions reduce to static solutions. For $n = -\frac{1}{2}$, our solutions become homogeneous either in the presence or absence of magnetic field. Therefore, inhomogeneity dies out for $n = -\frac{1}{2}$. In general, our models represent expanding, shearing and non-rotating universe. The magnetic field imposed the restriction on constants such as $a$, $\alpha$ and $s$. It is observed that in the presence of magnetic field, the rate of expansion of the universe is faster than the rate of expansion in the absence of magnetic field. The idea of primordial magnetism is appealing because it can potentially explain all large-scale fields seen in the universe today, especially those found in remote protogalaxies (primeval galaxies). As a result, the literature contains many studies examining the role and the implications of magnetic fields for cosmology.

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# CILINDRIČNO-SIMETRIČNI NEHOMOGENI KOZMOLOŠKI MODELI S MAGNETSKIM POLJEM, STRUNAMA, PERFEKTNOM RASPODJELOM FLUIDA I PROMJENLJIVOM PERMEABILNOŠĆU

Istražujemo novu vrstu cilindrično-simetričnih nehomogenih kozmoloških modela s magnetskim poljem, perfektnim fluidom, strunama i promjenljivom permeabilnošću. Pretpostavljamo da je $F_{12}$ jedina neiščezavajuća sastavnica tenzora elektromagnetskog polja $F_{ij}$. Maxwellove jednadžbe daju da je $F_{12}$ funkcija samo $x$, dok magnetska permeabilnost može ovisiti o $x$ i o $t$. Radi postizanja određenosti rješenja, pretpostavili smo separabilne metričke koeficijente u obliku $A = f(x)\ell(t)$, $B = g(x)k(t)$, $C = g(x)\nu(t)$. Također smo riješili Einsteinove jednadžbe polja sa strunskim izvorom bez prisustva magnetskog polja. Raspravljamo neka fizička i geometrijska svojstva modela u prisustvu i bez magnetskog polja.