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HIGHER-DIMENSIONAL NON-SINGULAR COSMOLOGY DOMINATED BY A VARYING COSMOLOGICAL CONSTANT

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Two of the greatest problems in theoretical physics today are the accelerated expansion of the universe and the singularity problem at the origin of time. This crisis has been attacked head on, but no convincing, well-developed and well-motivated solutions have emerged. While much work in literature has focused on the search for new matter sources that yield accelerating solutions, and to avoid concurrently the initial singularity, more recently complementary approach of examining whether alternative theories of gravity might be responsible for cosmic acceleration and whether the singularity problem may be avoided was developed. The work done in this paper is just a kind of the simple modified gravitational physics to solve both problems simultaneously. In the present work, we analyze all kinematic expressions for a closed but effectively flat universe when the cosmological constant and the matter density decays, respectively, like $\Lambda = C\,\ddot{a}/a$ and $\rho(a) = \beta G^{-1}(a) a^{-2}/(8\pi)$, where C and β are real positive parameters, $a(a)$ is the scale factor and G is the gravitational coupling constant which is assumed to depend on the scale factor as predicted by several models. It is found for some constrained parameters that the universe is closed but effectively flat, free from the initial singularity, dominated by dark energy and is accelerated in time. Expressions for some observable quantities were derived and are found to be compatible with available recent astrophysical findings.

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1. Introduction: The recent available astronomical observations favour a spatially flat universe undergoing a phase of accelerated expansion

The recent available astronomical observations favour a spatially flat universe undergoing a phase of accelerated expansion, dominated by a mysterious form of repulsive energy dubbed *dark energy* (DE) violating the strong energy condition and can be represented with a good approximation by a homogeneous Friedmann-Robertson-Walker (FRW) model [1 − 7]. Recent observations of more than 50 types SNeIa with redshift in the range $-0.10 \leq z \leq 0.83$ strongly favour a positive but tiny lambda with negative equation of state parameter (EoSP), i.e. negative pressure [8 − 12].

Many models were build to explain these facts, including the ΛCDM which consists of a mixture of the cosmological constant Λ and CDM or WIMPS [13], K-essence with modified kinetic energy [14 − 17], Chaplygin gas [18 − 23], generalized Chaplygin gas model [24 − 26], holographic dark energy [27 − 29], varying massive neutrinos mass [30, 31], quintom field [32 − 34] and so on. Despite the fact that most of these theoretical models possess many advantages in the explanation of the cosmological accelerated expansion, most of them are based on a particular choice of the scalar field potential and they predict degenerate luminosity distance history of the universe and thus cannot be distinguished by supernovae measurements alone. However, many theorists still hope to explain the problem without invoking the existence of scalar fields. Although significant efforts have been devoted for this attempt, we still have not succeeded yet to provide convincing cosmological models. However, the rapid progress of alternative theories of gravity coming from string/M-theory (extra-dimensional theories (10D) including braneworld models (5D)) has provided a new perspective for solving many of the cosmological problems, in particular the cosmological constant problem [35]. A deeper insight into DE may lead to interesting results which may be derived from extra-dimensional dynamical compactification theories (from 10D string theory to 4D spacetime). In other words, as has been emphasized, the 4D spacetime could have been preceded by a higher-dimensional spacetime compactified by a certain physical mechanism similar to the 3-brane world mechanisms where Standard Model matter fields are confined to a 4D membrane embedded in a higher-dimensional spacetime without conflict with observations [36, 37]. In brief, cosmological theories with extra-dimensions are of great importance in cosmology. In this paper, extra-dimensions will be our main concern.

Further, observing that the cosmological constant has dimension of inverse length square, several ansatzs have been proposed in which the cosmological constant decays with time [38, 39]. One of the motivations for introducing a decaying-lambda is to reconcile the age parameter and the density parameter of the universe with recent observational data. Since spacetime was strongly curved at early times (Planck times), one naturally expects an initial huge value of the cosmological constant of the order of $l_P^{-2} = G^{-1}$, G is the gravitational coupling constant. As the universe expands in time, the cosmological constant should decay, thereby leading to the tiny value observed presently. Therefore, if the vacuum density decays with time, one should expect that the gravitational constant varies also with time. The variation of G is also expected if we consider it a renormalized coupling constant of a quantum theory of gravity [40]. It should be mentioned that models with variation of Λ and G simultaneously have been discussed in literature [41]. Within the structure of extra-dimensions, the decaying of the effective cosmological constant plays a vital role [42 − 47].

One of the interesting ansatzs introduced in the literature is $\Lambda \propto \ddot{a}/a$, where a is the scale factor of the FRW metric [48]. It has been discussed by several authors in literature [38, 39, 49 − 54].

In this paper we discuss many interesting features of this phenomenological decaying law within the framework of higher-dimensional spacetime. By making the assumption that the extra-dimensions compactify as the visible dimensions expand like, the phenomenological law $\Lambda \propto \ddot{a}/a$ may explain why the effective cosmological constant is reduced from a large value at early times to a sufficiently small value at late times in agreement with the observational upper limit. In this work we make another assumption that is common in modern cosmology: the matter density decreases like $\rho \propto G^{-1}(a)a^{-2}$ as we accept in this paper the fact that the gravitational constant varies in time and thus depends on the scale factor. Our main aim in this work is to explore the cosmological consequences of the above assumptions on the evolution of the extra-dimensional world. We look up for the general solution of the scale factor and the gravitational coupling constant. Further, we discuss whether the results obtained agree with the observed accelerating universe and we hunt for the equation of state parameter of the present model. Many cosmological tests are discussed in some details.

2. Setup: Action, equations of motion and cosmological solutions

We consider the $(n + 2)$-dimensional homogeneous universe described by the combination of the standard $(1 + 3)$ FRW metric and n extra dimensions as [54]

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(\mathrm{d}\theta_1^2 + \sin^2\theta_1\mathrm{d}\theta_2^2 + \ldots + \sin^2\theta_{n-1}\mathrm{d}\theta_n^2)\right], \quad (1)$$

where $\widetilde{\gamma}_{pq}$ is the maximally symmetric metric in n-dimensions. For the perfect fluid distribution, the Einstein field equations with the effective cosmological constant and gravitational constant may be written as

$$R_{ij} - \frac{1}{2}g_{ij}R + \Lambda_{\mathrm{effective}}\, g_{ij} = -8\pi G T_{ij} = -8\pi G\big[(p+\rho)u_i u_j - p g_{ij}\big], \quad (2)$$

where G is the higher-dimensional gravitational coupling constant, p and ρ are, respectively, the pressure and density of the cosmic perfect fluid and finally g_{ij} is the metric tensor. The variations of the gravitational constant, the cosmological constant and the fluid density in cosmological time will in turn influence the expansion of the universe through the generalization of the Friedmann equations which take the form

$$\frac{n(n+1)}{2}\left[\frac{\dot{a}^2}{a^2}+\frac{k}{a^2}\right]=8\pi G(a)\rho(a)+\Lambda(a)\,, \tag{3}$$

$$n\frac{\ddot{a}}{a}+\frac{n(n-1)}{2}\left[\frac{\dot{a}^2}{a^2}+\frac{k}{a^2}\right]=-8\pi G(a)p(a)+\Lambda(a)\,. \tag{4}$$

$k=0,\pm 1$ is the curvature parameter. The Bianchi identity

$$\left(R^{\mu\nu}-\frac{1}{2}g^{\mu\nu}R\right)_{;\mu}=-\big(8\pi G T^{\mu\nu}+\Lambda g^{\mu\nu}\big)_{;\mu}=0 \tag{5}$$

leads to [55]

$$\dot{\rho}+(n+1)(p+\rho)\frac{\dot{a}}{a}=0\,, \tag{6}$$

$$\dot{\Lambda}+8\pi\dot{G}\rho=0\,. \tag{7}$$

In fact, we followed Berman's idea and we conjecture that the variation of Λ is cancelled by the variation of the gravitational coupling constant. It is notable here that our approach suffers from a lack of Lagrangian approach. There is no recognized way to present a consistent Lagrangian cosmological model satisfying the necessary conditions discussed in the paper. As stated previously, we will adopt the phenomenological decay laws: $\Lambda=C\,\ddot{a}/a$ and $\rho(a)=\beta G^{-1}(a)a^{-2}/(8\pi)$, where C and β are real positive parameters. Overdin and Cooperstock [34] have pointed out that the model with $\Lambda\propto H^2$ is equivalent to $\Lambda=C\,\ddot{a}/a$ [48]. For a complete determinacy of the system, we will adopt the EoS $p=\gamma(t)\rho$, where $\gamma(t)$ is a time-dependent parameter. It is noteworthy that quintessence cosmological models involving scalar fields give rise to time-dependent equation of state parameter (EoSP) [56–65]. Consequently, Eqs. (3) and (4) take the forms

$$a\ddot{a}-\frac{n(n+1)}{2C}\,\dot{a}^2=\frac{1}{C}\left[\frac{n(n+1)k}{2}-\beta\right], \tag{8}$$

$$\gamma(t)=-\frac{n(n+1)}{2\beta}\,\dot{a}^2+\frac{C-n}{\beta}a\ddot{a}-\frac{n(n-1)k}{2\beta}\,. \tag{9}$$

The first integral of Eq. (8) is

$$\dot{a}^2=Ea^{n(n+1)/C}-2\left[\frac{k}{2}-\frac{\beta}{n(n+1)}\right], \tag{10}$$

where E is an integration constant. Following the arguments of Refs. [51, 54], the constantβ plays the role of the curvature parameter k. Accordingly, an interesting non-singular solution sorting from Eq. (10) for some range of parameters may be obtained if we assume that $2\beta = n(n+1)k$ which is given by $a(t) = (A+MDt)^{1/M}$, where A and D are integration constants, $M = 1 + \omega$ and $\omega = -n(n+1)/(2C) = -\beta k/C \neq -1$. Notice that for $2C > n(n+1)$, we get $0 < M < 1$ and $2 \leq n < \left(\sqrt{4C+1} - 1\right)/2$, i.e. $C \geq 6$. This shows that there is a possibility of the universe passing through a minimum at the origin of time. In this context, the effective curvature parameter depends on the number of extra-dimensions through the relation $k_{\text{effective}} = 2\beta/(n(n+1))$. In other words, the universe is closed but effectively flat. Consequently, the cosmological constant decays like

$$\Lambda = \frac{CD^2(1-M)}{(A+MDt)^2} \approx CMD^3(1-M)t^{-2} \tag{11}$$

for large time and thus a positive cosmological constant corresponds to $M < 1$, i.e. $\omega < 0$, and consequently $k > 0$ (closed spacetime). The cosmological constant is a decreasing function of time, non-singular at $t = 0$ (large value it might have had) and approaches a small positive value at the present epoch, i.e. at $t = t_0$. At the origin of time, $\Lambda_0 = CD^2(1-M)/A$ and thus $\Lambda = \Lambda_0(A+MDt)^{-2}$. The equation of state parameter in its turn behaves as

$$\gamma(t) = \left(\frac{2(C-n)(n+1)}{2C} - n+1\right)\frac{2CD^2}{(n+1)k(2CA+(2C-n(n+1))Dt)^{-2\beta k/(C-\beta k)}} - \frac{n-1}{n+1}. \tag{12}$$

For $0 < M < 1$, $\beta > 0$ and $C < \beta k$, the universe is non-singular and is accelerating in time and the equation of state parameter decreases in time and tends for very large time to $\gamma = (1-n)/(1+n) > -1$, i.e. the universe is dominated by dark energy. For very large n, $\gamma \to -1$, which corresponds to the famous cosmological constant. Thus, the present accelerated expansion of the universe may be attributed to the ever growing gravity and to the ever decreasing of the matter density through the law $\rho(a) = \beta G^{-1}(a)a^{-2}/(8\pi)$ and not only to the presence of phantom energy.

Substituting Eq. (11) in Eq. (7), one gets easily

$$G = G_0 \exp\left\{\frac{CD^2M}{\beta}\left[(A+MDt)^{2(1-M)/M} - 1\right]\right\}, \tag{13}$$

which corresponds to an ever-increasing gravitational constant. Here $G_0 = G(t = 0)$. The perfect fluid density decays consequently like

$$\rho(t) = \rho(0)\left(\frac{A}{A+MDt}\right)^{2/M} \exp\left\{-\frac{CD^2M}{\beta}\left[(A+MDt)^{2(1-M)/M} - A^{2(1-M)/M}\right]\right\}, \tag{14}$$

with $\rho_0 = \rho(t = 0)$. Thus, the density decreases exponentially in time. It is worth mentioning, the form of G represented by Eq. (13) is obtained within the framework of the quantum corrections to the gravitational potential, in particular in the finite grand unified theories (GUTs) [66]. It is interesting to find an exponential decrease of the matter density while the universe is expanding as a power-law and not exponentially.

The density parameter due to vacuum contribution is

$$\Omega_\Lambda = \frac{2\Lambda}{n(n+1)H^2} = \frac{2C(1-M)}{n(n+1)}\,, \tag{15}$$

where $H = \dot{a}/a$ is the Hubble parameter. The density parameter of the universe in contrast is given by

$$\Omega_m = \frac{16\pi G\rho}{n(n+1)H^2} = \frac{2\beta}{n(n+1)\dot{a}^2} = \frac{2\beta}{n(n+1)D^2(A+MDt)^{2(1-M)/M}}\,, \tag{16}$$

which tends to zero at large times and as a result, the universe is vacuum-dominated. As pointed previously, astrophysical measurements indicate the expansion of the universe is accelerating with time. The energy density is dominated by a vacuum density with $\Omega_\Lambda = 0.7$. At late times, the scale factor tends to infinity and consequently the radiation and matter energy density are driven to zero by the accelerated expansion of the spacetime. It is thus natural that $\Omega_m \to 0$ as $a \to \infty$. Thus our universe is simultaneously closed and asymptotic to a de Sitter space. A vacuum-dominated closed Friedmann universe was found to play a crucial role in holographic theory [67, 68].

For the illustration purpose, we plot the variations of the scale factor (Fig. 1), the energy density (Fig. 2), the equation of state parameter (Fig. 3), the cosmological constant (Fig. 4) and the gravitational coupling constant (Fig. 5) with time and for different values of n. Notice that for $k = 1$, we get $3 \leq \beta < C/2$. To do this, we choose $A = D = 1$, $C = 20$, and $\beta = 3$, i.e. $2 \leq n < 4$ or (a) $n = 2$ (4D) and (b) $n = 3$ (5D).

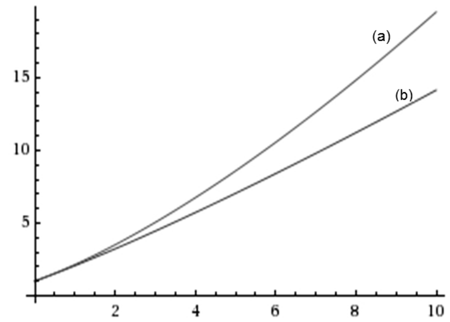

Fig. 1. Plot of the scale factor $a(t) = (A + MDt)^{1/M}$ for $A = D = 1$, $C = 20$, and $\beta = 30$, (a) $n = 2$, $a(t) = (17t/20+1)^{20/17}$ (4D) and (b) $n = 3$, $a(t) = (7t/10+1)^{10/7}$ (5D).

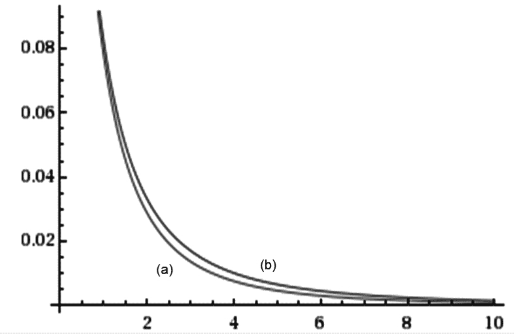

Fig. 2. Plot of the energy density $\rho(t)$, Eq. (14), for $A = D = 1$, $C = 20$, and $\beta = 30$, (a) $n = 2$, $\rho(t) = \exp\left[-\frac{17}{600}\left(\frac{17t}{20}+1\right)^{16/17} - 1\right]\left\{\frac{17t}{20}+1\right\}^{40/17}$ (4D) and (b) $n = 3$ $\exp\left[-\frac{7}{300}\left(\frac{7t}{10}+1\right)^{6/7} - 1\right]\left\{\frac{7t}{10}+1\right\}^{20/7}$ (5D).

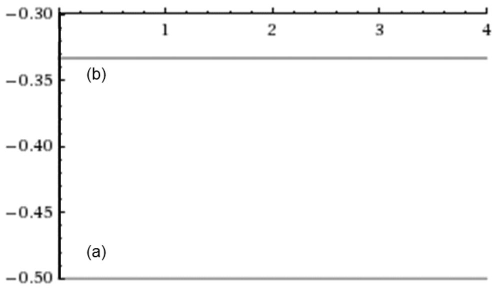

Fig. 3. Plot of $\gamma(t)$, Eq. (12), for $A = D = 1$, $C = 20$, and $\beta = 30$, (a) $n = 2$, $\gamma(t) = \frac{68}{3(34t+40)^6} - \frac{1}{3}$ (4D) and (b) $n = 3$, $\gamma(t) = \frac{14}{(34t+40)^6} - \frac{1}{2}$ (5D).

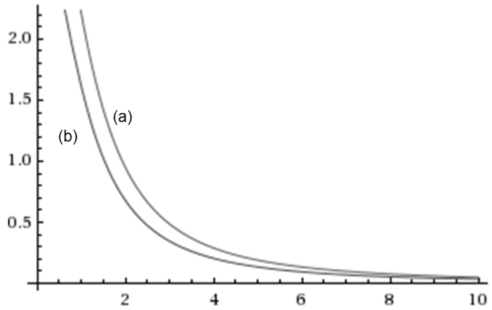

Fig. 4. Plot of Λ, Eq. (11), for $A = D = 1$, $C = 20$, and $\beta = 30$, (a) $n = 2$, $\Lambda = \frac{60}{17t^2+20}$ (4D) and (b) $n = 3$, $\Lambda = \frac{20}{4t^2+5}$ (5D).

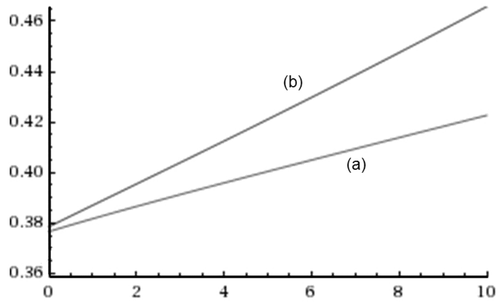

Fig. 5. Plot of G, Eq. (13), for $A = D = 1$, $C = 20$, and $\beta = 30$, (a) $n = 2$, $G = \exp\left[\frac{17}{600}\left(\frac{17t}{20}+1\right)^{16/17}-1\right]$ (4D) and (b) $n = 3$, $G = \exp\left[\frac{7}{300}\left(\frac{7t}{10}+1\right)^{6/7}-1\right]$ (5D).

We may derive now the relationships between the observational parameters, including the age of the universe, proper and luminosity distances and angular distance parameter. For the illustration purpose, we choose $C = 20$. First note that the age of the universe in the present model is calculated easily from the scale factor $a(t) = (A + MDt)^{1/M}$ as:

$$t_0 = \frac{1}{M}\frac{1}{H_0} - \frac{A}{MD} \approx \frac{1}{M}\frac{1}{H_0}, \quad 0 < M < 1\,. \tag{17}$$

The above mentioned observations restrict the dimensionless age parameter H_0t_0 to $H_0t_0 > 1$. Further, if we adopt the conservative bounds $0.85 \lesssim H_0t_0 \lesssim 1.95$ [69−71], it is easily seen that the present non-singular cosmological model solves the age conflict if the allowed value of M is constrained to $0.5 \lesssim M \lesssim 1$. For this range, n is constrained to $2 \leq n \lesssim 4$, i.e. $n = 2, 3, 4$. The value of the cosmological constant, the density parameter due to vacuum contribution and the deceleration parameter at the present time, for $C = 20$, are restricted to satisfy, respectively,

$$0 < \Lambda_0 \lesssim \frac{n(n+1)H_0^2}{2}\,, \tag{18}$$

$$0 < \Omega_{\Lambda 0} \lesssim 1\,, \tag{19}$$

$$-0.5 \lesssim q_0 = M - 1 \lesssim -n(n+1)/40\,, \tag{20}$$

which are in agreement with recent observational limits.

Another interesting neoclassical test concerns the horizon at time t_0, the proper distance traveled by light and emitted at $t = t_1$which is given by

$$d(z) = a(t)\lim_{t_1\to 0}\int \frac{\mathrm{d}t}{a(t)} \approx H_0^{-1}\frac{1}{M-1}\left[1-(1+z)^{1-M}\right], \tag{21}$$

where $1+z = \dot{a}/a$ is the redshift. For small values of z, this equation reduces easily to

$$H_0 d(z) \approx z - \frac{1}{2M} z^2 + \ldots = z - \frac{1}{2(q+1)} z^2 + \ldots . \tag{22}$$

It is observed that for $0.5 \lesssim M \lesssim 1$ and when $z \to \infty$,

$$d(z) \approx \frac{H_0^{-1}}{1-M} (1+z)^{1-M} \approx \frac{H_0^{-1}}{1-M} \left[1 + (1-M)z - \frac{M(1-M)}{2} z^2 + \ldots \right]. \tag{23}$$

As long as z increases, the distance increases and there is no event horizon for M very close to unity. The thrust of this result is that an observer in a power-law-expanding non-singular universe sees only events that take place at a distance no farther away than $H_0^{-1}/(1-M) \gg H_0^{-1}$ for $M \lesssim 1$, i.e. the causal communication between two observers exists. Other kinetic tests may be derived including the luminosity distance-redshift d_L *(the ratio of the detected energy flux and the apparent luminosity)*, the angular distance-redshift d_A *(the measure of how large objects appear to be)* and the look back time-redshift $t_1(z=0) - t_{\max}$ *(the difference between the age of the universe at the present time and the age of the universe when photons were emitted at redshift z)*. Following Ref. [27], it is easy to prove that for $a(t) = (A + MDt)^{1/M} \approx (MDt)^{1/M}$, i.e. larger time,

$$d_L(z) \approx H_0^{-1} \frac{z+1}{M-1} \big[1 - (1+z)^{M-1} \big], \quad 0.5 \lesssim M \lesssim 1 \,, \tag{24}$$

$$d_A(z) = \frac{d(z)}{1+z} = \frac{d_L(z)}{(1+z)^2} \,, \tag{25}$$

$$H_0(t_1 - t_{\max}) \approx \frac{1}{MH_0} \big[1 - (1+z)^{-M} \big]. \tag{26}$$

For $M \lesssim 1$ and small redshift, Eqs. (24)−(26) may be approximated to

$$d_L(z) \approx H_0^{-1} \left[z + \tfrac{1}{2}(1-q) z^2 \ldots \right], \quad -0.5 \lesssim q \lesssim 0 \,, \tag{27}$$

$$d_A(z) = \frac{d_L(z)}{(1+z)^2} \approx z H_0^{-1} \,, \tag{28}$$

$$H_0(t_1 - t_{\max}) \approx z - \left(1 + \frac{q}{2} \right) z^2 + \ldots , \tag{29}$$

which are better approximations to recent available data than the results obtained in Refs. [54, 72].

These relations, in particular the angular distance-redshift and the luminosity distance-redshift are difficult to test observationally for two main reasons: first, objects of fixed size like spherical galaxies do not possess sharp edges used in practice for measuring angular size and consequently one has rather to measure isophotal diameters, while objects with well-defined linear dimensions such as double radio sources, are highly dynamical and as a result their intrinsic size is unknown. Therefore, these neoclassical tests are in fact difficult to use in practice [73].

Finally, notes that the main difference between the present work and the one discussed in Ref. [54] is that in the latter, cosmological implications of the decay law $\Lambda = C\,\ddot{a}/a$ were analyzed in the framework of higher-dimensional universe dominated by dust, while in our framework the universe is dominated by dark energy.

The recent observational available data for an accelerated expansion state of the present universe, obtained from distant SNeIa gave a strong support to the search of alternative cosmologies. Recently, there have been a number of different attempts to modify Einstein's gravity to yield accelerated expansion at late times. Unfortunately, many of the theoretical models discussed in literature are plagued with theoretical problems, in particular the singularity problem at the origin of time.

3. *Conclusions*

In the present work we have analyzed all kinematic expressions for a closed but effectively flat universe when the cosmological constant an the matter density decays, respectively, like $\Lambda = C\,\ddot{a}/a$ and $\rho(a) = \beta G^{-1}(a)a^{-2}/(8\pi)$. As shown in this work, these decay laws represents interesting elements accounting for this unexpected observational result. It was found that the universe is non-singular at the origin of time and is dominated by dark energy with equation of state parameter depending on the dimensionality through the relation $\gamma = -(n-1)/(n+1)$. At large times, the cosmological constant decays to zero from an initially large value it might have had in the early past. The dynamics is accompanied with ever-increasing gravitational constant where the present relative variation rate found is in agreement with recent astrophysical data. It is worth-mentioning that the form of the gravitational constant found in this work is obtained in finite grand unified theories (GUTs). The exponential decay of the matter density leads generically to $\Omega_{m0} \approx 0$ at late times and thus provides a possible solution to the "Ω-problem" for low density cosmological scenarios. In contrast to arguments sorting from quantum cosmology, our model does not predict $a^2\rho(a) = \mathit{constant}$ for a time-dependent cosmological constant. The product is constant only at the origin of time.

Kinematic tests, like luminosity distance, angular diameter and look-back time-redshift constrain perceptively the decaying laws introduced in this work. By making the assumption that the extra-dimensions compactify as the visible dimensions expand, we found that dark energy is needed for the dynamical suppression. The model is free from phantom fields and free from the naked initial singularity in the course of its evolution, i.e. the behaviour of the scale factor is given by the expression $a(t) = (A + MDt)^{1/M}$, $0.5 \lesssim M \lesssim 1$. The horizon distance is infinite at the initial time and no horizon problem appears in this special case. The structure formation problem deserves a special attention and deeper investigations in the future.

The model presented here constitutes in our opinion a very interesting and appealing alternatives to the standard FRW cosmological scenario with an initial singularity, which seems to be eliminated in our framework. However, the initial singularity problem remains a serious problem in cosmology to build a valid cosmology at any period of time. There are some indications that theories like string and superstring may lead to non-singular cosmological models. It will be of interest to explore in the future the effects of the decaying laws presented here in those fundamental theories [74 – 82]. Further consequences and numerical tests are under progress.

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VIŠEDIMENZIJSKA NESINGULARNA KOZMOLOGIJA U KOJOJ PREVLADAVA PROMJENLJIVA KOZMOLOŇA KONSTANTA

Dva među najvećim problemima današnje teorijske fizike su ubrzano širenje svemira i singularnost u njegovu početku. Ti su problemi žestoko napadnuti, ali još nisu iskrsnula uvjerljiva, te dobro osnovana i razvijena rješenja. Dok su objavljeni mnogi radovi koji se usredotočuju na traženje novih vrsta tvari koje daju rješenja s ubrzanim širenjem i istovremenim izbjegavanjem početne singularnosti, u nedavno se vrijeme razvijaju dodatni pristupi u istraživanju jesu li druge teorije gravitacije odgovorne za ubrzanje svemira i može li se izbjeći problem singularnosti. Ovaj je rad upravo drugi pristup kako bi se jednostavnom izmijenjenom gravitacijskom fizikom oba problema istovremeno riješila. Analiziramo sve kinematičke izraze za zatvoren ali u biti ravan svemir, pretpostavljajući da svemirska stalnica i gustoća tvari opadaju kao $\Lambda = C\,\ddot{a}/a$ odnosno $\rho(a) = \beta G^{-1}(a)a^{-2}/(8\pi)$, gdje su C i β realni pozitivni parametri, dok je $a(t)$ množitelj sumjernosti a G gravitacijska stalnica vezanja za koju se pretpostavlja da ovisi o $a(t)$ kao u više modela svemira. Nalazimo za neka ograničenja parametara da je model svemira zatvoren ali u biti ravan, bez početnog singulariteta, u kojemu prevladava tamna tvar i ubrzan je u vremenu. Izveli smo izraze za neke veličine koje se mogu promatrati i nalazimo da su usklađeni s dostupnim nedavnim astrofizičkim nalazima.