Printed ISSN 1330–0016
Online ISSN 1333–9133
CD ISSN 1333–8390
CODEN FIZBE7

FIVE-DIMENSIONAL LRS BIANCHI TYPE-1 STRING COSMOLOGICAL MODEL IN GENERAL RELATIVITY

G. MOHANTY[a] and G. C. SAMANTA[b]

[a]*P. G. Department of Mathematics, Sambalpur University, Jyoti Vihar-768019, India*
703, Rameswarpatna, Bhubaneswar-751002, India
E-mail address: ganeshwarm@yahoo.com

[b]*Department of Mathematics, Gandhi Institute for Technological Advancement (GITA), Madanpur-752054, India* *E-mail address: gauranga1981@yahoo.co.in*

Received 29 March 2010; Revised manuscript received 16 January 2011
Accepted 19 January 2011 Online 15 April 2011

Field equations in the presence of cosmic string source are obtained in general theory of relativity in Kaluza-Klein type space-time. An exact inflationary string cosmological model is obtained in which the sum of tension density and rest energy density of strings vanishes. Some physical and geometrical properties of the model are discussed.

PACS numbers: 04.50.+h, 11.10.Lm, 98.80.Cq UDC 524.83
Keywords: Kaluza-Klein type space-time, string cosmology, general relativity

1. *Introduction*

The Kaluza-Klein theory [1, 2] is very important because it has a particularly elegant presentation in terms of geometry. In certain sense, it looks like an ordinary gravity in free space. A number of authors [3 − 5] studied physics of the universe in higher-dimensional space-time. On the other hand, higher-dimensional space-time is an active field of research in the attempts to unify gravity with other gauge interactions (Scherk and Schwarz [6]). Theoretically the present 4D stage of the universe might have been preceded by a multidimensional phase. The concept of extra dimensions is more important in cosmology particularly at the early stage of the universe.

The concept of string theory was developed to describe the early stage of the universe. Kibble [7] and Vilenkin [8] believed that string may be one of the sources for density perturbation that is required for the formation of large-scale structure of the universe. The construction of string cosmological models was initiated by Vilenkin [9 − 13]. The gravitational effects of strings were studied by several researchers [14, 11, 15]. Letelier [16] and Stachel [17] have given the general relativistic treatment for strings. Using

$$T^i_j = \rho u^i u_j - \lambda w^i w_j \tag{1}$$

as the expression for the energy momentum tensor of a cloud of massive strings, Letelier [10] obtained relativistic cosmological models in Bianchi and Kantowski-Sachs space-times. In Eq. (1), ρ is the proper energy density for a cloud of strings with particle attached to them, λ is the string tension density, u^i is the five-velocity vector of the particles and w^i is a unit space-like vector representing the direction of the string. Thus

$$u^i u_i = -w^i w_i = -1, \qquad u^i w_i = 0\,. \tag{2}$$

If the particle density of the configuration is denoted by ρ_p, then we have

$$\rho = \rho_p + \lambda\,. \tag{3}$$

Nowadays, it is interesting to study string cosmology in five-dimensional space-time in the framework of general relativity. Some of authors [18 − 23] studied string cosmology in higher-dimensional space-time. In this paper, we constructed a five-dimensional locally rotationally symmetric Bianchi type-I string cosmological model in general theory of relativity.

2. *Metric and field equations*

Here we consider the five-dimensional LRS Bianchi type-I space-time in the form

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + A^2\mathrm{d}x^2 + B^2(\mathrm{d}y^2 + \mathrm{d}z^2) + C^2\mathrm{d}m^2, \tag{4}$$

where A, B and C are functions of the 'cosmic time' t only.

We assume the coordinates to be co-moving so that

$$u^0 = 1, \qquad u^1 = u^2 = u^3 = u^4 = 0\,. \tag{5}$$

The Einstein's field equations are given by

$$R_{ij} - \frac{1}{2} g_{ij} R = -8\pi T_{ij}\,. \tag{6}$$

Using Eqs. (1), (2) and (5), the explicit forms of the field equations (6) for the metric (4) are obtained as

$$-\left(\frac{\dot{B}}{B}\right)^2 - 2\frac{\dot{A}\dot{B}}{AB} - 2\frac{\dot{B}\dot{C}}{BC} - \frac{\dot{A}\dot{C}}{AC} = -8\pi\rho\,, \tag{7}$$

$$2\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\left(\frac{\dot{B}}{B}\right)^2+2\frac{\dot{B}\dot{C}}{BC}=8\pi\lambda\,, \tag{8}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{A}\dot{C}}{AC}=0 \tag{9}$$

and

$$\frac{\ddot{A}}{A}+2\frac{\ddot{B}}{B}+2\left(\frac{\dot{B}}{B}\right)^2+2\frac{\dot{A}\dot{B}}{AB}=0\,. \tag{10}$$

Here and afterwards the dot represents ordinary differentiation w. r. t. time t. In order to derive some admissible exact solutions of the field equations (7) – (10), we use the following scale transformations

$$A=\mathrm{e}^{\alpha},\quad B=\mathrm{e}^{\beta},\quad \text{and}\quad C=\mathrm{e}^{\gamma}, \tag{11}$$

$$\mathrm{d}t=AB^2C\mathrm{d}T\,. \tag{12}$$

The field Eqs. (7) – (10) reduce to

$$-\beta'^2-2\alpha'\beta'-2\beta'\gamma'-\alpha'\gamma'=-8\pi\rho\mathrm{e}^{2\alpha+4\beta+2\gamma}, \tag{13}$$

$$2\beta''+\gamma''-\beta'^2-2\alpha'\beta'-2\beta'\gamma'-\alpha'\gamma'=8\pi\lambda\mathrm{e}^{2\alpha+4\beta+2\gamma}, \tag{14}$$

$$\alpha''+\beta''+\gamma''-2\alpha'\beta'-2\beta'\gamma'-\alpha'\gamma'-\beta'^2=0\,, \tag{15}$$

and

$$\alpha''+2\beta''-2\alpha'\beta'-2\beta'\gamma'-\alpha'\gamma'-\beta'^2=0\,. \tag{16}$$

Here and afterwards the prime stands for $\mathrm{d}/\mathrm{d}T$.

From the field equations (15) and (16) we obtain

$$\gamma=\beta+kT+k_1\,, \tag{17}$$

where k and k_1 are arbitrary constants of integration.

Using Eq. (17) in the field Eqs. (13) – (16), one can obtain solution for any arbitrary function β. In order to overcome the shortage of field equations, we consider here

$$\beta=k_2T+k_3\,, \tag{18}$$

where k_2 ($\neq 0$) and k_3 are arbitrary constants. Using Eq. (18) in Eq. (17), we get

$$\gamma=k_4T+k_5\,, \tag{19}$$

where k_4 ($\neq 0$) and k_5 arbitrary constants.

Using Eqs. (18) and (19) in the field equation (15), we obtain

$$\alpha'' - \alpha'(2k_2 + k_4) - k_2(2k_4 + k_2) = 0\,. \tag{20}$$

Equation (20) yields

$$\alpha = c_1 + c_2 \mathrm{e}^{(2k_2+k_4)T} - \frac{2k_4k_2 + k_2^2}{2k_2 + k_4}\,T\,. \tag{21}$$

Thus, the five-dimensional string cosmological model corresponding to the above solution is written as

$$\begin{aligned} \mathrm{d}s^2 = &-\exp\left(c_1 + c_2\mathrm{e}^{(2k_2+k_4)T} - \frac{2k_4k_2 + k_2^2}{2k_2 + k_4}\,T + 2k_2T + k_4T + 2k_3 + k_5\right)\mathrm{d}T^2 \\ &+\exp\left(2c_1 + 2c_2\mathrm{e}^{(2k_2+k_4)T} - \frac{4k_4k_2 + 2k_2^2}{2k_2 + k_4}\,T\right)\mathrm{d}X^2 \\ &+\exp\left(2k_2T + 2k_3\right)(\mathrm{d}Y^2 + \mathrm{d}Z^2) + \exp\left(2k_4T + 2k_5\right)\mathrm{d}M^2, \end{aligned} \tag{22}$$

The rest energy density (ρ), the string tension density (λ), the particle density (ρ_p), the scalar expansion (θ), the shear (σ), the spatial volume (V) and the deceleration parameter (q) for the model (22) are obtained as

$$\rho = \frac{(k_4 + 2k_2)^2 k_2 \mathrm{e}^{(2k_2+k_4)T}}{\exp\left(2c_1 + 2c_2\mathrm{e}^{(2k_2+k_4)T} - \dfrac{4kk_2 + 2k_2^2}{2k_2 + k_4}\,T + 4k_2T + 2k_4T + 4k_3 + 2k_5\right)}\,, \tag{23}$$

$$\lambda = -\frac{(k_4 + 2k_2)^2 k_2 \mathrm{e}^{(2k_2+k_4)T}}{\exp\left(2c_1 + 2c_2\mathrm{e}^{(2k_2+k_4)T} - \dfrac{4kk_2 + 2k_2^2}{2k_2 + k_4}\,T + 4k_2T + 2k_4T + 4k_3 + 2k_5\right)}\,, \tag{24}$$

$$\rho_p = \frac{2(k_4 + 2k_2)^2 k_2 \mathrm{e}^{(2k_2+k_4)T}}{\exp\left(2c_1 + 2c_2\mathrm{e}^{(2k_2+k_4)T} - \dfrac{4kk_2 + 2k_2^2}{2k_2 + k_4}\,T + 4k_2T + 2k_4T + 4k_3 + 2k_5\right)}\,, \tag{25}$$

$$\theta = \frac{c_2(k_4 + 2k_2)\mathrm{e}^{(2k_2+k_4)T} - \dfrac{2k_4k_2 + k_2^2}{2k_2 + k_4} + 2k_2 + k_4}{\exp\left(c_1 + c_2\mathrm{e}^{(2k_2+k_4)T} - \dfrac{2k_4k_2 + k_2^2}{2k_2 + k_4}\,T + 2k_2T + k_4T + 2k_3 + k_5\right)}\,, \tag{26}$$

$$\sigma^2 = \frac{1}{2}\left[\left[\frac{c_2(k_4+2k_2)\mathrm{e}^{(2k_2+k_4)T} - \dfrac{2k_4k_2+k_2^2}{2k_2+k_4}}{\exp\left(c_1 + c_2\mathrm{e}^{(2k_2+k_4)T} - \dfrac{2k_4k_2+k_2^2}{2k_2+k_4}T\right)}\right]^2\right.$$

$$+\frac{1}{4} + \frac{c_2(k_4+2k_2)\mathrm{e}^{(2k_2+k_4)T} - \dfrac{2k_4k_2+k_2^2}{2k_2+k_4}}{\exp\left(c_1 + c_2\mathrm{e}^{(2k_2+k_4)T} - \dfrac{2k_4k_2+k_2^2}{2k_2+k_4}T\right)} \tag{27}$$

$$\left.+2\left(\frac{k_2^2}{(k_2T+k_3)^2} + \frac{1}{4} + \frac{k_2}{k_2T+k_3}\right) + \left(\frac{k_4}{k_4T+k_5}\right)^2 + \frac{1}{4} + \frac{k_4}{k_4T+k_5}\right],$$

$$V = \exp\left(c_1 + c_2\mathrm{e}^{(2k_2+k_4)T} - \frac{2k_4k_2+k_2^2}{2k_2+k_4}T + 2k_2T + k_4T + 2k_3 + k_5\right) \tag{28}$$

and

$$q = -\left[1 + \frac{(2k_2+k_4)^2c_2\mathrm{e}^{(2k_2+k_4)T}}{\left((2k_2+k_4)c_2\mathrm{e}^{(2k_2+k_4)T} - \dfrac{2k_4k_2+k_2^2}{2k_2+k_4} + 2k_2 + k_4\right)^2}\right]. \tag{29}$$

3. *Discussion*

In the preceding section, we obtained a five-dimensional string cosmological model in general theory of relativity given by (22). At the initial epoch $T = 0$, the metric (22) becomes flat. For $k_2 = 0$, we observe that $\rho = \lambda = 0$. Hence the model (22) becomes vacuum cosmological model. For $k_2 = 0$ and $k_4 = 0$, we observe that the model (22) becomes flat and vacuum cosmological model. For $k_4 > 0$, the extra dimension expands indefinitely for very large time which is a physically unrealistic situation. Therefore, to get a physically realistic string cosmological model from the above discussion, we can consider the following relations among the integration constants: $c_2 > 0$, $2k_2 + k_4 > 0$, $k_2 > 0$ and $k_4 < 0$. As the time increases, three spatial coordinates expand indefinitely for $c_2 > 0$, $2k_2+k_4 > 0$, $k_2 > 0$, whereas the extra dimension contracts and becomes unobservable at infinite time for $k_4 < 0$. The model does not admit singularity throughout the evolution. We discuss below the behavior of the physical parameters involved in the model (22) to gain a further insight into the model.

(i) The proper energy density of the model (22) given by (23) satisfies the reality condition $\rho > 0$ if $k_2 > 0$ and $2k_2 + k_4 \neq 0$. At early era ($T = 0$), we have $\rho > 0$ and $\rho_p > 0$. We also get

$$\frac{\rho_p}{|\lambda|} = 2\,. \tag{30}$$

That indicates that strings and particles coexist, and the particles dominate over the strings.

(ii). At initial epoch $T = 0$, The scalar of expansion (θ) given by (26) is finite and $\theta \to 0$ as $T \to \infty$. Hence there is a finite expansion in the model.

(iii) Since $\lim_{T\to\infty} \dfrac{\sigma^2}{\theta^2} \neq 0$, the universe remains anisotropic throughout the evolution.

(iv) The spatial volume of the universe given by (28) increases with time.

(v) The deceleration parameter given by Fienstein et al. [24], (29) being negative, indicates that the cosmological model is inflationary.

(vi) From Eqs. (23) and (24), we obtain

$$\rho + \lambda = 0\,. \tag{31}$$

It is interesting to mention here that this equation of state comes naturally in our case, whereas in Refs. [25], [26], [27], [28] and [22] this equation was imposed to construct cosmological models.

4. *Conclusion*

Cosmologists believe in inflation even though there is no experimental evidence of its existence. This is so because no other mechanism resolves the problems of FRW cosmology so well. The inflationary models propounded by Guth [29] resolve the flatness and horizon problems of the standard cosmological model. In this paper, we obtained that the five-dimensional LRS Bianchi type-1 string cosmological model given by equation (22) is inflationary and we observe that the sum of the proper energy density (ρ) and the string tension density (λ) vanishes, i.e. $\rho + \lambda = 0$.

Acknowledgements

The authors are very much thankful to honorable referees for their valuable comments for the improvement of the paper.

References

[1] T. Kaluza, *Zum Unitätsproblems der Physik*, Sitzungsber. Preuss. Akad. Wiss. Phys. Math. K **1** (1921) 966.

[2] O. Klein, Z. Phys. **37** (1926) 895.

[3] D. Sahdev, Phys. Rev. D **30** (1984) 2495.

[4] V. M. Emelyanov, Yu. P. Nikitin, J. L. Rozentas and A. V. Berkov, Phys. Rep. **143** (1986) 1.

[5] S. Chatterjee and B. Bhui, Int. J. Theor. Phys. **32** (1993) 671.

[6] J. Scherk and J. H. Schwarz, Phys. Lett. B **82** (1979) 60.

[7] T. W. B. Kibble, J. Phys. A **9** (1976) 1387.

[8] A. Vilenkin, Phys. Rep. **121** (1985) 263.

[9] A. Vilenkin, Phys. Rev. D **23** (1981) 852.

[10] P. S. Letelier, Phys. Rev. D **28** (1983) 2414.

[11] J. R. Gott, Astrophys. J. **288** (1985) 422.

[12] K. D. Krori, T. Choudhary and C. R. Mahanta, Gen. Rel. Gravitation **22** (1990) 123.

[13] K. D. Krori, T. Choudhary and C. R. Mahanta, Gen. Rel. Gravitation **26** (1994) 265.

[14] A. Vilenkin, Phys. Rev. D **24** (1981) 1982.

[15] D. Garfinkle, Phys. Rev. D **32** (1985) 1323.

[16] P. S. Letelier, Phys. Rev. D **20** (1979) 1294.

[17] J. Stachel, Phys. Rev. D **21** (1980) 2171.

[18] S. Chatterjee, N. Banerjee and B. Bhui, Int. J. Modern Phys. D **2** (1993) 105.

[19] R. Venkateswarlu and K. Pavankumar, Astrophys. Space Sci. **298** (2005) 403.

[20] D. R. K. Reddy and R. L. Naidu, Astrophys. Space Sci. **307** (2007) 395.

[21] G. Mohanty, G. C. Samanta and K. L. Mahanta, Comm. Physics **17** (2007) 213.

[22] G. Mohanty and G. C. Samanta, Turk. J. Phys. **32** (2008) 251.

[23] G. Mohanty, R. R. Sahoo and B. K. Bishi, Astrophys. Space Sci. **319** (2009) 75.

[24] A. Fienstein, J. Ibanez and P. Labarge, J. Math. Phys. **36** (1995) 4962.

[25] D. R. K. Reddy, Astrophys. Space Sci. **286** (2003) 359.

[26] D. R. K. Reddy, Astrophys. Space Sci. **286** (2003) 365.

[27] D. R. K. Reddy and M. V. S. Rao, Astrophys. Space Sci. **302** (2006) 157.

[28] D. R. K. Reddy and M. V. S. Rao, Astrophys. Space Sci. **305** (2006) 83.

[29] A. Guth, Phys. Rev. D **23** (1981) 347.

PETDIMENZIJSKI LRS BIANCHIJEV MODEL SVEMIRA TIPA 1 SA STRUNAMA U OPĆOJ RELATIVNOSTI

Izveli smo jednadžbe polja opće teorije relativnosti u Kaluza-Kleinovom prostoru-vremenu uz prisustvo svemirskih struna. Dobili smo egzaktan naduvni svemirski model sa strunama u kojemu je zbroj gustoće naprezanja i energije mirovanja struna jednak nuli. Raspravljamo neka fizička i geometrijska svojstva modela