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A COSMOLOGICAL MODEL WITH VARYING G AND Λ TERM IN GENERAL RELATIVITY

R. K. TIWARI[a], S. S. TIWARI[a] and P. PANDEY[b]

[a]*Department of Mathematics, Government Model Science College, Rewa (M.P.), India*
[b]*Department of Physics, Government Model Science College, Rewa (M.P.), India*

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Einstein's field equations with variable gravitational constant and cosmological constants are considered in the presence of perfect fluid for Bianchi type-I universe by assuming the cosmological term proportional to R^{-m} (R is scale factor and m is a constant). The model approaches quasi-isotropic state. The cosmological term decreases with increasing time. We obtain that the present universe is accelerating with a large fraction of cosmological density in the from of cosmological term.

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1. *Introduction*

Cosmological models which are spatially homogeneous and anisotropic play a significant role in the description of the universe at its early stages of evolution. Bianchi type-I space-time is the simplest generalization of the Friedmann-Robertson-Walker (FRW) flat models. There is a significant observational evidence that the expansion of the universe in undergoing a late-time acceleration [1 − 9]. Nowadays, the problem of cosmological constant is one of the most salient and unsettled problems in cosmology. To resolve the problem of a huge difference between the effective cosmological constant observed today and the vacuum energy density predicted by the quantum field theory, several mechanisms have been proposed (Weinberg [10]). A possible way is to consider a varying cosmological term due to the coupling of dynamic degree of freedom with the matter fields of the universe. Models with dynamically decaying cosmological term representing the energy density of vacuum have been studied by several authors [11 − 16].

Cosmological scenarios with a time-varying Λ were proposed by several researchers. A number of models with different decay laws for the variation of cosmological term were investigated during the last two decades: Chen and Wu [17], Pavan [18], Carvalho et al. [19], Lima and Maia [20], Lima and Trodden [21], Arbab and Abdel-Rahman [22], Cunha and Santos [23], Carneiro and Lima [24].

On the other hand numerous modifications of general relativity to allow for a variable G based on different arguments have been proposed [25 – 32]. A lot of work has been done by Saha [33 – 36], in studying the anisotropic Bianchi type-I cosmological model in general relativity with varying G and Λ. Recently, the present author studied Bianchi type-I cosmological models with time dependent G and Λ [37].

In this paper, we study homogeneous Bianchi type-I space-time with variable G and Λ containing matter in the form of a perfect fluid. We obtain solution of the Einstein field equations by assuming the cosmological term proportional to R^{-m} (where R is scale factor and m is constant). The paper is organized as follows. Basic equations of the model are given in Sec. 2 and their solution in Sec. 3. We discuss the model and conclude our results in Sec. 4.

2. The metric and field equations

We consider the Bianchi type-I metric in the orthogonal form

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + A^2(t)\mathrm{d}x^2 + B^2(t)\mathrm{d}y^2 + C^2(t)\mathrm{d}z^2. \tag{1}$$

Matter content is taken to be perfect fluid given by the energy-momentum tensor

$$T_{ij} = (\rho + p)v_i v_j + p g_{ij}\,, \tag{2}$$

where v_i is the four velocity vector of the fluid satisfying

$$g_{ij}v^i v^j = -1\,, \tag{3}$$

and p and ρ are, respectively, the isotropic pressure and energy density of the fluid.

The Einstein's field equations with time-dependent G and Λ are

$$R_{ij} - \frac{1}{2}R_1^1 g_{ij} = -8\pi G(t)T_{ij} + \Lambda(t)g_{ij}\,. \tag{4}$$

For the metric (1) and energy-momentum tensor (2) in the co-moving system of coordinates, the field equation (4) yields

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = -8\pi G p + \Lambda\,, \tag{5}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} = -8\pi G p + \Lambda\,, \tag{6}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -8\pi G p + \Lambda\,, \tag{7}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} = 8\pi G\rho + \Lambda\,. \tag{8}$$

In view of vanishing of the divergence of Einstein tensor, we get

$$8\pi G\left[\dot{\rho}+(\rho+p)\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)\right]+8\pi\rho\dot{G}+\dot{\Lambda}=0\,. \tag{9}$$

The usual energy conservation equation $T^j_{i;j}=0$ yields

$$\dot{\rho}+(\rho+p)\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)=0\,. \tag{10}$$

Equation (9) together with (10) puts G and Λ in some sort of coupled field given by

$$8\pi\rho\dot{G}+\dot{\Lambda}=0\,, \tag{11}$$

implying that Λ is a constant whenever G is constant. Here and elsewhere, a dot stands for ordinary differentiation with respect to time t.

We define

$$R=(ABC)^{1/3} \tag{12}$$

as the average scale factor of Bianchi type-I universe. The Hubble parameter H, volume expansion θ, shear σ and the deceleration parameter q are given by

$$\theta=3H=\frac{3\dot{R}}{R},\qquad \sigma=\frac{k}{\sqrt{3}R^3},\quad k>0\ \text{(constant)},\qquad q=-1-\frac{\dot{H}}{H^2}=-\frac{R\ddot{R}}{\dot{R}^2}\,.$$

Einstein's field equations (5) – (8) can also be written in terms of Hubble parameter H, shear σ and deceleration parameter q as

$$H^2(2q-1)-\sigma^2=8\pi Gp-\Lambda\,, \tag{13}$$

$$3H^2-\sigma^2=8\pi G\rho-\Lambda\,. \tag{14}$$

On integrating Eqs. (5) – (8), we obtain

$$\frac{\dot{A}}{A}-\frac{\dot{B}}{B}=\frac{k_1}{R^3} \tag{15}$$

and

$$\frac{\dot{B}}{B}-\frac{\dot{C}}{C}=\frac{k_2}{R^3}\,, \tag{16}$$

where k_1 and k_2 are constants of integration. From Eq. (14), we obtain

$$\frac{3\sigma^2}{\theta^2}=1-\frac{24\pi G\rho}{\theta^2}-\frac{3\Lambda}{\theta^2}\,, \tag{17}$$

implying that $\Lambda \geq 0$, as

$$0 < \frac{\sigma^2}{\theta^2} < \frac{1}{3}\,, \qquad \text{and} \qquad 0 < \frac{8\pi G\rho}{\theta^2} < \frac{1}{3}\,.$$

Thus the presence of a positive Λ lowers the upper limit of anisotropy, whereas a negative Λ gives more room for anisotropy.

Equation (17) can also be written as

$$\frac{\sigma^2}{3H^2} = 1 - \frac{8\pi G\rho}{3H^2} - \frac{\Lambda}{3H^2} = 1 - \frac{\rho}{\rho_c} - \frac{\rho_v}{\rho_c}\,, \tag{18}$$

where $\rho_c = 3H^2/(8\pi G)$ is the critical density and $\rho_v = \Lambda/(8\pi G)$ is the vacuum density.

From (13) and (14), we get,

$$\frac{\mathrm{d}\theta}{\mathrm{d}t} = -12\pi G p - \frac{\theta^2}{2} + \frac{3\Lambda}{2} - \frac{3}{2}\sigma^2 = -12\pi G(\rho + p) - 3\sigma^2, \tag{19}$$

showing that the rate of volume expansion decreases during time evolution and presence of positive Λ, slows down the rate of this decrease, whereas a negative Λ would promote it.

3. *Solution of the field equations*

The system of equations (5) − (8) and (11) supply only five equations in seven unknowns (A, B, C, ρ, p, Λ and G). Two extra equations are needed to solve the system completely. We first take that the cosmological term is proportional to R^{-m}, i.e.,

$$\Lambda = \frac{a}{R^m}\,, \tag{20}$$

where a is a positive constant.

This variation law was proposed by Olson et al. [38], Pavon [18], Maia et al. [39]; Silveira et al. [40, 41] and Torres et al. [42]. Because observations suggest that Λ is very small in the present universe, a decreasing functional form permits Λ to be large in the early universe.

As the second condition, we assume that the volume expansion θ is proportional to the eigenvalues of the shear tensor σ_{ij}. It is believed that an evolution of one parameter should also be responsible for the evolution of the others [13]. Following Collins et al. [43], the volume expansion θ, having a constant ratio to the anisotropy, gives

$$A = B^{n_1} C^{n_2}\,, \tag{21}$$

where n_1 and n_2 are constants.

Without loss of generality we take $n_1 = n_2 = 1$ i.e.,

$$A = BC\,. \tag{22}$$

Using the condition (22) in (15) and (16), after suitable transformation, the metric (1) takes the following form

$$\mathrm{d}s^2 = -\mathrm{d}T^2 + T\mathrm{d}x^2 + T^{(2k_1+2k_2)/(4k_1+2k_2)}\mathrm{d}y^2 + T^{2k_1/(4k_1+2k_2)}\mathrm{d}z^2\,. \tag{23}$$

4. Discussion

For the model (23), the average scale factor R is given by

$$R = T^{1/3}\,. \tag{24}$$

The volume expansion θ, Hubble parameter H and shear σ for the model are

$$\theta = 3H = \frac{1}{T}\,, \tag{25}$$

$$\sigma = \frac{k}{\sqrt{3}\,T}\,. \tag{26}$$

Clearly, $\sigma/\theta = k/\sqrt{3}$. Therefore, the model does not approach isotropy. If k is small, the model is quasi-isotropic, i.e., $\sigma/\theta \approx 0$.

Using (20) in (5), (8) and (11), the pressure p, density ρ, gravitational constant G and the cosmological constant Λ are:

$$p = \frac{1}{8\pi k_3}\left[aT^{-m/3} + \frac{5k_1^2+5k_1k_2+k_2^2}{(4k_1+2k_2)^2T^2}\right] \Bigg/ \left[\frac{5k_1^2+5k_1k_2+k_2^2}{(4k_1+2k_2)^2} - aT^{(6-m)/3}\right]^{m/(m-6)}, \tag{27}$$

$$\rho = \frac{1}{8\pi k_3}\left[-aT^{-m/3} + \frac{5k_1^2+5k_1k_2+k_2^2}{(4k_1+2k_2)^2T^2}\right] \Bigg/ \left[\frac{5k_1^2+5k_1k_2+k_2^2}{(4k_1+2k_2)^2} - aT^{(6-m)/3}\right]^{m/(m-6)}, \tag{28}$$

$$G = k_3\left[\frac{5k_1^2+5k_1k_2+k_2^2}{(4k_1+2k_2)^2} - aT^{(6-m)/3}\right]^{m/(m-6)}, \tag{29}$$

and $\Lambda = aT^{-m/3}$, where k_3 is a positive integration constant.

In the model, we observe that at $T = 0$ the spatial volume V is zero and the expansion scalar θ is infinite, which shows that universe starts evolving with zero volume and an infinite rate of expansion at $T = 0$. The scale factors also vanish at $T = 0$ and hence the model has a point-type singularity at initial epoch. Initially, at $T = 0$, the energy density ρ, pressure p, shear σ and the cosmological term Λ are all infinite, but G is finite. As T increases, the spatial volume increases, but the expansion rate decreases as time increases.

As $T \to \infty$, the spatial volume V becomes infinitely large. All parameters θ, ρ, p, σ and Λ, tend asymptotically to zero, but G is increasing for $m < 6$ and decreasing for $m > 6$. A partial list of cosmological models in which the gravitational constant G is a decreasing function of time are contained in papers of Grøn [44], Helling et al. [45] and Rowan-Robinson [46]. The possibility of G increasing with time, at least in some stages of the development of the universe, has been investigated by Abdel-Rahman [27], Chow [47], Levit [48] and Milne [49].

For $m = 6$, the result $\Lambda \propto 1/T^2$ give Berman [50], Berman and Som [51], Berman et al. [52] and Bertolami [53, 54]. This form of Λ is physically reasonable as observations suggest that Λ is very small in the present universe. A decreasing functional form permits Λ to be large in the early universe.

Recent cosmological observations [1 − 5, 55 − 57] suggest the positive cosmological constant Λ with the magnitude $\Lambda(Gh/c^3) \approx 10^{-123}$.

The density parameter is

$$\Omega = \frac{\rho}{\rho_c} = 3\left[\frac{5k_1^2+5k_1k_2+k_2^2}{(4k_1+2k_2)^2} - aT^{(6-m)/3}\right],$$

and the ratio between the vacuum and critical density is given by

$$\frac{\rho_V}{\rho_c} = 3aT^{(6-m)/3}.$$

It is interesting that if we take $m = 2$, then the above results reduce to the result of Hoyle et al. [58].

5. *Conclusion*

We investigated the Bianchi type-I cosmological model with variable G and Λ in the presence of perfect fluid with cosmological term proportional to R^{-m} (R is scale factor and m is constant) suggested by Silveira et al. [40, 41] and others. Initially, the model has a point-type singularity, gravitational constant $G(t)$ is constant (for $m < 6$) and the cosmological constant Λ is infinite. When the time increases, Λ decreases but G increases for $m < 6$ and decreases for $m > 6$.

It is interesting that Beesham [59], Lima and Carvalho [60], Kalligas et al. [61] and Lima [62] have also derived the Bianchi type-I cosmological models with variable G and Λ assuming a particular from for G and by taking an equation of state, whereas we have neither assumed an equation of state nor a particular form of G. The model does not approach isotropy if k is small and the model is quasi-isotropic, i.e., $\sigma/\theta = 0$.

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KOZMOLOŠKI MODEL S PROMJENLJIVIM G I Λ ČLANOM U OPĆOJ RELATIVNOSTI

Proučavamo Einsteinove jednadžbe polja s promjenljivom gravitacijskom konstantom i kozmološkim konstantama u prisustvu perfektne tekućine za Bianchijev svemir tipa I pretpostavljajući kozmološki član razmjeran R^{-m} (R je faktor sumjeravanja a m stalnica). Model teži kvaziizotropnom stanju. Kozmološki član pada s porastom vremena. Dobivamo da se sadašnji svemir ubrzava s velikim udjelom kozmološke gustoće u vidu kozmološkog člana.