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LETTER TO THE EDITOR

MODIFIED GAUSS-BONNET-BRANS-DICKE COSMOLOGY WITH FREEZING SCALAR POTENTIALS

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In this letter, we discuss the late-time dynamics of a modified Gauss-Bonnet gravity theory à la Brans-Dicke, dominated by freezing potentials $V(\phi) \propto \phi^{-n}$, $n \in \mathbb{R}$. For a certain choice of the parameter n, it is observed that the universe is dominated by dark energy, accelerated in time and controlled by the Gauss-Bonnet invariant term. Besides, the Brans-Dicke parameter $\omega \gg 1$ compatible with some recent reports where this bound is updated by several thousands following from the bound on "$\tilde{\gamma} - 1$" from the Cassini mission, $\tilde{\gamma}$ being the PPN parameter. Many additional interesting features are raised and discussed in some details.

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1. Introduction

One of the most important discoveries over the past few years is the fact that we live in an accelerated almost spatially flat universe with a density parameter $\Omega_k = -0.015^{+0.020}_{-0.016}$ (within a 2% margin of error). More precisely, recent cosmological observations indicate that there are two periods of accelerated expansion in the history of our universe: cosmic inflation in the early universe (high energy limits) and acceleration in the current expansion of the universe (low energy limits). These facts are based on a number of cosmological and astrophysical observations from the CMBR dataset of the Three-Year WMAP, data from Supernova Legacy Survey of type SNeIa and large galaxy [1 − 7]. The simplest model that can explain this unexpected dynamics and the present observable features of the universe is the ADCM model [8] which consists of a mixture of cosmological constant Λ and CDM (or WIMPS) composed of weakly-interacting massive particles which must be relics of a grand unified phase of the universe. Other popular models were also proposed and these include scalar field driven dark energy or quintessence with a very shallow many-forms potential (self-interacting) [9], viscous fluid [10], Chaplygin gas [11, 12], generalized Chaplygin gas model [13 – 16], holographic dark energy [17, 18], mass-varying neutrino dark energy [19, 20], quintom cosmology [21 – 24], and so on. Most of these models have important drawbacks and suffer from serious fine-tuning problems, e.g. fine tuning of parameters for different types of potentials which model quintessence, stability of radiative corrections from the matter sector, etc.

There exist many phenomenological attempts (entirely in terms of a modified gravity theory with higher-order corrections coming from superstrings or M-theory) proposed to avoid these problems, which in reality can give rise to inflationary solutions and consequently result in a better explanation of the cosmic acceleration of the universe and the nature of the dark energy. Some nice alternative scalar theories include string-inspired dilaton gravities, a time-varying energy density, M/string theory, higher derivative gravity theories [25 – 29], Gauss-Bonnet (GB) cosmology [30 – 33], non-minimal coupling theories in all their aspects and forms [34, 35] and so on. Despite the fact that these scalar-tensor theories of gravity have the potential to provide a linkage between the accelerated expansion of the universe and of fundamental physics, many models apparently suffer from instabilities or are incompatible with solar system measurements. The choice of possibilities reflects the undisputable fact that the true nature of the DE has not been convincingly explained yet.

While it is not easy to satisfy all known solar system tests at once, we wish to construct an explicit model which describes the realistic universe expansion history (radiation and matter epochs, transition to acceleration and accelerating era). An interesting proposed theory as gravitational dark energy is scalar-Gauss-Bonnet gravity with simple higher-order correction which is closely related with low-energy effective heterotic string. It has been demonstrated that some scalar-GB gravities may be compatible with the known classical history of the universe expansion, and GB assisted dark energy may be constructed. More recently, some observations indicate that many significant constraints of the GB gravity can be derived from both solar system measurements and table-top laboratory experiments [36 – 42]. In reality, scalar-tensor theories of gravity with and without non-minimal coupling to the spacetime curvature in the gravitational action offer a suitable framework to investigate various properties of the dynamics of the universe not found within the standard cosmological model [28, 43 – 52].

The present paper is devoted to the study of some new aspects of a modified GB theory. We expect that this model may have many interesting features for producing in a natural way an epoch of accelerated expansion of the universe. Besides, the cosmology model should contain a sufficiently long matter-dominated epoch that takes place before acceleration in order to guarantee a decelerated epoch and large structure and galaxy formation. We will concentrate on the late-time dynamical epoch. Since the Gauss-Bonnet term is a topological invariant in four dimensions it does not contribute on its own to the Einstein field equations. It contributes to the field equations if it couples to a spin-zero field. In other words, coupling the later to a scalar field may produce a non-trivial effect, which could act as effective dark energy.

Let us start from a four-dimensional modified gravity theory à la Brans-Dicke,

$$S = \int \sqrt{-g}\,\mathrm{d}x^4 \left(\frac{1}{2}\xi F(\phi)R - \frac{1}{2}\omega\partial_\mu\phi\partial^\mu\phi - V(\phi)\right.$$
$$\left. + \frac{1}{2}f(\phi)\big(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\big)\right) + \frac{1}{2}\int \mathrm{d}x^4\sqrt{-g}\,L_m\,, \tag{1}$$

where $F(\phi)$ and $f(\phi)$ are the generic functions of the scalar field, ξ is a free parameter in the theory, g is the metric, R is the scalar curvature, ω is the Brans-Dicke coupling parameter, ϕ is the scalar field, $R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ is the GB invariant and L_m is the Lagrangian for ordinary matter. $f(\phi)$, $V(\phi)$ and $F(\phi)$ are given by the power-law functions $f(\phi) = f_0\phi^m$, $V(\phi) = V_0\phi^n$ and $F(\phi) = F_0\phi^r$, with the constant parameters f_0, V_0 and F_0 assumed equal to be equal to one for mathematical simplicity, while $(m,n,r) \in \mathbb{R}$. The fact that the corrections to Einstein gravity are second order in curvature suggests they will automatically be small. The GB term may couple with the scalar kinetic terms and thus may play an important role for other classes of gravitational models. One of the main advantages of such models is the fact that they exhibit power-law couplings and potentials and admit tracker behaviour.

The variations of the action with respect to the metric $g_{\mu\nu}$ and the scalar field ϕ yield, after some algebra, the following field equations

$$\xi F(\phi)\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right) - \omega\nabla_\mu\phi\nabla_\nu\phi - \nabla_\mu\nabla_\nu F(\phi)$$
$$+g_{\mu\nu}\left(\Box E(\phi) + 4R\,\Box f(\phi) - 8R^{\sigma\tau}\nabla_\sigma\nabla_\tau f(\phi) + \frac{\omega}{2}\nabla_\sigma\phi\nabla^\sigma\phi + V(\phi)\right)$$
$$-4\big(R\nabla_\mu\nabla_\nu f(\phi) + 2R_{\mu\nu}\Box f(\phi) + 2R_{(\mu}{}^{\sigma\tau}{}_{\nu)}\nabla_\sigma\nabla_\tau f(\phi) - 8R_{\sigma(\mu}\nabla^\sigma\nabla_{\nu)}f(\phi)\big)$$
$$= T_{\mu\nu} + \left(\nabla_\mu\phi\nabla_\nu\phi - \frac{1}{2}g_{\mu\nu}\nabla_\lambda\phi\nabla^\lambda\phi - V(\phi)g_{\mu\nu}\right), \tag{2}$$

$$\omega\Box\phi - \frac{\mathrm{d}V(\phi)}{\mathrm{d}\phi} + \frac{\mathrm{d}F(\phi)}{\mathrm{d}\phi}\frac{R}{2} + \frac{\mathrm{d}f(\phi)}{\mathrm{d}\phi}\big(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\big) = 0\,, \tag{3}$$

with stress-energy tensor $T_{\mu\nu} = (p_m + \rho_m)u_\mu u_\nu + p_m g_{\mu\nu}$, where p_m and ρ_m are the pressure and density of the perfect fluid and u_μ is the fluid rest-frame four-velocity.

In this work, the spatially flat Friedmann-Robertson-Walker (FRW) model with metric,

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\big[\mathrm{d}r^2 + r^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2)\big], \tag{4}$$

is considered, $a(t)$ being the scale curvature. The (t,t) component in the action takes the form

$$\frac{\omega}{2}\dot{\phi}^2 - 3\xi\dot{\phi}\frac{\mathrm{d}F}{\mathrm{d}\phi}H - 3\xi H^2F(\phi) + V(\phi) - 24\dot{\phi}\frac{\mathrm{d}f}{\mathrm{d}\phi}H^3 + \rho_m = 0\,, \tag{5}$$

where $H = \dot{a}/a$ is the Hubble parameter and ρ_m is the matter density. Equation (2) gives

$$\omega(\ddot{\phi} + 3H\dot{\phi}) + \frac{\mathrm{d}V}{\mathrm{d}\phi} - 24\frac{\mathrm{d}f}{\mathrm{d}\phi}(\dot{H}H^2 + H^4) - 3\xi\frac{\mathrm{d}F}{\mathrm{d}\phi}(2H^2 + \dot{H}) = 0\,. \tag{6}$$

Besides, we assume that the conservation equation of matter $\nabla_\nu T^{\mu\nu} = 0$ holds for the present theory. Therefore, the conservation equation takes the common form

$$\dot{\rho}_m + 3H(p_m + \rho_m) = 0\,. \tag{7}$$

By assuming that the baryonic matter obeys the equation of state $p_m = (\gamma - 1)\rho_m$, where γ is a real constant, Eq. (7) takes now the simplest form $\dot{\rho}_m + 3H\gamma\rho_m = 0$. To explore the solutions of Eqs. (5) and (6), we assume that (late-time behavior) the scale factor behaves as $a = a_0 t^q$ (power-law), where a_0 is a positive constant equal to one and q is assumed to be positive to correspond to expanding universe. Moreover, we propose the power-law behavior of the scalar field $\phi = \phi_0 t^p$, where p is a real constant with as well $\phi_0 = 1$. It is easy to check from Eq. (6) that a consistent relation is obtained if the scalar curvature decays like $R = t^{-2}$. Consequently, Eq. 6) takes the form

$$\omega\big(p(p-1)t^{p-2} + 3pqt^{p-2}\big) + nt^{p(n-1)} - 24mt^{p(m-1)-4}(q^4 - q^3) - 3\xi rqt^{p(r-1)-2}(2q-1) = 0, \tag{8}$$

Thus consistency is obtained for $n = 2(p-1)/p$, $m = 2(p+1)/p$ and $r = 2$. Further, Eq. (5) gives

$$\frac{\omega}{2}p^2t^{2(p-1)} - 3\xi pqrt^{pr-2} - 3\xi q^2t^{pr-2} + t^{2(p-1)} - 24pq^3mt^{pm-4} + \rho_m = 0\,, \tag{9}$$

from which a desirable consistent solution is obtained if the density of matter varies like $\rho_m = t^{2p-2}$. Hence, the continuity equation gives $3\gamma q = 2 - 2p$ augmented by $r = 2$. We may now discuss the cases illustrated in Table 1.

TABLE 1. Values of the parameters p, m, r and $3\gamma q$ for different values of n.

n	p	m	r	$3\gamma q$
4 (Chaotic potential)	-1	0	2	4
-1 (Freezing potential)	2/3	5	2	2/3
-2 (Freezing potential)	1/2	6	2	1
-4 (Freezing potential)	1/3	10	2	4/3

Notice that $R\phi^2 = t^{2p-2}$, just like the Gauss-Bonnet term which also varies like $R^2 f(\phi) = t^{-4+pm} = t^{2p-2}$ and, therefore, the modified gravity model introduced here reveals an interesting property which corresponds to the contribution of higher-order curvature terms in late-time dynamics. For illustration purpose, we discuss the case of $n = -2$ and we leave the rest of cases to the interested reader. It is noteworthy that freezing potentials play a crucial role in different aspects of quintessence and modern cosmology [53−55]. For $n = -2$, we obtain $3\gamma q = 1$ and, accordingly, accelerated expansion occurs if for instance $q > 1$ or $\gamma < 1/3$. This case corresponds to the equation of state parameter $w = \gamma - 1 < -2/3$ which lies within the observational limits [1−7]. In Fig. 1, we plot in 3D the function $R^2 f(\phi) = R\phi^2 = t^{2p-2} = t^{2a}$, $a = p - 1$.

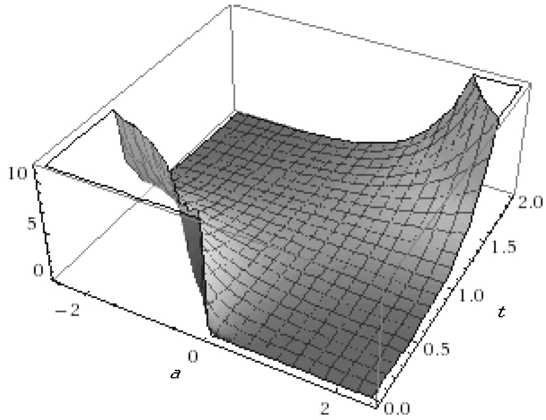


Fig. 1. Plot in 3D of the function $R^2 f(\phi) = R\phi^2 = t^{2p-2} = t^{2a}$, $a = p - 1$.

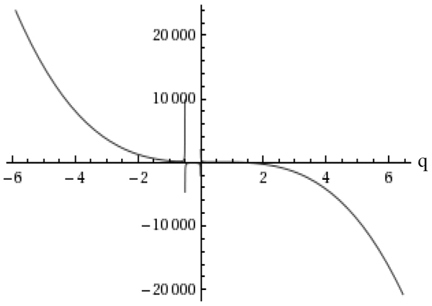
After simple algebraic manipulation, Eqs. (8) and (9) give

$$\omega = -96q^3 + 120q^2 - \frac{24}{2q+1} + \frac{8}{q}\,, \tag{10}$$

$$\xi = \frac{2 - 27q^3}{3q(q+1)} - \frac{18q^4 - 24q^3 + 15q^2 - q + 1}{q(2q+1)}\,, \tag{11}$$

with $q \neq (-1, -1/2, 0)$. Note that in the string theory, $\omega = -1$, then $q \approx 1.25$ and $\xi \approx -6$, but experimental observations require $|\omega| > 40000$, following from the bound "$\tilde{\gamma} - 1$" from the Cassini mission, $\tilde{\gamma}$ being the PPN parameter [56]. For $\omega = -40000$, we find $q \approx 8$ and $\xi \approx 5120$. It is noteworthy that for $r = 2$, the inverse of the parameter ξ does not correspond to the gravitational coupling constant. This special case matches an accelerated expansion dominated by dark energy with the equation of state parameter $w = \gamma - 1 = -0.95$ and controlled by the Gauss-Bonnet invariant terms. In Figs. 2 and 3, we plot ω and ξ as given by Eqs. (10) and (11), respectively.

Besides, the scalar potential is given by $V(\phi) = \phi^{-2}$, i.e. freezing (see Figs. 4 and 5).



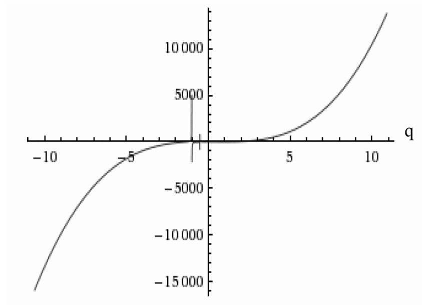
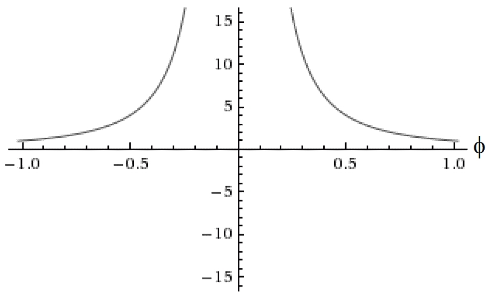


Fig. 2 (left). Plot of ω as given by Eq. (10).
Fig. 3. Plot of ξ as given by Eq. (11).



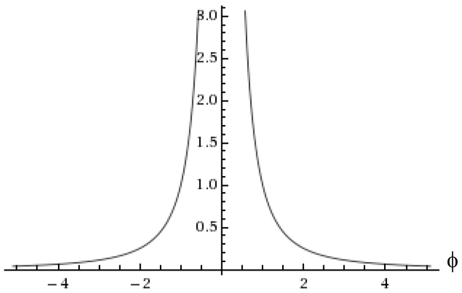


Fig. 4 (left). Plot of the function $V(\phi) = \phi^{-2}$ for $-1 < \phi < 1$.
Fig. 5. Plot of the function $V(\phi) = \phi^{-2}$ for $-4 < \phi < 4$.

It is noteworthy that the scalar field function $F(\phi) = \phi^2 \propto G^{-1}$, where G is the gravitational coupling constant, i.e. $G \propto \phi^{-2} = t^{-1}$, is similar to the Dirac argument [57]. In fact, there exist many experimental limits on the time variation of the gravitational constant: radar ranging data to the Viking landers on Mars [58], lunar laser ranging experiments [59, 60], measurements of the masses of young and old neutron stars in binary pulsars [61]. In general, it was recently observed that for late times, a modified cosmology with varying G is in accordance with the observed values of the cosmological parameters. More generally, in our approach, the present day variation of G, $(\dot{G}/G)_p = H_0/q = 3\gamma H + 0/(2-2p)$, thus we expect that for $\gamma \ll 1$ ($w \approx -1$), $(\dot{G}/G)_p \ll 1$ as recent astronomical limits suggest that $(\dot{G}/G)_p < 10^{-13}$.

In summary, the model described in this paper is interesting and may have appealing and important consequences on the physics of the early universe. The behavior of the discussed model is consistent with cosmological observations (e.g., Wilkinson Microwave Anisotropy Probe and supernovae test). However, this is a primitive model. This only shows that such investigations may be useful. Further consequences are under progress.

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IZMIJENJENA GAUSS-BONNET-BRANS-DICKEOVA KOZMOLOGIJA SA ZAMRZNUTIM SKALARNIM POTENCIJALIMA

Razmatramo kasnovremensku dinamiku izmijenjene Gauss-Bonnetove gravitacijske teorije u kojoj prevladavaju zamrznuti skalarni potencijali $V(\phi) \propto \phi^{-n}$, $n \in \mathbb{R}$. Primjećuje se kako za neki izbor parametra n u svemiru prevladava tamna tvar, ubrzana u vremenu i upravljana Gauss-Bonnetovim invarijantnim članom. K tome je Brans-Dickeov parametar $\omega \gg 1$ u skladu s nedavnim izvješćima gdje je ta granica obnovljena više tisuća puta na osnovi granice za "$\tilde{\gamma} - 1$" određene sondom Cassini, gdje je $\tilde{\gamma}$ PNP parametar. Više drugih pitanja se postavlja i podrobno raspravlja.