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SPATIALLY HOMOGENEOUS BIANCHI TYPE-V COSMOLOGICAL MODELS IN BRANS-DICKE THEORY

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We discuss the field equations of Brans-Dicke theory for a spatially homogeneous and anisotropic Bianchi type-V space-time in the presence of a perfect fluid and obtain their exact solutions by applying the law of variation of Hubble's parameter which yields a constant value of deceleration parameter. The corresponding cosmological models are divided into two categories, (i) singular models with power-law expansion, (ii) non-singular models with exponential expansion. The physical and kinematical behaviors of the cosmological models are discussed. These models are compatible with the results of recent observations.

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1. Introduction

The Brans-Dicke (BD) theory of gravity [1] has been extensively studied by many workers for the last four decades in different physical contexts. This theory is one of the simplest modifications of Einstein's general relativity as it involves probably the simplest form of non-linear kinetic terms for the BD scalar field which is not of quantum origin. This theory is arguably the most general choice as the scalar-tensor generalization of general relativity because of its simplicity and a possible reduction to general relativity in some limit. In fact, it is classical in nature

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and hence can be expected to serve as a very relevant candidate to play some role in the late-time evolution of the universe. The simplest inflationary models (Mathiazhagan and Johri [2]), extended inflation (La and Steinhardt [3]; Steinhardt and Accetta [4]) and hyper-extended inflation and extended chaotic inflation (Linde [5]) are based on BD theory and other general scalar-tensor theories. Obregon and Pimentel [6] presented exact cosmological models with particle creation taking BD scalar ϕ as a linear function of time. They found that the gravitational constant decreases linearly with time and the mass of the universe increases proportionally to the square of its age. Uehara and Kim [7] studied BD equations with cosmological constant and presented exact solutions for spatially flat Robertson-Walker metric in matter-dominated universe. Johri and Desikan [8] studied cosmological models with constant deceleration parameter q in the framework of BD theory and divided the resulting models in two categories: (i) singular models with expansion driven by big-bang impulses $q > 0$, (ii) non-singular models with expansion driven by creation of matter particles ($q < 0$). Barrow [9] presented a procedure to obtain exact solutions in BD theory for homogeneous and isotropic cosmological models in vacuum and with radiation as the matter content for all values of the curvature. Chauvet and Cervantes-Cota [10] discussed isotropization of Bianchi types-I, V and IX cosmological solutions in BD theory. Cervantes-Cota [11] obtained some exact solutions in BD theory for a Bianchi type-V metric having the properties of inflationary expansion, graceful exist and asymptotic evolution to a Friedmann-Robertson-Walker open model. Kim [12] considered BD theory as a unique unified model for dark matter-dark energy. He concluded that the BD scalar field interpolates smoothly between two late-time stages by speeding up the expansion rate of the matter-dominated era while slowing down that of the accelerated phase to some degree. Das and Banerjee [13] have shown that the BD scalar field itself can serve the purpose of providing an early deceleration and a late time acceleration of the universe without any need of quintessence field if one considers the transfer of energy between the dark matter and the BD scalar field.

Berman [14] presented a law of variation of Hubble's parameter for models of the universe. Singh and Kumar [15] obtained two categories of Bianchi type-II perfect fluid cosmological models with the help of special law of variation for Hubble's parameter. Motivated by these works, we study here Bianchi type-V anisotropic solutions in the BD theory because it is one of the simplest steps in increasing the complexity of the theory and permits the analysis of the anisotropic properties of the models. We apply the law of variation for Hubble's parameter to obtain two classes of solutions for specific values of the coupling constant ω of the BD theory. We first discuss this special law of variation of Hubble's parameter for a Bianchi type-V space-time which yields a constant value of the deceleration parameter and obtain two explicit forms of the average scale factor, one is of power-law type and the other of exponential form. Using these forms of the average scale factor, we present two categories of cosmological models, (i) singular models and (ii) non-singular, with constant deceleration parameter within the framework of Brans-Dicke theory with perfect fluid. We also discuss the physical and kinematical behaviors of the models.

2. Model and field equations

We consider the spatially homogeneous and anisotropic Bianchi type-V space-time of the form

$$\mathrm{d}s^2 = \mathrm{d}t^2 - A^2\mathrm{d}x^2 - e^{2mx}\left[B^2\mathrm{d}y^2 + C^2\mathrm{d}z^2\right], \tag{1}$$

where A, B and C are the cosmic scale-factors and m is a constant.

Brans-Dicke [1] field equations for combined scalar and tensor fields are

$$R_{ij} - \frac{1}{2}g_{ij}R + \omega\phi^{-2}\left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}\right) + \phi^{-1}\left(\phi_{;ij} - g_{ij}\Box\phi\right) = -8\pi\phi^{-1}T_{ij}, \tag{2}$$

and

$$\Box\phi = \phi^{;k}_{;k} = \frac{8\pi}{3+2\omega}T\,, \tag{3}$$

where $R_{ij} - \frac{1}{2}g_{ij}R$ is the Einstein tensor, T_{ij} is the energy momentum tensor of the matter and ω is the dimensionless coupling constant. The continuity equation reads as

$$T^j_{i;j} = 0\,. \tag{4}$$

Here comma and semi-colon denote partial and covariant differentiation, respectively.

The energy-momentum tensor for a perfect fluid is given by

$$T_{ij} = (\rho + p)\,u_i u_j - p g_{ij}\,, \tag{5}$$

where u^i is the fluid 4-velocity vector. In the co-moving system of coordinates, we have $u^i = (0,0,0,1)$. ρ and p are energy density and pressure, respectively. From Eqs. (1) and (5), the non-vanishing components of T^i_j in co-moving coordinates are

$$T^1_1 = T^2_2 = T^3_3 = -p, \qquad T^4_4 = \rho, \qquad T = \rho - 3p\,. \tag{6}$$

The field Eqs. (2) and (3) for the Bianchi type-V metric (1), in view of Eq. (6), are given as

$$\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} - \frac{3m^2}{A^2} + \frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 - \frac{1}{\phi}\left(\ddot{\phi} - \Box\phi\right) = \frac{8\pi\rho}{\phi}\,, \tag{7}$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} - \frac{m^2}{A^2} + \frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 - \frac{1}{\phi}\left(\frac{\dot{A}}{A}\dot{\phi} - \Box\phi\right) = \frac{-8\pi p}{\phi}\,, \tag{8}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} - \frac{m^2}{A^2} + \frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 - \frac{1}{\phi}\left(\frac{\dot{B}}{B}\dot{\phi} - \Box\phi\right) = \frac{-8\pi p}{\phi}\,, \tag{9}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} - \frac{m^2}{A^2} + \frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 - \frac{1}{\phi}\left(\frac{\dot{C}}{C}\dot{\phi} - \Box\phi\right) = \frac{-8\pi p}{\phi}\,, \tag{10}$$

$$2\frac{\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0\,, \tag{11}$$

$$\Box\phi = \ddot{\phi} + \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right)\dot{\phi} = \frac{8\pi}{2\omega+3}(\rho - 3p)\,, \tag{12}$$

where a dot $(\cdot)$ denotes differentiation with respect to time t.

We define the average scale factor a and the volume scalar V as

$$V = a^3 = ABC\,. \tag{13}$$

The dynamical scalars such as the expansion θ and the shear scalar σ^2 are defined as

$$\theta = u^i_{;i}\,, \tag{14}$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij}, \tag{15}$$

where

$$\sigma_{ij} = \frac{1}{2}(u_{i;\alpha}P^\alpha_j + u_{j;\alpha}P^\alpha_i) - \frac{1}{3}\theta P_{ij}\,. \tag{16}$$

The projection tensor P_{ij} has the form

$$P_{ij} = g_{ij} - u_i u_j\,. \tag{17}$$

For the metric (1), these dynamical scalars have expressions

$$\theta = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} = \frac{\dot{V}}{V}\,, \tag{18}$$

and

$$\sigma^2 = \frac{1}{2}\left[\left(\frac{\dot{A}}{A}\right)^2 + \left(\frac{\dot{B}}{B}\right)^2 + \left(\frac{\dot{C}}{C}\right)^2\right] - \frac{\theta^2}{6}\,. \tag{19}$$

We define the generalized mean Hubble's parameter H as

$$H = \frac{1}{3}\left(H_1 + H_2 + H_3\right), \tag{20}$$

where $H_1 = \dot{A}/A$, $H_2 = \dot{B}/B$ and $H_3 = \dot{C}/C$ are the directional Hubble's parameters in the directions of x, y and z, respectively.

From Eqs. (14), (18) and [20], we obtain

$$H = \frac{1}{3}\frac{\dot{V}}{V} = \frac{\dot{a}}{a} = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right). \tag{21}$$

The anisotropy parameter A_m is given as

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2, \tag{22}$$

where $\Delta H_i = H_i - H$, $(i = 1, 2, 3)$.

The deceleration parameter q in a cosmological model is defined as

$$q = -\frac{\ddot{a}a}{\dot{a}^2}. \tag{23}$$

From Eqs.(7) − (12), the energy density ρ and the pressure p, in terms of H, q, σ^2 and ϕ, are given by

$$8\pi\rho = \left[3H^2 - \sigma^2 - \frac{3m^2}{A^2} + 3H\frac{\dot{\phi}}{\phi} + \frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2\right]\phi\,, \tag{24}$$

and

$$8\pi p = \left[H^2(2q-1) - \sigma^2 + \frac{m^2}{A^2} - 2H\frac{\dot{\phi}}{\phi} - \frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 - \frac{\ddot{\phi}}{\phi}\right]\phi\,. \tag{25}$$

3. *Solution of the field equations*

From Eq. (11), we have

$$A^2 = BC\,. \tag{26}$$

Now, following the approach of Saha and Rikhvitsky [16], Singh and Chaubey [17] and Singh et al. [18], we solve the field equations. Subtracting Eqs. (8) and (9), Eqs. (9) and (10), and Eqs. (8) and (10), we get the following three relations

$$\frac{B}{A} = d_1 \exp\left(k_1 \int \frac{\mathrm{d}t}{a^3\phi}\right), \tag{27}$$

$$\frac{C}{B} = d_2 \exp\left(k_2 \int \frac{\mathrm{d}t}{a^3\phi}\right), \tag{28}$$

and

$$\frac{C}{A} = d_3 \exp\left(k_3 \int \frac{\mathrm{d}t}{a^3\phi}\right), \tag{29}$$

where d_1, d_2, d_3 and k_1, k_2, k_3 are constants of integration. From Eqs. (27) – (29), the metric functions can be written explicitly as

$$A = l_1 a \exp\left(X_1 \int \frac{\mathrm{d}t}{a^3\phi}\right), \tag{30}$$

$$B = l_2 a \exp\left(X_2 \int \frac{\mathrm{d}t}{a^3\phi}\right), \tag{31}$$

$$C = l_3 a \exp\left(X_3 \int \frac{\mathrm{d}t}{a^3\phi}\right), \tag{32}$$

where

$$l_1 = \sqrt[3]{({d_1}^2 d_2)^{-1}}, \qquad l_2 = \sqrt[3]{d_1 {d_2}^{-1}}, \qquad l_3 = \sqrt[3]{d_1 {d_2}^2},$$

and

$$X_1 = \frac{-(2k_1 + k_2)}{3}, \qquad X_2 = \frac{-(k_2 - k_1)}{3}, \qquad X_3 = \frac{(k_1 + 2k_2)}{3}.$$

Here the constants X_1, X_2, X_3 and l_1, l_2, l_3 satisfy the relations

$$X_1 + X_2 + X_3 = 0 \qquad \text{and} \qquad l_1 l_2 l_3 = 1\,. \tag{33}$$

From Eq. (26) and Eqs. (30) – (32), we obtain

$$X_1 = 0, \quad X_2 = -X_3 = X, \qquad l_1 = 1, \quad l_2 = l_3^{-1} = M\,,$$

where X and M are constants. Then the expressions of the metric functions given in Eqs. (30) – (32) reduce to

$$A = a, \tag{34}$$

$$B = M a \exp\left(X \int \frac{\mathrm{d}t}{a^3\phi}\right), \tag{35}$$

$$C = M^{-1} a \exp\left(-X \int \frac{\mathrm{d}t}{a^3\phi}\right). \tag{36}$$

Now, in order to get the exact solutions of the metric functions, we make certain physically valid assumptions. It has been shown by Johri and Desikan [8] that for the flat Robertson-Walker Brans-Dicke models, the necessary and sufficient for the deceleration parameter to be constant, is a power-law relation between the scale-factor a and the scalar field ϕ of the form $\phi = ba^{\alpha}$. In order to find physically realistic solution of the field equations, here we assume that

$$\phi = ba^{-2}, \tag{37}$$

where b is a proportionality constant.

We also assume that the Hubble's parameter H is related to the average scale factor a by the relation

$$H = la^{-n}, \tag{38}$$

where $l > 0$ and $n(\geq 0)$ are constants. This type of relation gives a constant value of the deceleration parameter which is a very useful tool in solving the field equations. Earlier Berman [14], and Berman and Gomide [19] have also considered such type of assumption for solving FRW cosmological models. Later on, many workers (see Singh and Kumar [15] and references therein) have used the concept of constant deceleration parameter for the solution of Einstein field equations in FRW models. Singh and Kumar [15, 20] and Kumar and Singh [21] have extended this work to anisotropic Bianchi types-I and II cosmological models. Also, Singh et al. [18] have further extended it to Bianchi type-V models. Here we use the same concept of the variation law of Hubble's parameter which yields a constant value of the deceleration parameter.

From Eqs. (21) and (38), we obtain

$$\dot{a} = la^{-n+1}, \tag{39}$$

$$\ddot{a} = -l^2\,(n-1)\,a^{-2n+1}. \tag{40}$$

Using Eqs. (39) and (40) into Eq. (23), we get

$$q = n - 1\,. \tag{41}$$

We find that the deceleration parameter is constant. The sign of q indicates the behavior of the model. The positive sign of $q\,(0 \leq n \leq 1)$ corresponds to the decelerating model of the universe and the negative sign of q shows inflation.

Integrating Eq. (39), we obtain two different values of the average scale factor for $n \neq 0$ and $n = 0$ as

$$a = (nlt + c_1)^{1/n}\,, \qquad n \neq 0\,, \tag{42}$$

$$a = c_2 \exp{(lt)}\,, \qquad n = 0\,, \tag{43}$$

where c_1 and c_2 are constants of integration.

Now we solve the quadrature equations (34) − (36) for the metric functions with the help of Eqs. (37), (42) and (43). In this way, we obtain two categories of cosmological models, (i) singular model for $n \neq 0$, (ii) non-singular model for $n = 0$ as follows.

3.1. *Model of the Universe with $n \neq 0$*

From Eqs. (37) and (42), the expression for the BD scalar field ϕ is given by

$$\phi = b(nlt + c_1)^{-2/n}. \tag{44}$$

Using Eqs.(42) and (44) in Eqs. (34) − (36), the exact solutions of the metric functions A, B, C can be obtained as

$$A = (nlt + c_1)^{1/n}\,, \tag{45}$$

$$B = M\,(nlt + c_1)^{1/n} \exp\left[\frac{X}{lb(n-1)}\,(nlt + c_1)^{(n-1)/n}\right], \qquad n \neq 1\,, \tag{46}$$

$$C = M^{-1}\,(nlt + c_1)^{1/n} \exp\left[-\frac{X}{lb(n-1)}\,(nlt + c_1)^{(n-1)/n}\right], \qquad n \neq 1\,. \tag{47}$$

The volume scale factor V can be written as

$$V = (nlt + c_1)^{3/n}\,. \tag{48}$$

The dynamical scalars θ and σ^2 have the values given by

$$\theta = 3l\,(nlt + c_1)^{-1}\,, \tag{49}$$

$$\sigma^2 = \frac{X^2}{b^2}\,(nlt + c_1)^{-2/n}\,. \tag{50}$$

The directional Hubble's parameters H_1, H_2 and H_3 are obtained as

$$H_1 = l\,(nlt + c_1)^{-1}\,, \tag{51}$$

$$H_2 = l\,(nlt + c_1)^{-1} + \frac{X}{b}\,(nlt + c_1)^{-1/n}\,, \tag{52}$$

$$H_3 = l\,(nlt + c_1)^{-1} - \frac{X}{b}\,(nlt + c_1)^{-1/n}\,, \tag{53}$$

and the generalized mean Hubble's parameter is

$$H = l\,(nlt + c_1)^{-1}\,. \tag{54}$$

With the help of Eqs. (51) – (54), the anisotropy parameter of the model is obtained as

$$A_m = \frac{2X^2}{3b^2l^2}\left(nlt + c_1\right)^{2(n-1)/n} . \tag{55}$$

From Eqs. (24) and (25), the energy density ρ and the pressure p are calculated respectively, by using the above physical parameters as

$$8\pi\rho = b(2\omega - 3)l^2\left(nlt + c_1\right)^{-2(n+1)/n} - b\left(\frac{X^2}{b^2} + 3m^2\right)\left(nlt + c_1\right)^{-2/n} , \tag{56}$$

$$8\pi p = -b\left(2\omega + 3\right)l^2\left(nlt + c_1\right)^{-2(n+1)/n} + b\left(m^2 - \frac{X^2}{b^2}\right)\left(nlt + c_1\right)^{-2/n} . \tag{57}$$

It can be observed here that the wave Eq. (12) is satisfied if $\omega = 3(n-2)/2(3-n)$, $(n \neq 3)$ and $X^2 = 3m^2b^2$.

From the set of solutions obtained in this section, it is easy to see that in this model, at the initial epoch $t \to t_s$, $t_s = -c_1/nl$, the physical parameters such as θ, σ^2, ρ, p, H_1, H_2, H_3 and H are all infinite, whereas the volume scalar vanishes. The scalar function ϕ is also infinite at this epoch. The infinite density and pressure show that the model has a point singularity at $t = t_s$. The metric functions A and B, at this point singularity, vanish for $n \geq 0$ but the metric function C becomes indeterminate for $0 < n < 1$ and zero for $n > 1$. The anisotropy parameter is infinite for $n < 1$ but it will vanish for $n > 1$ at this epoch. At the final stage of expansion, as $t \to \infty$, the proper volume, the metric functions; A, B, $C(0 < n < 1)$ and the anisotropy parameter $A_m(n > 1)$ diverge, and all other functions vanish. The scalar field function will be zero for large time. The metric function C becomes indeterminate for $n > 1$. From all these observations, we infer that this model starts evolving with zero volume with infinite density and pressure at $t = t_s$ and expands with cosmic time t. The model also indicates that as t increases, the expansion scalar, the shear scalar and the anisotropy parameter for $n > 1$ decrease which show that for the large time the expansion will completely be finished and the model will attain isotropy. The isotropy condition for the large time of the model can also be verified for $\lim \sigma^2/\theta \to 0$ as $t \to \infty$ for $n < 2$.

3.2. Model of the Universe with $n = 0$

In this case, the expression for the BD scalar field ϕ, by using Eq. (43) into Eq. (37), is given as

$$\phi = \frac{b}{{c_2}^2}\exp\left(-2lt\right) , \tag{58}$$

which is a decreasing function of time. The exact solutions of the metric functions in Eqs. (34) – (36), with the help of Eqs. (43) and (58), can be obtained as

$$A = c_2 \exp\left(lt\right) , \tag{59}$$

$$B = Mc_2 \exp\left[lt - \frac{X}{blc_2}\exp(-lt)\right], \tag{60}$$

$$C = M^{-1}c_2 \exp\left[lt + \frac{X}{blc_2}\exp(-lt)\right]. \tag{61}$$

The volume scale factor V can be written as

$$V = {c_2}^3 \exp(3lt) . \tag{62}$$

The dynamical scalars θ and σ^2 have the values given by

$$\theta = 3l , \tag{63}$$

and

$$\sigma^2 = \frac{X^2}{b^2{c_2}^2}\exp(-2lt) . \tag{64}$$

The directional Hubble's parameters are obtained as

$$H_1 = l , \tag{65}$$

$$H_2 = l + \frac{X}{bc_2}\exp(-lt) , \tag{66}$$

$$H_3 = l - \frac{X}{bc_2}\exp(-lt) , \tag{67}$$

whereas H is given by

$$H = l . \tag{68}$$

The anisotropy parameter is obtained as

$$A_m = \frac{2X^2}{3l^2b^2{c_2}^2}\exp(-2lt) . \tag{69}$$

Putting the values of H, σ^2, ϕ, q and A from above equations into Eqs. (24) and (25), the ρ and p are calculated, respectively, as

$$8\pi\rho = \frac{bl^2(2\omega - 3)}{{c_2}^2}\exp(-2lt) - \frac{b}{{c_2}^4}\left(3m^2 + \frac{X^2}{b^2}\right)\exp(-4lt) , \tag{70}$$

$$8\pi p = \frac{-bl^2(2\omega + 3)}{{c_2}^2}\exp(-2lt) + \frac{b}{{c_2}^4}\left(m^2 - \frac{X^2}{b^2}\right)\exp(-4lt) . \tag{71}$$

We can easily verify that the wave equation (12) is satisfied with these solutions for $\omega = -1$ and $X^2 = 3m^2b^2$.

We observe that the physical quantities ρ, p, σ^2 and A_m are all infinite when $t \to -\infty$, which shows that there is no finite physical singularity in the model. As $t \to -\infty$, the volume scalar and the metric functions A and B vanish, whereas the metric function C becomes indeterminate. The scalar function ϕ is infinite for $t \to -\infty$. Thus the model has singularity in the infinite past. The expansion scalar is constant throughout the time of evolution of the universe. The directional Hubble's parameter H_1 and the generalized Hubble's parameter H are constants throughout the domain $-\infty < t < \infty$, but two other directional Hubble's parameters are time dependent and they have infinite values at this very epoch. We now study the model for the late time of its evolution. As $t \to \infty$, ρ, p, σ^2 and A_m will all become zero. The volume scalar and all three metric functions will diverge for the large time. The scalar function ϕ becomes zero as $t \to \infty$. The values of the directional Hubble's parameters H_2 and H_3 will be constant for the late time of the evolution.

After going through the behavior of physical and kinematical parameters for early and late times of evolution of the universe, we find that the universe is infinitely old and has exponential inflationary phase without any finite physical singularity. It is inferred from all these observations that the universe starts evolving with zero volume from an infinite past and expands with constant rate and will finally become isotropic with zero pressure and density for large time. The isotropy of the universe for the large time can also be verified for $\lim \sigma^2/\theta \to 0$ as $t \to \infty$.

4. *Conclusions*

We have presented two categories of exact solutions of the field equations of Brans-Dicke theory for a spatially homogeneous and anisotropic Bianchi type-V space-time by applying the law of variation for Hubble's parameter which yields a constant value of the deceleration parameter. The first category of solutions corresponds to the singular cosmological model with power-law expansion, whereas the second category of solutions corresponds to the non-singular model with exponential expansion. These solutions are valid for restricted values of the coupling parameter ω. These models are compatible with the results of recent observations. We have also discussed the physical and kinematical properties of the cosmological models.

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PROSTORNO HOMOGENI BIANCHIJEVI KOZMOLOŠKI MODELI TIPA V U BRANS-DICKEOVOJ TEORIJI

Raspravljamo jednadžbe polja Brans-Dickeove teorije za prostorno homogen i neizotropan Bianchijev prostor-vrijeme u prisutnosti perfektne tekućine te izvodimo njihova egzaktna rješenja primjenom varijacija Hubbleovog parametra, što vodi na stalnu vrijednost parametra usporavanja. Odnosni kozmološki modeli dijele se u dvije podvrste: (i) singularni modeli sa širenjem s potencijom i (ii) nesingularni modeli s eksponencijalnim širenjem. Raspravljaju se njihova fizička i kinematička svojstva. Ti su modeli u skladu s ishodima nedavnih opažanja.