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$D^{2/3}$ GENERATES MORE QUANTUM VACUUM ENERGY

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Dedicated to the memory of Andrea Raspini

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It has been shown that within the framework of the fractional spectral triplet action in noncommutative geometry, constructed recently by the author in the sense of Erdélyi-Kober fractional integral considerations, more quantum vacuum energy per a single mode is generated in the background of the spatially flat Friedmann-Robertson-Walker spacetime geometry. Our analyses are based on Broda et al. formalism which uses the coordinate gauge freedom transformations in the Euclidean version of the formalism of effective action. The result is estimated for a particular value of the fractional integral exponent which corresponds to the Raspini fractional Dirac operator of the order 2/3.

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1. Introduction

Fractional calculus is a special field of applied mathematics which was developed mainly in the 19^{th} century. Over the last decades the usefulness of this special mathematical framework in applications as well as its merits in pure mathematics has become more and more evident [1−5]. Because of the integral in the definition of the fractional order derivatives, it is obvious that these derivatives are non-local operators, which explains one of their most significant uses in applied sciences: the fractional derivatives possess the memory effect. In other words, at a certain point in time or space, the fractional or non-integer derivative contains information about the function at earlier points in time or space, respectively. Although assumptions about the existence of different types of fractional order derivative and integral, the Riemann-Liouville (RL) and the modified Erdélyi-Kober (EK) operators are still the most frequently used and have been popularized when fractional integration is performed. They are defined respectively as follows:

A-Riemann-Liouville fractional integral

If $f(t) \in \mathbb{C}^1([t_0, b]) : t \in [t_0, b]$, the left and right Riemann-Liouville fractional derivative (LRFD) of orders $\alpha, \beta \in \mathbb{R},\ \alpha, \beta < 1$ are defined by

$$D^{\alpha}_{a_+} f(t) = \frac{1}{\Gamma(1-\alpha)} \frac{\mathrm{d}}{\mathrm{d}t} \int\limits_a^t f(\tau)(t-\tau)^{-\alpha} \mathrm{d}\tau \,, \tag{1}$$

$$D^{\beta}_{b_-} f(t) = \frac{1}{\Gamma(1-\beta)} \left(-\frac{\mathrm{d}}{\mathrm{d}t} \int\limits_t^b f(\tau)(\tau-t)^{-\beta} \mathrm{d}\tau \right) , \tag{2}$$

where τ is the intrinsic time and t the observer time. This multi-time characteristic is important in applications and is the main ingredient of the theory being developed by Udriste [6].

B-Erdélyi-Kober Fractional Integral

For $\bar{G} \in L_p(0, \infty)$, the right and left modified Erdélyi-Kober operators of fractional integration for order $0 < \alpha < 1$ are defined through association of power weights as follows [7,8]:

$$I_{\mathrm{right}}(\alpha, \chi : m)\bar{G}(t) = \frac{m}{\Gamma(\alpha)} t^{-\chi - m\alpha + m - 1} \int\limits_{t_0=0}^{t} \tau^{\chi}(t^m - \tau^m)^{\alpha-1} \bar{G}(\tau) \mathrm{d}\tau \,, \tag{3}$$

$$I_{\mathrm{left}}(\alpha, \xi : m)\bar{G}(t) = \frac{m}{\Gamma(\alpha)} t^{\xi} \int\limits_t^{\infty} \tau^{-\xi - m\alpha + m - 1}(\tau^m - t^m)^{\alpha-1} \bar{G}(\tau) \mathrm{d}\tau \,, \tag{4}$$

where $\Re e(\alpha) > 0$, $\Re e(\chi) > -1/q$, $\Re e(\xi) > -1/p$, $1/p + 1/q = 1$, $p \geq 1$ and $m > 0$.

Although various fields of application of fractional derivatives and integrals are already well established, some others have just started, in particular the study of fractional problems in classical and quantum field theory. Nevertheless, the emerging fractional field theory is still an open problem under development. During the last decade, there are a number of papers devoted to investigation of fractional powers of second order classical differential operators of mathematical physics based on the use of the Riemann-Liouville fractional derivative, in particular the wave operator, Klein-Gordon and Dirac operators, and Schrödinger operators [9 − 17]. Of interest for us is the case of the fractional Dirac operator which plays in reality a crucial role in physics and geometry, in particular in Connes noncommutative geometry (NCG). NCG attracts an ever increasing attention of researchers, especially after the greatest success of unifying the forces of nature into a single gravitational action-the spectral action [18 − 20]. The Einstein-Hilbert spectral action was found to be approximated by the trace of a simple function of the Dirac operator.

In a more recent work [21], a fractional version of the square of the Dirac operator was constructed and accordingly the fractional spectral action principle (FSAP) was investigated and constructed as well [22, 23]. We summarize the basic results as the following:

1. For a fractional spectral triplet action based on the RL fractional integral, both the cosmological constant and the first-order scalar curvature are absent. The complexified action is dominated only by higher-order curvature terms and their complexified counterpart with corrections which are small for low curvature geometries.

2. For the case of the FSAP based on the Erdélyi-Kober fractional integral, the range of solutions is larger than that from its previous build from the Riemann-Liouville fractional integral. A cosmological constant may be present in the theory in particular if Raspini's Dirac operator of order 2/3 is taken into account [24, 25]. This result strongly suggests that the fractional spectral action principle in the sense of Erdélyi-Kober is more generalized.

In the present paper we extend the 2^{nd} formalism to the case of quantum vacuum energy within the framework of fractional effective action in the background of Friedmann-Robertson-Walker (FRW) spacetime geometry.

We will deal for simplicity with fractional time-variable. The quantum vacuum energy is in fact a difficult problem in modern cosmology: the vacuum energy is set to be constant with time while the matter energy density is a decreasing quantity, their ratio must be set to a specific infinitesimally small value 10^{-120} in the early Universe so as to nearly coincide today, i.e. a huge vacuum energy which by about 120 orders of magnitude exceeds the experimental limit [26]. Despite the fact that our novel approach does appeal to exotic assumptions, we will show that the idea discussed here will adhere to the standard quantum field theory formalism as closely as possible and in with a much better estimation than the one found recently in literature, mainly the Broda et al. approach [27].

The paper is organized as follows: in Section 2, we briefly recall the necessary definitions and properties of the fractional spectral action principle in the sense of Erdélyi-Kober. Our results are stated, proved and illustrated in Section 3. We end with Section 4 with conclusions and perspectives

2. *Fractional spectral action principle*

Before starting with our calculation, this is the right place to say several words about the fractional spectral triplet in the sense of Erdélyi-Kober within the framework of noncommutative geometry. Connes noncommutative geometry generalizes $(C^\infty(M), L^2(M,S), D)$ to a spectral triplet $(\mathcal{A}, \mathcal{H}, D)$, where $\mathcal{A}$ is a smooth algebra acting on a separable Hilbert space $\mathcal{H} = L^2(M,S)$, D is the hermitian self-adjoint operator that refers to as Dirac operator acting on $\mathcal{H}$ such that $\|[\mathcal{D}, \pi(x)]\| = \|[\mathrm{grad}\, \pi(x)]\|_\infty$, $\pi \in C(M)$. M is an orientable, connected, compact, N-dimensional differentiable unbounded manifold. The algebra $\mathcal{A} = C^\infty(M)$ of smooth functions on a compact boundaryless N-dimensional spin manifold M acts in $\mathcal{H}$ by multiplication operators as follows: $(fg)(x) = f(x)g(x)$, $\forall x \in M$. The total Riemannian spin geometry of M can be reconstructed from $(\mathcal{A}, \mathcal{H}, D)$. A positive functional on the affine space set F containing all possible Dirac operators is needed to obtain the dynamics on the gravitational field. This is the celebrated spectral action principle which states that the bosonic action depends only on the spectrum of the covariant Dirac operator as $S = \mathrm{Tr}F[D^2/\varepsilon^2] : F \to \mathbb{R}^+$ where F is any regular and fast decreasing function at infinity with $F(x) \geq 0$ if $x \geq 0$ and ε is a UV cutoff in the units length to the power two (Planck's length) [18−20, 28].

We summarize our basic results making use of equations (1)−(4) [22].

Definition II-1: *In the fractional spectral triplet $(\mathcal{A}, \mathcal{H}, D^\alpha)$ in the sense of Erdélyi-Kober, where D^α is the fractional Dirac operator of order α, the left function $\bar{G}$ at infinity is defined, under appropriate conditions, by*

$$\left[\bar{G}^{\alpha,n/2-k-1,m}_{2k,\,\mathrm{left}}\right](t) = \lim_{t\to\infty} \frac{1}{\Gamma(\alpha)} \int\limits_0^t \tau^{n/2-k-1}(t^m - \tau^m)^{\alpha-1}\bar{G}(\tau)\mathrm{d}\tau$$
$$= \frac{1}{m} \lim_{t\to\infty} I_{\mathrm{left}}(\alpha, \chi/2 - k - 1 : m)\bar{G}(t) \cdot t^{n/2-k+m\alpha-m}\,. \tag{5}$$

Corollary 1: *The trace of the fractional bosonic action in the Erdélyi-Kober fractional action framework takes in n-dimensional the form*

$$\mathrm{Tr}F\left[\frac{D^{2\alpha}}{\varepsilon^2}\right] = \frac{1}{m}\lim_{t\to\infty} \sum_{n-2<2k\leq n} \varepsilon^{n-2k} I_{\mathrm{left}}(\alpha, \chi/2 - k - 1 : m)\bar{G}$$
$$\times t^{n/2-k+m\alpha-m} \int\limits_M a_{2k}(D^{2\alpha})\sqrt{g}\,\mathrm{d}^n x + \mathrm{O}\left(\frac{1}{\varepsilon^2}\right), \tag{6}$$

where $a_{2k}(D^{2\alpha})$ is the Seeley-de Witt coefficient that occurs in the expansion of

$$\mathrm{Tr}\left[\exp(-\tau D^{2\alpha})\right] = \sum_{n-2<2k\leq n} \tau^{k-n/2} \int\limits_M a_{2k}(D^{2\alpha})\sqrt{g}\,\mathrm{d}^n x + \mathrm{O}(\tau), \tag{7}$$

when $\tau \to 0$ and $\alpha \leq \frac{1}{m}(k - n/2) + 1$ to avoid divergence.

Corollary 2: *In the EK-fractional spectral action principle, the Seeley-de Witt coefficient that occurs in the expansion of*

$$\mathrm{Tr}\left[\exp(-\tau D^{2\alpha})\right] = \sum_{n-2<2k\leq n} \tau^{k-n/2} \int\limits_M a_{2k}(D^{2\alpha})\sqrt{g}\,\mathrm{d}^n x + \mathrm{O}(\tau) \tag{8}$$

admits two possible conditions $n - 2m < 2k \leq n$ for $0 < \alpha \leq \frac{1}{m}(k - n/2) + 1$.

The fractional spectral action in four-dimensions in the sense of Erdélyi-Kober exists unless $2 - m < k \leq 2$. For $m = 2$, $k = 1, 2$, and consequently

$$\mathrm{Tr}F\left[\frac{D^{2\alpha}}{\varepsilon^2}\right] = \varepsilon^2 F_2^\alpha \int\limits_M a_2(D^{2\alpha})\sqrt{g}\,\mathrm{d}^4 x + \varepsilon^0 F_4^\alpha \int\limits_M a_4(D^{2\alpha})\sqrt{g}\,\mathrm{d}^4 x\,, \tag{9}$$

where we have neglected the higher-order non-physical terms. The first integral is recognized as the scalar curvature gravitational action and besides the cosmological constant is absent. It is an easy exercise to prove that only for $m = 3$, for which $-1 < k \leq 2$ $(k = 0, 1, 2)$ and $0 < \alpha \leq 1/3$, the cosmological constant may appear in the theory. The special value of $\alpha = 1/3$ indicates that we are dealing with a fractional Dirac operator of order $2/3$.

3. Main results

It is commonly expected that the effective cosmological constant can be induced by the 0^{th} term of the Seeley-de Witt coefficient. For a Planckian UV cutoff, i.e. $\varepsilon = \hbar G/c^3$ [29] where $\hbar$, G and c are, respectively, the reduced Planck's constant, gravitational coupling constant and velocity of light, the 0^{th} fractional term yields Casimir-like density contribution of the form [22]

$$\tilde{a}_0 = \mp \frac{1}{4} \frac{c^7}{16\pi^2 \hbar G^2} \frac{m}{\Gamma(\alpha)} t^{-\chi - m\alpha + m - 1} \int_{t_0=0}^{t} \tau^{\chi} (t^m - \tau^m)^{\alpha-1} \sqrt{g}\, \mathrm{d}\tau \iiint \mathrm{d}^3 x \,. \quad (10)$$

Here the positive sign corresponds to a boson, whereas the negative sign corresponds to a fermion. For the case of a spatially flat Friedmann-Robertson-Walker spacetime with metric

$$\mathrm{d}s^2 = -\mathrm{d}\tau^2 + a^2(t)\big[\mathrm{d}r^2 + r^2(\mathrm{d}\theta^2 + \sin^2\theta\, \mathrm{d}\phi^2)\big], \quad (11)$$

we can expand the scale factor $a(t)$ around $t = 0$ like

$$a(t) = a(0) + \left(\frac{\dot{a}}{a}\right)_0 \tau + \frac{1}{2}\left(\frac{\dot{a}}{a}\right)_0^2 \tau^2 + \mathrm{O}(\tau^3) = 1 + H_0 \tau - \frac{1}{2} q_0 H_0^2 \tau^2 + \mathrm{O}(\tau^3)\,, \quad (12)$$

where $H_0 \equiv \dot{a}/a$ is the Hubble expansion rate at the present time $\tau = 0$ and q_0 is the present deceleration parameter. We set $a(0) = 1$ in order to normalize the coordinates to unity. Accordingly, the radical square of the metric is

$$\sqrt{g} = \big[a^2(\tau)\big]^{3/2} = \big(1 + 2H_0\tau + (1 - q_0)H_0^2\tau^2 + \mathrm{O}(\tau^3)\big)^{3/2}. \quad (13)$$

Following the arguments of Ref. [27], one can effortlessly show that the infinitesimal gauge transformation of the metric $\delta g_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$ with the gauge parameter $\xi_\mu = (H_0 x^2/2 - H_0 \tau x^i)$ cancels in Eq. (13) the linear part in time [30]. Therefore, Eq. (10) is reduced to

$$\tilde{a}_0 \approx \mp \frac{1}{4} \frac{c^7}{16\pi^2 \hbar G^2} \frac{m}{\Gamma(\alpha)} t^{-\chi - m\alpha + m - 1} \int_{t_0=0}^{t} \tau^{\chi} (t^m - \tau^m)^{\alpha-1} \left[1 + \frac{3}{2}(1 - q_0) H_0^2 \tau^2\right] \mathrm{d}\tau \iiint \mathrm{d}^3 x \,. \quad (14)$$

By using some standard procedure in quantum field theory, we can subtract the number one in the bracket which corresponds to the term uncoupled to gravitational field and furthermore, we can discard the spatial volume as the integrand is time-dependent. Besides, as the calculation is perturbative in time, the density must be a fractional time average about the Planck's infinitesimal time $T_P = \sqrt{\hbar G/c^5}$, i.e.

$$\frac{1}{T_P} \frac{m}{\Gamma(\alpha)} T_P^{-\chi - m\alpha + m - 1} \int_0^{T_P} \tau^{\chi} (T_P^m - \tau^m)^{\alpha-1} \mathrm{d}\tau (\cdot)\,. \quad (15)$$

Therefore

$$\rho \approx \mp \frac{1}{4} \frac{c^7}{16\pi^2 \hbar G^2} \frac{3}{2} (1 - q_0) H_0^2 \lim_{T \to T_P} \frac{1}{T} \frac{m}{\Gamma(\alpha)} T^{-\chi - m\alpha + m - 1} \int_0^{T} \tau^{\chi+2} (T^m - \tau^m)^{\alpha-1} \mathrm{d}\tau \,. \quad (16)$$

Now, for $m = 3$ and $0 < \alpha \leq 1/3$, we find

$$\rho \approx \mp \frac{9}{8} \frac{c^7}{16\pi^2 \hbar G^2} (1 - q_0) H_0^2 \frac{1}{\Gamma(\alpha)} \lim_{T \to T_P} \frac{1}{T} T^{3-\chi-3\alpha} \int_0^T \tau^{\chi+2} (T^3 - \tau^3)^{\alpha-1} \mathrm{d}\tau \,. \tag{17}$$

The results for the densities for different values of the parameter χ are shown in Table 1.

TABLE 1. Density for different values of the parameter χ.

χ	$\int_0^T \tau^{\chi+2}(T^3 - \tau^3)^{-3/2}\mathrm{d}\tau$	ρ
-1	$\frac{2\pi}{3\sqrt{3}}$	$\approx \mp \frac{9}{8} \frac{c^7}{16\pi^2\hbar G^2} \frac{2\pi}{3\sqrt{3}} (1 - q_0) \frac{1}{\Gamma(1/3)} H_0^2 T_P^2$
0	T	$\approx \mp \frac{9}{8} \frac{c^7}{16\pi^2\hbar G^2} (1 - q_0) \frac{1}{\Gamma(1/3)} H_0^2 T_P^2$
1	$\frac{\sqrt{\pi} T^2 \Gamma(4/3)}{2^{2/3}\Gamma(5/6)}$	$\approx \mp \frac{9}{8} \frac{c^7}{16\pi^2\hbar G^2} (1 - q_0) \frac{1}{\Gamma(1/3)} \frac{\sqrt{\pi}\Gamma(4/3)}{2^{2/3}\Gamma(5/6)} H_0^2 T_P^2$
$-\frac{4}{3}$	$\frac{\Gamma(4/3)\Gamma(5/9)}{\Gamma(8/9)\sqrt[3]{T}}$	$\approx \mp \frac{9}{8} \frac{c^7}{16\pi^2\hbar G^2} (1 - q_0) \frac{\Gamma(4/3)\Gamma(5/9)}{\Gamma(1/3)\Gamma(8/9)} H_0^2 T_P^2$
-2	$\frac{\sqrt[3]{2}\Gamma(1/3)\Gamma(7/8)}{T\sqrt{\pi}}$	$\mp \frac{9}{8} \frac{c^7}{16\pi^2\hbar G^2} (1 - q_0) \frac{\sqrt[3]{2}\Gamma(7/6)}{\sqrt{\pi}} H_0^2 T_P^2$

Using the well-known relation $\rho_0 = 8\pi G \rho_{\mathrm{crit}}/(3c^2)$ [31], we obtain the results for ρ shown in Table 2.

TABLE 2. Density as a function of the critical density and the deceleration parameter for different values of the parameter χ.

$\chi = -1$	$\rho \approx \mp 0.009(1 - q_0)\rho_{\mathrm{crit}}$
$\chi = 0$	$\rho \approx \mp 0.022(1 - q_0)\rho_{\mathrm{crit}}$
$\chi = 1$	$\rho \approx \mp 0.019(1 - q_0)\rho_{\mathrm{crit}}$
$\chi = -\frac{4}{3}$	$\rho \approx \mp 0.03(1 - q_0)\rho_{\mathrm{crit}}$
$\chi = -2$	$\rho \approx \mp 0.04(1 - q_0)\rho_{\mathrm{crit}}$

Now, for $q_0 \approx -0.7$ [32], we find for $\chi = -2$ and $\rho \approx \mp 0.068\rho_{\mathrm{crit}}$ per a single mode which is a much better estimation than the value deduced in Broda et al. work which is $\rho \approx \mp 0.01\rho_{\mathrm{crit}}$. The experimental value is $\rho \approx \mp 0.076\rho_{\mathrm{crit}}$. It is noteworthy that for $\chi = -2$, the integral $\int_0^T \tau^{\chi+2}(T^3 - \tau^3)^{-3/2}\mathrm{d}\tau$ does not converge. Our result may be improved by a factor four if we make use of the regularization procedure in quantum field theory [29]. It is noteworthy that all previous analyses were explored for the case of the fractional Dirac operator $D^{2/3}$ and for the special value $\chi = -2$.

4. Conclusions and perspectives

In summary, making use of the coordinate gauge freedom discussed recently by Broda et al. within the framework of the fractional spectral triplet action principle based on the Erdélyi-Kober fractional integral approach, we have estimated the vacuum density of the accelerated universe. It has been shown that fractional estimation of the quantum vacuum energy can give a highly reasonable result if compared to observations, better than the one derived by Broda et al. The result is estimated for the particular value $\alpha = 1/3$ which corresponds to a fractional Dirac equation of the order 2/3. We pointed out that our estimated result is only a simple example. Our main aim was to show the reader the importance of fractional calculus in its broad range and its potential application in high energy physics. No claim about the originality of the results presented in this paper is made, but it is felt that the elementary use of operators of Erdélyi-Kober fractional integration to obtain them might appeal to both the applied mathematician and the theoretical physicist. Still, there are many open questions and serious problems to solve on the way to build a successful universal fractional quantum field theory. The results obtained in this work are promising.

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$D^{2/3}$ STVARA VIŠE KVANTNE VAKUUMSKE ENERGIJE

Pokazuje se da se u okviru razlomnog spektralnog tripletnog djelovanja u nekomutativnoj geometriji, koji je nedavno uveo autor primjenom Erdélyi-Koberovog razlomnog integrala, dobiva veća kvantna vakuumska energija po modu uz pozadinu prostorno ravne Friedmann-Robertson-Walkerove geometrije prostora-vremena. Ova se analiza zasniva na nedavnom radu Broda i sur. u kojoj se primjenjuju transformacije u Euklidskoj inačici formalizma efektivnog djelovanja s koordinatnom baždarnom slobodom. Ishodi računa se procjenjuju za posebnu vrijednost razlomnog integralnog eksponenta koji odgovara razlomnom Diracovom operatoru reda 2/3 koji je uveo A. Raspini.