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CLASSICAL ELECTRON MODEL WITH NON-STATIC CONFORMAL SYMMETRY

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Lorentz proposed a classical model of electron in which it was assumed the electron to have only 'electromagnetic mass'. We modelled electron as a charged anisotropic perfect fluid sphere admitting non-static conformal symmetry. We find that the pressure and density fail to be regular at the origin, but the effective gravitational mass is regular everywhere and vanishes in the limit $r \to 0$, i.e. it does not have the problem of singularity. Further, we have matched the interior metric with the exterior (Reissner-Nordström) metric and determine the values of the parameters k and r_0 (occurring in the solutions) as functions of mass, charge and radius of the spherically-symmetric charged object, i.e. electron.

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1. Introduction

In recent years, Herrera and Varela [1] discussed the electron model assuming the equation of state $p_{\mathrm{radial}} + \rho = 0$ with ad hoc anisotropy. Ray et al. [2] studied the electron model admitting a one-parameter group of conformal motion. We also notice that a new electromagnetic mass model admitting Chaplygin gas equation of state with a particular specialization was developed [3]. In the present study, we extend both the work in Refs. [1] and [2].

Valuable studies performed in the last two decades point out that a very interesting topic are the charged imperfect fluid spheres with a space-time geometry that admits a conformal symmetry, both in the static and the generalized non-static cases [4−9] (and references cited therein).

These classes of solutions are used in relativistic astrophysics for developing the star models. Herrera and his collaborators [5−7] searched for exact solutions of the field equations for static spheres considering that the static and spherically symmetric fluid geometry also presents a conformal symmetry. Ponce de Leon [8] gave regular static solutions with anisotropic pressure in the approach of a conformally flat sphere. Maartens and Maharaj [4] studied the problem more deeply and obtained regular solutions of the Einstein-Maxwell equations for charged imperfect fluids with conformal symmetry.

We investigate a new electron model assumig a charged anisotropic perfect fluid sphere admitting non-static conformal symmetry. The paper is organized as follows: in Section 2 we give the basic equations which are the Einstein-Maxwell field equations combined with the electromagnetic tensor field with anisotropic fluid. In Section 3 the solutions and some particular cases are given together with the graphical representation of the results. In Section 4 we match interior metric with the exterior (Reissner-Nordström) metric and determine the parameters k and r_0 in terms of mass, charge and radius of the spherically symmetric charged objects i.e. electron. Section 5 is devoted to a discussion of the results.

2. *Basic equations for the new electron mass model with non-static conformal symmetry*

The mass of the electron originates from its electromagnetic field itself. The electron is modelled as a charged fluid sphere obeying Einstein-Maxwell equations. To perform our study, we consider the Einstein-Maxwell field equations combined with the electromagnetic tensor field with anisotropic fluid, which together with an appropriate equation of state (EOS), $p_{\mathrm{radial}} + \rho = 0$, yield the basic equations used for developing a new electron model with non-static conformal symmetry.

The most general energy momentum tensor compatible with spherical symmetry is

$$T^{\mu}_{\nu} = (\rho + p_r)u^{\mu}u_{\nu} + p_r g^{\mu}_{\nu} + (p_t - p_r)\eta^{\mu}\eta_{\nu}\,, \tag{1}$$

with $u^{\mu}u_{\mu} = -\eta^{\mu}\eta_{\mu} = 1$, where ρ is the matter density, and p_r and p_t are the radial pressure and the transverse pressure of the fluid, respectively.

The static spherically symmetric space-time is given by the line-element (in geometrized units with $G = 1 = c$)

$$\mathrm{d}s^2 = -e^{\nu(r)}\mathrm{d}t^2 + e^{\lambda(r)}\mathrm{d}r^2 + r^2(\mathrm{d}\theta^2 + sin^2\theta\mathrm{d}\phi^2)\,, \tag{2}$$

where the functions of radial coordinate r, $\nu(r)$ and $\lambda(r)$ are the metric potentials.

For this metric, the Einstein Maxwell field equations are

$$e^{-\lambda}\left[\frac{\lambda'}{r}-\frac{1}{r^2}\right]+\frac{1}{r^2}=8\pi\rho+E^2, \tag{3}$$

$$e^{-\lambda}\left[\frac{1}{r^2}+\frac{\nu'}{r}\right]-\frac{1}{r^2}=8\pi p_r-E^2, \tag{4}$$

$$\frac{1}{2}e^{-\lambda}\left[\frac{1}{2}(\nu')^2+\nu''-\frac{1}{2}\lambda'\nu'+\frac{1}{r}(\nu'-\lambda')\right]=8\pi p_t+E^2, \tag{5}$$

where p_i, ρ and $E(r)$ represent fluid pressures (radial and transverse), matter-energy density and electric field, respectively.

The electric field is expressed by

$$(r^2E)'=4\pi r^2\sigma e^{\lambda/2}, \tag{6}$$

where σ represents the charge density of the spherical distribution.

The equation (6) gives the following form for the electric field

$$E(r)=\frac{1}{r^2}\int_0^r 4\pi r^2\sigma\, e^{\lambda/2}\,\mathrm{d}r=\frac{q(r)}{r^2}\,, \tag{7}$$

with $q(r)$ the total charge of the sphere under consideration.

Like most of the researchers who studied the electron model, we assume the equation of state

$$p_r=-\rho\,. \tag{8}$$

The assumption (8) is consistent with the 'Causality Condition' $|\mathrm{d}p/\mathrm{d}\rho|\le 1$ [10]. It is very common for cosmologists to use the matter distribution obeying this type of equation of state with the equation of state parameter $\omega=-1$ (known as false vacuum or ρ - vacuum) to explain the acceleration phase of the Universe [11].

3. Solutions

Now, we consider electron as a charged fluid sphere under conformal motion through the non-static conformal Killing vector as [4]

$$L_\xi g_{ij}=g_{ij;k}\xi^k+g_{kj}\xi^k_{;i}+g_{ik}\xi^k_{;j}=\psi g_{ij}\,, \tag{9}$$

where L represents the Lie derivative operator, ξ is the four vector along which the derivative is taken, ψ is the conformal factor, and g_{ij} are the metric potentials [9].

For a vanishing ψ, the equation above yields the Killing vector, the case $\psi = const.$ corresponds to the homothetic vector and for $\psi = \psi(x,t)$, we obtain conformal vectors. In this way the conformal Killing vector allows a more general study of the space-time geometry.

The proposed charged fluid (electromagnetic mass) space-time is mapped conformally onto itself along the direction ξ.

Here one takes ξ non-static, but ψ is static as

$$\xi = \alpha(t,r)\partial_t + \beta(t,r)\partial_r \,, \tag{10}$$

$$\psi = \psi(r)\,. \tag{11}$$

The above equations give the following set of expressions [4]

$$\alpha = A + \frac{1}{2}kt, \tag{12}$$

$$\beta = \frac{1}{2}Bre^{-\lambda/2}, \tag{13}$$

$$\psi = Be^{-\lambda/2}, \tag{14}$$

$$e^{\nu} = C^2 r^2 exp\left[-2kB^{-1}\int \frac{e^{\lambda/2}}{r}\,\mathrm{d}r\right], \tag{15}$$

where C, k, A, B are constants. According to Ref. [4], one can set $A = 0$ and $B = 1$ by re-scaling.

The equation of state (8) implies

$$\nu = -\lambda\,. \tag{16}$$

Equations (15) and (16) yield

$$-\lambda' = \frac{2}{r} - 2k\,\frac{e^{\lambda/2}}{r}\,. \tag{17}$$

Solving this equation, one gets

$$e^{\lambda/2} = \frac{1}{(k - r/r_0)}\,, \tag{18}$$

where r_0 is an integration constant.

We discuss the model by assuming the following assumption,

$$\sigma e^{\lambda/2} = \sigma_0 r^s, \tag{19}$$

where σ_0, λ and s represent the charge density at the center of the system, metric potential and a constant, respectively. Usually, the term $\sigma e^{\lambda/2}$ inside the integral sign in Eq. (7) is known as volume charge density. One can interpret the assumption (19) as the volume charge density being polynomial function of r [12].

Finally, we obtain the following set of solutions for different parameters

$$e^{\lambda} = e^{-\nu} = \frac{1}{\left(k - r/r_0\right)^2}, \tag{20}$$

$$\psi = \left(k - \frac{r}{r_0}\right), \tag{21}$$

$$E(r) = \left[\frac{4\pi\sigma_0}{(s+3)}\right] r^{s+1}, \tag{22}$$

$$q(r) = \left[\frac{4\pi\sigma_0}{(s+3)}\right] r^{s+3}, \tag{23}$$

$$8\pi\rho = \left(k - \frac{r}{r_0}\right)^2 \left[\frac{2}{r(kr_0 - r)} - \frac{1}{r^2}\right] + \frac{1}{r^2} - \left[\frac{16\pi^2\sigma_0^2}{(s+3)^2}\right] r^{2s+2}, \tag{24}$$

$$8\pi p_r = \left(k - \frac{r}{r_0}\right)^2 \left[\frac{1}{r^2} - \frac{2}{r(kr_0 - r)}\right] - \frac{1}{r^2} + \left[\frac{16\pi^2\sigma_0^2}{(s+3)^2}\right] r^{2s+2}, \tag{25}$$

$$8\pi p_t = \frac{1}{r_0^2}\left[3 - \frac{2(kr_0 - r)}{r}\right] - \left[\frac{16\pi^2\sigma_0^2}{(s+3)^2}\right] r^{2s+2}. \tag{26}$$

It is worthwhile to calculate effective gravitational mass which is due to the contribution of the energy density ρ of the matter and the electric energy density $E^2/(8\pi)$ and can be expressed as

$$M = \int_0^r 4\pi r^2 \left[\rho + \frac{E^2}{8\pi}\right] \mathrm{d}r = \frac{k}{r_0} r^2 + (1 - k^2)\frac{r}{2} - \left(1 + \frac{1}{2}k^2\right)\frac{r^3}{3r_0^2}. \tag{27}$$

It is noted that the metric potentials $\nu(r)$ & $\lambda(r)$, ψ, electric field $E(r)$, electric charge $q(r)$, matter energy density and pressures are depended on several unknown parameters such as σ_0, s, k and r_0. So, to get the exact values of these parameters, one has to match the solutions with Reissner-Nordström metric as will be discussed later.

We have $p_r = -\rho$, which implies that a negative value of the matter energy density ρ determines a positive value of the radial pressure p_r.

Here, the NEC energy condition $\rho + p_r \geqslant 0$ is satisfied (for equality with zero). The violation of NEC implies the breakdown of causality in general relativity and the violation of the second law of thermodynamics [13]. The condition $\rho + p_t > 0$ is satisfied for

$$\left(k - \frac{r}{r_0}\right)^2 \left[\frac{2}{r(kr_0 - r)} - \frac{1}{r^2}\right] + \frac{1}{r^2} + \frac{1}{r_0^2}\left[3 - \frac{2(kr_0 - r)}{r}\right] > \left[\frac{32\pi^2\sigma_0^2}{(s+3)^2}\right] r^{2s+2}\,.$$

This means, there exists a limiting value of the radial coordinate for which $\rho + p_t > 0$.

One can note that apparently there is no singularity at $r = 0$ for the metric coefficients σ, E if $s > -1$.

But the Kretschmann scalar

$$K = R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} =$$

$$\frac{4}{r^4 r_0^4}[10r^2k^2r_0^2 - 12r^3kr_0 + 6r^4 + r_0^4 - 2r_0^4k^2 + 4r_0^3kr - 2r_0^2r^2 + k^4r_0^4 - 4k^3r_0^3r]$$

indicates that there is a point of divergence at $r = 0$. The pressure and density also fail to be regular at the origin, but the effective gravitational mass is always positive and will vanish as $r \to 0$, i.e., it does not have to tolerate the problem of singularity.

4. *Matching with Reissner Nordström metric*

To match interior metric with the exterior (Reissner-Nordström) metric, we impose only the continuity of g_{tt}, g_{rr} and $\partial g_{tt}/\partial r$, across the surface S at $r = a$

$$1 - \frac{2m}{a} + \frac{Q^2}{a^2} = \left(k - \frac{a}{r_0}\right)^2, \tag{28}$$

$$\frac{m}{a^2} - \frac{Q^2}{a^3} = -\frac{1}{r_0}\left(k - \frac{a}{r_0}\right). \tag{29}$$

From, the above three equations, one can find the values of the unknown k and r_0 in terms of mass, charge and radius of electron as

$$r_0 = -\frac{\left[1 - \frac{2m}{a} + \frac{Q^2}{a^2}\right]^{1/2}}{\left(-\frac{m}{a^2} + \frac{Q^2}{a^3}\right)}\,, \tag{30}$$

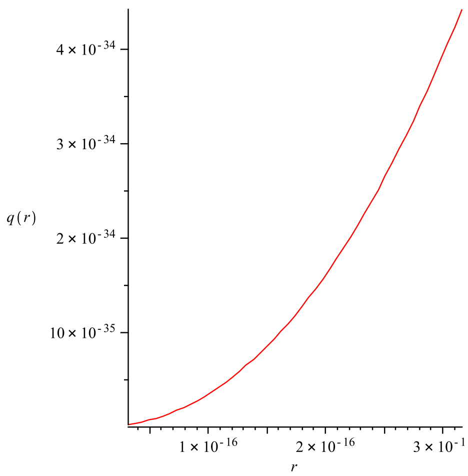
$$k = \frac{\left[1 - \frac{3m}{a} + \frac{2Q^2}{a^2}\right]}{\left[1 - \frac{2m}{a} + \frac{Q^2}{a^2}\right]^{1/2}}\,. \tag{31}$$

According Herrera et al. [1], we assume the following values for the electrons: radius $a \sim 10^{-16}$ cm, the inertial mass $m \sim 10^{-56}$ cm and charge $Q \sim 10^{-34}$ cm in relativistic units. So the matching conditions imply

$$r_0 = -10^{20}\,, \tag{32}$$

$$k = 1\,. \tag{33}$$

In Fig. 1, Fig. 2, Fig. 3, Fig. 4, Fig. 5 and Fig.6, we plot electric charge $q(r)$, electric field $E(r)$, matter energy density ρ, radial pressure p_r, effective gravitational mass of the electron M and the transverse pressure of the fluid p_t against the parameter r, assuming the above values of the parameters r_0 and k.



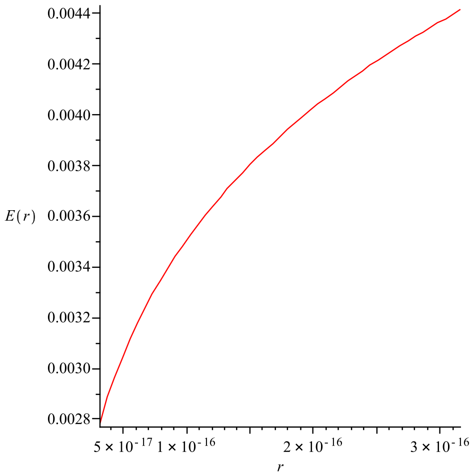
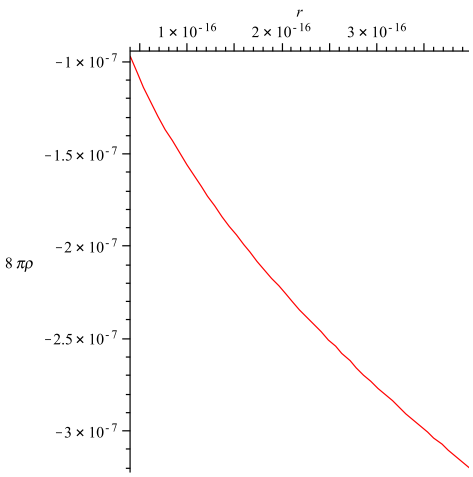


Fig. 1 (left). Dependence of the electric charge on the radial coordinate r for $s = -.8$ and $\sigma_0 = 1$.

Fig. 2. Dependence of the electric field strength on the radial coordinate r for $s = -.8$ and $\sigma_0 = 1$.



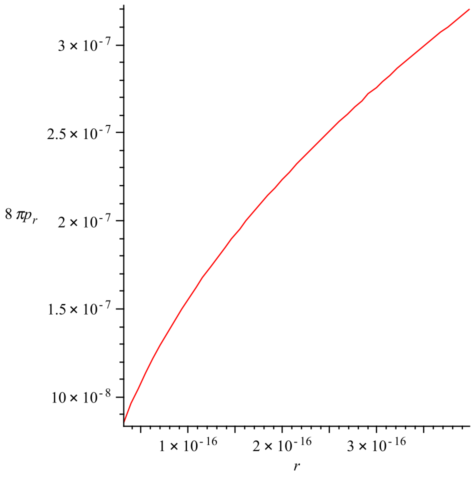
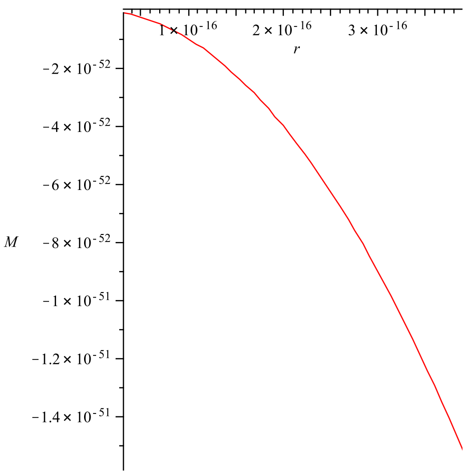


Fig. 3 (left). Dependence of the energy density on the radial coordinate r *for* $s = -.8$ *and* $\sigma_0 = 1$.

Fig. 4. Dependence of the radial pressure on the radial coordinate r *for* $s = -.8$ *and* $\sigma_0 = 1$.



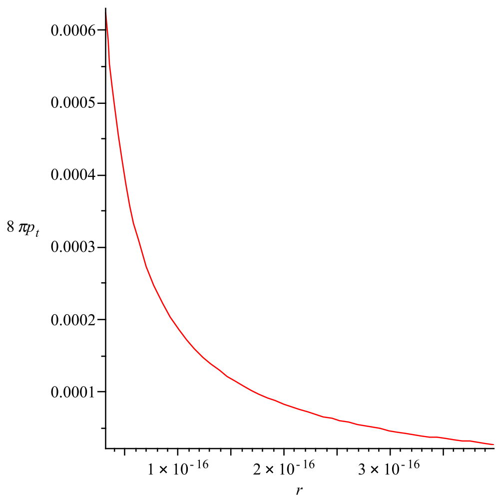


Fig. 5 (left). Dependence of the effective gravitational mass function on the radial coordinate r.

Fig. 6. Dependence of the transverse pressure on the radial coordinate r *for* $s = -.8$ *and* $\sigma_0 = 1$.

5. *Discussion*

A new electron model as a charged anisotropic perfect fluid sphere, admitting non-static conformal symmetry, is investigated and in this view we performed our calculations with the Einstein-Maxwell field equations combined with the electromagnetic tensor field of an anisotropic fluid and with an appropriate equation of state, $p_{\text{radial}} + \rho = 0$. For describing the behavior of our model, we assume that $\sigma e^{\lambda/2} = \sigma_0 r^s$ and obtain the expressions for the metric potentials $\nu(r)$ and $\lambda(r)$, ψ, the electric field $E(r)$, the electric charge $q(r)$, the matter energy density ρ, the radial pressure p_r, the transverse pressure of the fluid p_t, and the effective gravitational mass of the electron M. It has been found that the pressure and density fail to be regular at the origin, but the effective gravitational mass is regular everywhere and vanishes at the limit $r \to 0$ i.e. it does not have to tolerate the problem of singularity. We extend our study by matching interior metric with the exterior (Reissner-Nordström) metric, and also establish the values of the parameters k and r_0 in terms of the mass, charge and radius of the electron. Using the values obtained for r_0 and k, one finds that the charge of the fluid is nearly equal to $q \sim 10^{-34}$ cm in relativistic units, which is equivalent to the charge of electron (experimentally obtained [1]). The matching of the interior metric with the exterior (Reissner-Nordström) metric, $1 - \dfrac{2m}{a} + \dfrac{Q^2}{a^2} = 1 - \dfrac{2M}{a}$, where M is the effective gravitational mass, implies $M = m - \dfrac{Q^2}{2a}$. If we use the above values of inertial mass, charge and radius of the electron, one gets the value of the effective gravitational mass within the fluid sphere as $\sim -10^{-52}$cm. Using the above obtained values of r_0 and k, one finds that the effective gravitational mass of the charged fluid is nearly equal to $\sim -10^{-52}$ (see Fig. 5).

In the present work, we used the non-static conformal symmetry technique to study charged fluid of radius $\sim 10^{-16}$. It may be interesting to extrapolate the present investigation to the astrophysical bodies, especially quarks or strange stars.

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KLASIČAN MODEL ELEKTRONA S NESTATIČKOM KONFORMNOM SIMETRIJOM

Lorentz je predložio klasičan model elektrona u kojemu se pretpostavlja da ima samo ‘elektromagnetsku’ masu. Mi smo razvili model elektrona kao nabijene anizotropne perfektne tekućine s nestatičkom konformnom simetrijom. Nalazi se da tlak i gustoća nisu konačni u ishodištu, dok je gravitacijska masa svugdje konačna i nestaje u ishodištu, tj. ne nalazimo problem singularnosti. Nadalje, pripojili smo unutarnju metriku vanjskoj (Reissner-Nordströmovoj) metrici i odredili vrijednosti parametara k i r_0 koji se susreću u rješenjima za masu, naboj i polumjer sferno simetričnih nabijenih čestica, tj. elektrona.