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LYRA'S COSMOLOGY OF INHOMOGENEOUS UNIVERSE WITH ELECTROMAGNETIC FIELD

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The plane-symmetric inhomogeneous cosmological models of perfect fluid distribution with electro-magnetic field are obtained in the framework of Lyra's geometry. To get the deterministic solution, I assume $A = f(x)\nu(t)$, $B = g(x)\mu(t)$ and $C = h(x)\mu(t)$, where A, B and C are metric coefficients. It has been found that the solutions generalize the solution obtained by Pradhan, Yadav and Singh (2007) and are consistent with the recent observations of type Ia supernovae. A detailed study of physical and kinematical properties of the models have been carried out.

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1. *Introduction*

In recent years, our knowledge of cosmology has improved remarkably by various experimental and theoretical results. The universe is spherically symmetric and the matter distribution in it is on the whole isotropic and homogeneous. But during the early stage of evolution, it is unlikely that it could have such a smoothed-out picture, so I consider plane symmetry which provides an opprotunity for the study of inhomogeneity. The present study deals with plane-symmetric inhomogeneous models within the framework of Lyra's geometry in the presence of electromagnetic field. The essential difference between between the cosmological theories based on Lyra's geometry and Riemannian geometry lies in the fact that the constant vector displacement field β arises naturally from the concept of gauge in Lyra's geometry, where as the cosmological constant Λ was introduced in an ad hoc fashion in the usual treatment. Currently, the study of gauge function and cosmological constant have gained renewed interest due to their application in structure formation in the universe.

Einstein introduced his general theory of relativity in which gravitation is described in terms of geometry of space-time. Einstein's idea of geometrizing gravitation in the form of general theory of relativity inspired the idea of geometrizing other physical fields. Shortly after Einstein's general theory of relativity, Weyl [1] suggested the first so-called unified field, which is a geometrized theory of gravitation and electromagnetism. But this theory was never taken seriously because it was based on the concept of non-integrability of length transfer. Lyra [2] proposed a modification of Riemannian geometry by introducing a gauge function which removes the non-integrability condition of the length of a vector under parallel transport. In consecutive investigations, Sen [3], and Sen and Dunn [4] proposed a new scalar-tensor theory of gravitation and constructed an analog of the Einstein field equations based on Lyra's geometry. It is thus possible [3] to construct a geometrized theory of gravitation and electromagnetism much along the lines of Weyl's "unified" field theory however, without the inconvenience of non-integrability of length transfer.

Halford [5] has pointed out that the constant vector displacement field ϕ_i in Lyra's geometry plays the role of the cosmological constant Λ in the normal general relativistic treatment. It is shown by Halford [6] that the scalar-tensor treatment based on Lyra's geometry predicts the same effects, within observational limits, as the Einstein's theory. Several authors, Sen and Vanstone [7], Bhamra [8], Karade and Borikar [9], Kalyanshetti and Wagmode [10], Reddy and Innaiah [11], Beesham [12], Reddy and Venkateswarlu [13], Soleng [14], studied cosmological models based on Lyra's manifold with a constant displacement field vector. However, this restriction of the displacement field to be constant is merely one of convenience and there is no a priori reason for it. Beesham [15] considered FRW models with time-dependent displacement field. He has shown that by assuming the energy density of the universe to be equal to its critical value, the models have the $k = -1$ geometry. Singh and Singh [16] – [19], Singh and Desikan [20] studied Bianchi type-I, -III, -Kantowaski-Sachs and a new class of cosmological models with time dependent displacement field and made a comparative study of Robertson-Walker models with constant deceleration parameter in Einstein's theory with cosmological term and in the cosmological theory based on Lyra's geometry. Soleng [14] pointed out that the cosmologies based on Lyra's manifold with a constant gauge vector ϕ will either include a creation field and be equal to Hoyle's creation-field cosmology [21] – [23] or contain a special vacuum field which together with the gauge vector term may be considered as a cosmological term. In the latter case, the solutions are equal to the general relativistic cosmologies with a cosmological term.

The occurrence of magnetic fields on galactic scale is well-established fact today, and their importance for a variety of astrophysical phenomena is generally acknowledged as pointed out by Zel'dovich et al. [24]. Also Harrison [25] has suggested that magnetic field could have a cosmological origin. As a natural consequence, we should include magnetic fields in the energy-momentum tensor of the early universe. The choice of anisotropic cosmological models in Einstein system of field equations leads to the cosmological models more general than the Robertson-Walker model [26]. Strong magnetic fields can be created due to adiabatic compression in clusters of galaxies. Primordial asymmetry of particle (say electron) over antiparticle (say positron) have been well established as CP (charge-parity) violation. Asseo and Sol [27] speculated that the large-scale inter-galactic magnetic field is of primordial origin and at present measures 10^{-8} G, and gives rise to a density of the order of 10^{-35} gcm^{-3}. The present day magnitude of the magnetic energy is very small in comparison with the estimated matter density, but it might not have been negligible during early stage of evolution of the universe. FRW models are approximately valid as the present day magnetic field is very weak. The existence of a primordial magnetic field is limited to Bianchi type-I, -II, -III, -VI$_0$ and -VII$_0$ models, as shown by Hughston and Jacobs [28]. Large-scale magnetic fields give rise to anisotropies in the universe. The anisotropic pressure created by the magnetic fields dominates the evolution of the shear anisotropy and it decays slower than if the pressure were isotropic [29, 30]. Such fields can be generated at the end of an inflationary epoch [31] − [33]. Anisotropic magnetic-field models have significant contribution in the evolution of galaxies and stellar objects. Recently, Pradhan et al. [34], Casama et al. [35], Rahaman et al. [36], Bali and Chandani [37], Kumar and Singh [38], Singh [39] and Rao, Vinutha and Santhi [40] studied cosmological models based on Lyra's geometry in various contexts. More recently, Rahaman et al. [41, 42] and Pradhan et al. [43, 44] obtained some inhomogeneous cosmological models in Lyra's geometry. Motivated by these researches, in this paper I study the plane-symmetric inhomogeneous cosmological models in the presence of magnetic field in the framework of Lyra's geometry and also discuss the thermodynamical behaviour of the universe.

2. *The metric and field equations*

We consider the plane-symmetric metric in the form

$$\mathrm{d}s^2 = A^2(\mathrm{d}x^2 - \mathrm{d}t^2) + B^2\mathrm{d}y^2 + C^2\mathrm{d}z^2, \tag{1}$$

where A, B and C are functions of x and t. The energy momentum tensor is taken as

$$T_i^j = (\rho + p)u_i u^j + p g_i^j + E_i^j, \tag{2}$$

where ρ and p are, respectively, the energy density and pressure of the cosmic fluid, and u_i is the fluid four-velocity vector satisfying the condition

$$u^i u_i = -1, \quad u^i x_i = 0\,. \tag{3}$$

In Eq. (2), E_i^j is the electromagnetic field given by Lichnerowicz [45]

$$E_i^j = \bar{\mu}\left[h_l h^l\left(u_i u^j + \frac{1}{2}g_i^j\right) - h_i h^j\right], \tag{4}$$

where $\bar{\mu}$ is the magnetic permeability and h_i the magnetic flux vector defined by

$$h_i = \frac{1}{\bar{\mu}}\,{}^*F_{ji}u^j, \tag{5}$$

where the dual electromagnetic field tensor $^*F_{ij}$ is defined by Synge [46]

$$^*F_{ij} = \frac{\sqrt{-g}}{2} \epsilon_{ijkl} F^{kl}. \tag{6}$$

Here F_{ij} is the electromagnetic field tensor and ϵ_{ijkl} is the Levi-Civita tensor density.

The co-ordinates are considered to be co-moving so that $u^1 = 0 = u^2 = u^3$ and $u^4 = \dfrac{1}{A}$. If we consider that the current flows along the z-axis, then F_{12} is the only non-vanishing component of F_{ij}. The Maxwell's equations

$$F_{ij;k} + F_{jk;i} + F_{ki;j} = 0\,, \tag{7}$$

$$\left[\frac{1}{\bar{\mu}} F^{ij}\right]_{;j} = 4\pi J^i, \tag{8}$$

require that F_{12} is the function of x alone. We assume that the magnetic permeability is the function of both x and t. Here the semicolon represents the covariant differentiation.

The field equations, in normal gauge for Lyra's manifold, obtained by Sen [4] are

$$R_{ij} - \frac{1}{2} g_{ij} R + \frac{3}{2} \phi_i \phi_j - \frac{3}{4} g_{ij} \phi_k \phi^k = -8\pi T_{ij}, \tag{9}$$

where ϕ_i is the displacement vector defined as

$$\phi_i = (0, 0, 0, \beta(t))\,, \tag{10}$$

and other symbols have their usual meaning as in Riemannian geometry.

For the line-element (1), the field Eq. (9) with Eqs. (2) and (10) lead to the following system of equations

$$\frac{1}{A^2}\left[-\frac{B_{44}}{B} - \frac{C_{44}}{C} + \frac{A_4}{A}\left(\frac{B_4}{B} + \frac{C_4}{C}\right) + \frac{A_1}{A}\left(\frac{B_1}{B} + \frac{C_1}{C}\right) - \frac{B_4 C_4}{BC} + \frac{B_1 C_1}{BC}\right] - \frac{3}{4}\beta^2 = 8\pi\left(p + \frac{F_{12}^2}{2\bar{\mu} A^2 B^2}\right), \tag{11}$$

$$\frac{1}{A^2}\left[\frac{A_{11}}{A} + \frac{C_{11}}{C} - \frac{A_{44}}{A} - \frac{C_{44}}{C} - \left(\frac{A_1}{A}\right)^2 + \left(\frac{A_4}{A}\right)^2\right] - \frac{3}{4}\beta^2 = 8\pi\left(p + \frac{F_{12}^2}{2\bar{\mu} A^2 B^2}\right), \tag{12}$$

$$\frac{1}{A^2}\left[\frac{A_{11}}{A}+\frac{B_{11}}{B}-\frac{A_{44}}{A}-\frac{B_{44}}{B}-\left(\frac{A_1}{A}\right)^2+\left(\frac{A_4}{A}\right)^2\right]$$

$$-\frac{3}{4}\beta^2 = 8\pi\left(p+\frac{F_{12}^2}{2\bar{\mu}A^2B^2}\right), \tag{13}$$

$$\frac{1}{A^2}\left[-\frac{B_{11}}{B}-\frac{C_{11}}{C}+\frac{A_1}{A}\left(\frac{B_1}{B}+\frac{C_1}{C}\right)+\frac{A_4}{A}\left(\frac{B_4}{B}+\frac{C_4}{C}\right)-\frac{B_1C_1}{BC}+\right.$$

$$\left.\frac{B_4C_4}{BC}\right]+\frac{3}{4}\beta^2 = 8\pi\left(\rho+\frac{F_{12}^2}{2\bar{\mu}A^2B^2}\right), \tag{14}$$

$$\frac{B_{14}}{B}+\frac{C_{14}}{C}-\frac{A_1}{A}\left(\frac{B_4}{B}+\frac{C_4}{C}\right)-\frac{A_4}{A}\left(\frac{B_1}{B}+\frac{C_1}{C}\right)=0\,, \tag{15}$$

where the sub-indices 1 and 4 in A, B, C and elsewhere denote ordinary differentiation with respect to x and t, respectively.

3. Solutions and field equations

Equations (11) - (13) lead to

$$\left(\frac{A_4}{A}\right)_4-\frac{B_{44}}{B}+\frac{A_4}{A}\left(\frac{B_4}{B}+\frac{C_4}{C}\right)-\frac{B_4C_4}{BC}=$$

$$\left(\frac{A_1}{A}\right)_1+\frac{C_{11}}{C}-\frac{A_1}{A}\left(\frac{B_1}{B}+\frac{C_1}{C}\right)-\frac{B_1C_1}{BC}=a\ (\text{constant}) \tag{16}$$

and

$$\frac{8\pi F_{12}^2}{\bar{\mu}B^2}=\frac{B_{44}}{B}-\frac{B_{11}}{B}+\frac{C_{11}}{C}-\frac{C_{44}}{C}\,. \tag{17}$$

Equations (10) - (14) represent a system of five equations in seven unknowns A, B, C, ρ, p, β and F_{12}. For the complete determination of these unknowns, one more condition is needed. As in the case of general-relativistic cosmologies, the introduction of inhomogeneities into the cosmological equations produces a considerable increase in mathematical difficulty: non-linear partial differential equations must now be solved. In practice, this means that we must proceed either by means of approximations which render the non-linearities tractable, or we must introduce particular symmetries into the metric of the space-time in order to reduce the number of degrees of freedom which the inhomogeneities can exploit. In the present case, we assume that the metric is Petrov type-II non-degenerate. This requires that

$$\left(\frac{B_{11}+B_{44}+2B_{14}}{B}\right)-\left(\frac{C_{11}+C_{44}+2C_{14}}{C}\right)=$$
$$\frac{2(A_1+A_4)(B_1+B_4)}{AB}-\frac{2(A_1+A_4)(C_1+C_4)}{AC}\,. \tag{18}$$

Let us consider that

$$A=f(x)\nu(t)\,,$$
$$B=g(x)\mu(t)\,,$$
$$C=h(x)\mu(t)\,. \tag{19}$$

Using (19) in (14) and (17), we get

$$\frac{(g_1/g)+(h_1/h)}{(f_1/f)}=\frac{(2\mu_4/\mu)}{(\mu_4/\mu)-(\nu_4/\nu)}=b(\text{constant}) \tag{20}$$

and

$$\frac{(g_{11}/g)+(h_{11}/h)}{(g_1/g)-(h_1/h)}-\frac{2f_1}{f}=2\left(\frac{\mu_4}{\mu}-\frac{\nu_4}{\nu}\right)=L\ (\text{constant})\,. \tag{21}$$

Equation (20) leads to

$$f=n(gh)^{\frac{1}{b}},\ b\neq 0\,, \tag{22}$$

and

$$\mu=m\nu^{\frac{b}{b-2}}, \tag{23}$$

where m and n are constants of integration.

From Eqs. (15), (18) and (19), we have

$$\frac{1}{b}\frac{g_{11}}{g}+\left(\frac{1+b}{b}\right)\frac{h_{11}}{h}-\frac{2}{b}\left(\frac{g_1^2}{g^2}+\frac{h_1^2}{h^2}\right)-\frac{(2+b)}{b}\frac{g_1h_1}{gh}=a \tag{24}$$

and

$$\frac{2}{b}\left(\frac{\mu_{44}}{\mu}+\frac{\mu_4^2}{\mu^2}\right)=-a\,. \tag{25}$$

Let us assume

$$g=e^{U+W},\quad h=e^{U-W}. \tag{26}$$

Equations (20) and (25) lead to

$$W_1 = M \exp\left(Lx + \frac{2(2-b)}{b} U\right), \tag{27}$$

where M is an integrating constant. From Eqs. (23), (25) and (26), we have

$$\left(\frac{2+b}{b}\right) U_{11} - \frac{4}{b} U_1^2 - 2bM \exp\left(Lx + \frac{2(2-b)}{b} U\right) -$$

$$ML \exp\left(Lx + \frac{2(2-b)}{b} U\right) + 2M^2 \exp\left(2Lx + \frac{4(2-b)}{b} U\right) = a\,. \tag{28}$$

Equation (27) leads to

$$U = \frac{Lbx}{2(b-2)}, \quad b \neq 2\,. \tag{29}$$

Equations (26) and (28) lead to

$$W = Mx + \log N, \tag{30}$$

where N is the constant of integration.

Equation (24) leads to

$$\mu = \left[\begin{array}{ll} \ell \cosh^{\frac{1}{2}}(\sqrt{|\alpha|}\, t + t_0) & \text{when } ab < 0\,, \\ \ell \cos^{\frac{1}{2}}(\sqrt{\alpha}\, t + t_0) & \text{when } ab > 0\,, \\ (c_1 t + t_0)^{\frac{1}{2}} & \text{when } ab = 0\,, \end{array}\right. \tag{31}$$

where $\alpha = ab$, ℓ is constant and c_1, t_0 are constants of integration. Now we consider the following three cases.

4. *Case (i):* $ab < 0$

In this case we obtain

$$f = n \exp\left(\frac{Lx}{(b-2)}\right), \tag{32}$$

$$\mu = \ell \cosh^{\frac{1}{2}}(\sqrt{|\alpha|}\, t + t_0)\,, \tag{33}$$

$$\nu = r \cosh^{\frac{(b-2)}{2b}}(\sqrt{|\alpha|}\, t + t_0)\,, \tag{34}$$

$$g = N \exp\left(\frac{Lbx}{2(b-2)} + Mx\right), \tag{35}$$

$$h = \frac{1}{N}\exp\left(\frac{Lbx}{2(b-2)} - Mx\right), \tag{36}$$

where $r = \left(\frac{\ell}{m}\right)^{\frac{(b-2)}{b}}$. Therefore, we have

$$A = E_0 \exp\left(\frac{Lx}{(b-2)}\right)\cosh^{\frac{(b-2)}{2b}}(\sqrt{|\alpha|}\,t + t_0), \tag{37}$$

$$B = G_0 \exp\left(\frac{Lbx}{2(b-2)} + Mx\right)\cosh^{\frac{1}{2}}(\sqrt{|\alpha|}\,t + t_0), \tag{38}$$

$$C = H_0 \exp\left(\frac{Lbx}{2(b-2)} - Mx\right)\cosh^{\frac{1}{2}}(\sqrt{|\alpha|}\,t + t_0), \tag{39}$$

where $E_0 = nr$, $G_0 = N\ell$, $H_0 = \ell/N$.

After using suitable transformation of coordinates, the metric (1) reduces to the form

$$\begin{aligned} \mathrm{d}s^2 = E_0^2 \exp\left(\frac{2LX}{(b-2)}\right)\cosh^{\frac{(b-2)}{b}}(\sqrt{|\alpha|}\,T)(\mathrm{d}X^2 - \mathrm{d}T^2) + \\ \exp\left(\frac{LbX}{(b-2)} + 2MX\right)\cosh(\sqrt{|\alpha|}\,T)\mathrm{d}Y^2 + \\ \exp\left(\frac{LbX}{(b-2)} - 2MX\right)\cosh(\sqrt{|\alpha|}\,T)\mathrm{d}Z^2. \end{aligned} \tag{40}$$

For the specification of displacement vector $\beta(t)$ within the framework of Lyra geometry and for realistic models of physical importance, we consider the following two cases by taking β as constant and also β as a function of time.

4.1. β is a constant, $\beta = \beta_0$

Using Eqs. (37), (38) and (39) in Eqs. (10) and (13), the expressions for the pressure p and density ρ for the model (40) are given by

$$\begin{aligned} 8\pi p = \frac{1}{E_0^2}\exp\left(\frac{2LX}{2-b}\right)\cosh^{\frac{2-b}{b}}\left(\sqrt{|\alpha| T}\right) \times \\ \left[|\alpha|\left\{\frac{(b+1)}{4b}\tanh^2\left(\sqrt{|\alpha| T}\right) - 1\right\} + \frac{b(b+4)L^2}{4(b-2)^2} - M^2 + \frac{MLb}{(b-2)}\right] - \frac{3}{4}\beta_0^2, \end{aligned} \tag{41}$$

$$8\pi\rho = \frac{1}{E_0^2}\exp\left(\frac{2LX}{2-b}\right)\cosh^{\frac{2-b}{b}}\left(\sqrt{|\alpha|T}\right)\times$$

$$\left[\frac{|\alpha|(3b-4)}{4b}\tanh^2\left(\sqrt{|\alpha|T}\right) + \frac{b(4-3b)L^2}{4(b-2)^2} - M^2 + \frac{MLb}{(b-2)}\right] + \frac{3}{4}\beta_0^2. \tag{42}$$

From Eq. (17), the non-vanishing component F_{12} of the electromagnetic field tensor is obtained as

$$F_{12} = \sqrt{\frac{\bar{\mu}MLb}{4\pi(2-b)}}\; G_0\; \exp\left\{\left(\frac{LB}{b-2} + 2M\right)\frac{X}{2}\right\}\cosh\left(\sqrt{|\alpha|}\,T\right). \tag{43}$$

From the above equation, it is observed that the electromagnetic field tensor increases with time.

The reality conditions (Ellis [47])

$$(i)\ \rho + p > 0, \quad (ii)\ \rho + 3p > 0$$

lead to

$$\frac{(4b-3)|\alpha|}{4b}\tanh^2\left(\sqrt{|\alpha|}\,T\right) > |\alpha| + 2M^2 - \frac{2MLb}{(b-2)} + \frac{L^2 b(b-4)}{2(b-2)^2}\,, \tag{44}$$

and

$$\exp\left(\frac{2LX}{2-b}\right)\left[\frac{(6b-1)|\alpha|}{4b}\tanh^2\left(\sqrt{|\alpha|}\,T\right) - 3|\alpha| + \frac{4bL^2}{(b-2)^2} - 4M^2\right.$$

$$\left. + \frac{4MLb}{(b-2)}\right] > \frac{3}{2}\beta_0^2\cosh^{\frac{b-2}{b}}\left(\sqrt{|\alpha|}\,T\right). \tag{45}$$

The dominant energy conditions (Hawking and Ellis [48])

$$(i)\ \rho - p \geq 0, \quad (ii)\ \rho + p \geq 0$$

lead to

$$\exp\left(\frac{2LX}{2-b}\right)\left[\frac{(2b-5)|\alpha|}{4b}\tanh^2\left(\sqrt{|\alpha|}\,T\right) + |\alpha|\right]$$

$$+\frac{3}{2}\beta_0^2 E_0^2\cosh^{\frac{b-2}{2}}\left(\sqrt{|\alpha|}\,T\right) \geq \exp\left(\frac{2LX}{2-b}\right)\frac{L^2 b^2}{(b-2)^2}\,, \tag{46}$$

and

$$\frac{(4b-3)|\alpha|}{4b}\tanh^2\left(\sqrt{|\alpha|}\,T\right) \geq |\alpha| + 2M^2 - \frac{2MLb}{(b-2)} + \frac{L^2 b(b-4)}{2(b-2)^2}\,. \tag{47}$$

The conditions (45) and (46) impose the restriction on β_0.

4.2. β is a function of t

In this case, to find the explicit value of displacement field $\beta(t)$, we assume that the fluid obeys an equation of state of the form

$$p = \gamma\rho\,, \tag{48}$$

where γ ($0 \leq \gamma \leq 1$) is a constant. Here we consider three cases of physical interest.

4.2.1. **Empty universe**

Let us consider $\gamma = 0$. In this case $p = \rho = 0$. Thus, from Eqs. (11) and (14), we obtain

$$\beta^2(t) = \frac{2}{3E^2}\exp\left(\frac{2LX}{2-b}\right)\cosh^{\frac{2-b}{b}}\left(\sqrt{|\alpha|}\,T\right)\times$$
$$\left[\frac{L^2b^2}{(b-2)^2} - \frac{|\alpha|}{2}\tanh^2\left(\sqrt{|\alpha|}\,T\right) - |\alpha|\right]. \tag{49}$$

Halford [6] has pointed out that the constant vector displacement field ϕ_i in Lyra's geometry plays the role of cosmological constant Λ in the normal general relativistic treatment. From Eq. (49), it is observed that the displacement vector $\beta(t)$ is a decreasing function of time.

4.2.2. **Zel'dovich universe**

Let us consider $\gamma = 1$. In this case

$$\rho = p\,. \tag{50}$$

Using Eqs. (37), (38) and (39) in Eqs. (11) and (14), we obtain

$$\beta^2(t) = \frac{2}{3E^2}\exp\left(\frac{2LX}{2-b}\right)\cosh^{\frac{2-b}{b}}\left(\sqrt{|\alpha|}\,T\right)\times$$
$$\left[\frac{L^2b^2}{(b-2)^2} - \frac{|\alpha|}{2}\tanh^2\left(\sqrt{|\alpha|}\,T\right) - |\alpha|\right]. \tag{51}$$

From Eq. (51), it is observed that displacement vector β is a decreasing function of time. The expressions for the pressure p and energy density ρ are given by

$$8\pi p = 8\pi\rho = \frac{1}{E^2}\exp\left(\frac{2LX}{2-b}\right)\cosh^{\frac{2-b}{b}}\left(\sqrt{|\alpha|}\,T\right)\times$$
$$\left[\frac{(2b+1)|\alpha|}{b}\tanh^2\left(\sqrt{|\alpha|}\,T\right) + \frac{b(4-b)L^2}{4(b-2)^2} + \frac{MLb}{(b-2)} - M^2 - \frac{3|\alpha|}{2}\right]. \tag{52}$$

The reality conditions (Ellis 1973)

$$(i)\ \rho + p > 0, \quad (ii)\ \rho + 3p > 0$$

lead to

$$\frac{(2b+1)|\alpha|}{b} \tanh^2\left(\sqrt{|\alpha|}\,T\right) > \frac{b(4-b)L^2}{4(b-2)^2} - \frac{MLb}{(b-2)} + M^2 + \frac{3|\alpha|}{2}\,. \tag{53}$$

4.2.3. Radiating universe

Let us consider $\gamma = \frac{1}{3}$. In this case,

$$\rho = 3p\,. \tag{54}$$

In this case, using Eqs. (37) - (39) in Eqs. (11) - (14), the expressions for $\beta(t)$, p and ρ are obtained as

$$\beta^2(t) = \frac{2}{3E^2}\exp\left(\frac{2LX}{2-b}\right)\cosh^{\frac{2-b}{b}}\left(\sqrt{|\alpha|}\,T\right)\times$$

$$\left[\frac{L^2b^2}{2(b-2)^2} + 2M^2 - \frac{(b+4)|\alpha|}{2}\tanh^2\left(\sqrt{|\alpha|}\,T\right) - \frac{(b-2)|\alpha|}{2b}\right], \tag{55}$$

$$8\pi p = \frac{1}{E^2}\exp\left(\frac{2LX}{2-b}\right)\cosh^{\frac{2-b}{b}}\left(\sqrt{|\alpha|}\,T\right)\left[\frac{(2b+5)|\alpha|}{4b}\tanh^2\sqrt{|\alpha|}\,T\right.$$

$$\left. + \frac{bL^2}{(b-2)} + \frac{MLb}{4(b-2)} + \frac{(b-2)|\alpha|}{2b} - 2M^2 - |\alpha|\right], \tag{56}$$

$$8\pi\rho = \frac{3}{E^2}\exp\left(\frac{2LX}{2-b}\right)\cosh^{\frac{2-b}{b}}\left(\sqrt{|\alpha|}\,T\right)\left[\frac{(2b+5)|\alpha|}{4b}\tanh^2\sqrt{|\alpha|}\,T\right.$$

$$\left. + \frac{bL^2}{(b-2)} + \frac{MLb}{4(b-2)} + \frac{(b-2)|\alpha|}{2b} - 2M^2 - |\alpha|\right]. \tag{57}$$

From Eq. (55), it is observed that displacement vector β is a decreasing function of time. The reality conditions (Ellis [47])

$$(i)\ \rho + p > 0, \quad (ii)\ \rho + 3p > 0,$$

and the dominant energy conditions (Hawking and Ellis [48])

$$(i)\ \rho - p \geq 0, \quad (ii)\ \rho + p \geq 0$$

lead to

$$\frac{(2b+5)|\alpha|}{4b} \tanh^2 \sqrt{|\alpha|}\, T > \frac{bL^2}{(2-b)} + \frac{MLb}{(2-b)} + \frac{(2-b)|\alpha|}{2b} + 2M^2 + |\alpha|$$

and

$$\frac{(2b+5)|\alpha|}{4b} \tanh^2 \sqrt{|\alpha|}\, T \geq \frac{bL^2}{(2-b)} + \frac{MLb}{(2-b)} + \frac{(2-b)|\alpha|}{2b} + 2M^2 + |\alpha|\,, \tag{58}$$

respectively.

The expressions for the expansion θ, shear scalar σ^2, deceleration parameter q and proper volume V for the model (40) are given by

$$\theta = \frac{(3b-2)\sqrt{|\alpha|}}{2bE} \exp\left(\frac{LX}{2-b}\right) \cosh^{\frac{2-b}{2b}}\left(\sqrt{|\alpha|}\, T\right) \tanh^2(\sqrt{|\alpha|}\, T)\,, \tag{59}$$

$$\sigma^2 = \frac{|\alpha|}{3b^2E^2} \exp\left(\frac{2LX}{2-b}\right) \cosh^{\frac{2-b}{b}}\left(\sqrt{|\alpha|}\, T\right) \tanh^2(\sqrt{|\alpha|}\, T)\,, \tag{60}$$

$$q = -\frac{8b(b-1)E^2\left(1 + \dfrac{b-2}{b}\tanh^2(\sqrt{|\alpha|}\, T)\right)}{9(3b-2)^2 \exp\left(\dfrac{2LX}{2-b}\right) \cosh^2\left(\sqrt{|\alpha|}\, T\right) \tanh^4\left(\sqrt{|\alpha|}\, T\right)}\,, \tag{61}$$

$$V = \sqrt{-g} = E^2 \exp\left(\frac{(b+2)LX}{(b-2)}\right) \cosh^{\frac{2(b-1)}{b}}\left(\sqrt{|\alpha|}\, T\right). \tag{62}$$

From Eqs. (59) and (60), we obtain

$$\frac{\sigma^2}{\theta^2} = \frac{4}{3(3b-2)^2} = \text{constant}\,. \tag{63}$$

The Hubble parameter H is given by

$$H = \frac{3(3b-2)\sqrt{|\alpha|}}{2bE} \exp\left(\frac{LX}{2-b}\right) \cosh^{\frac{2-b}{2b}}\left(\sqrt{|\alpha|}\, T\right) \tanh^2(\sqrt{|\alpha|}\, T)\,. \tag{64}$$

The sign of q indicates whether the model inflates or not. A positive sign of q corresponds to a standard decelerating model, whereas the negative sign $-1 \leq q < 0$, indicates inflation. Recent observations show that the deceleration parameter of the universe is in the range $-1 \le q < 0$ and the present universe is undergoing an accelerated expansion [49, 50]. This behaviour is clearly shown in Fig. 1, as a representative case with appropriate choice of constants and other physical parameters, using reasonably well known situation. Also, the current observation of SNe Ia and CMBR favour an accerating model ($q < 0$). From Eq. (61), it can be seen that the deceleration parameter $q < 0$ when

$$T < \frac{1}{\sqrt{|\alpha|}} \tanh^{-1} \sqrt{\frac{b}{(2-b)}}\,.$$

It follows that our models of the universe are consistent with recent observations. Generally, model (40) represents expanding, shearing and anisotropic universe in which the flow vector is geodesic.

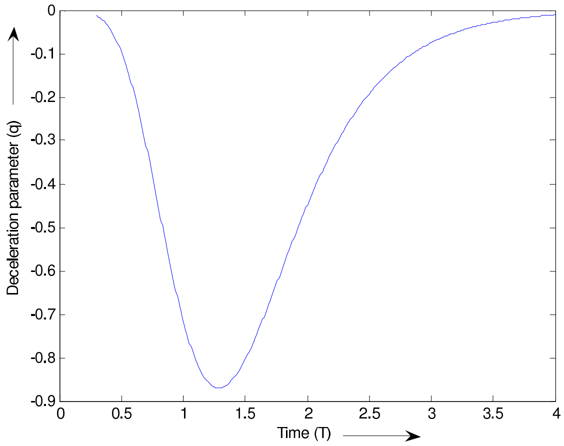

Fig. 1. Plot of the deceleration parameter q vs. time T.

4.3. Thermodynamical behaviour and entropy of universe

From the thermodynamics [51, 52], we apply the combination of the first and second law of thermodynamics to the system with volume V. As we know,

$$\c{T} \mathrm{d}S = \mathrm{d}(\rho V) + p \mathrm{d}V\,, \tag{65}$$

where $\c{T}$ and S represent the temperature and entropy, respectively.

Equation (65) may be written as

$$\c{T} \mathrm{d}S = \mathrm{d}[(\rho + p)V] - V \mathrm{d}p\,. \tag{66}$$

The integrability condition is necessary to define a perfect fluid as a thermodynamical system [52] – [55]. It is given by

$$\mathrm{d}p = \frac{\rho + P}{\text{Ţ}} \mathrm{d}\text{Ţ} \,. \tag{67}$$

Plugging Eq. (67) into Eq. (66), we have the differential equation

$$\mathrm{d}S = \frac{1}{\text{Ţ}} \mathrm{d}[(\rho + p)V] - (\rho + p)V \frac{\mathrm{d}\text{Ţ}}{\text{Ţ}^2} \,. \tag{68}$$

We rewrite Eq. (68) as

$$\mathrm{d}S = \mathrm{d}\left[\frac{(\rho + p)V}{\text{Ţ}} + c\right], \tag{69}$$

where c is a constant.

Hence the entropy is defined as

$$S = \frac{\rho + p}{\text{Ţ}} V \,. \tag{70}$$

Let the entropy density be s, so that

$$s = \frac{S}{V} = \frac{\rho + p}{\text{Ţ}} = \frac{(1 + \gamma)\rho}{\text{Ţ}} \,, \tag{71}$$

where $p = \gamma\rho$ and $0 < \gamma \leq 1$.

If we define the entropy density in terms of temperature, then the first law of thermodynamics may be written as

$$\mathrm{d}(\rho V) + \gamma\rho \mathrm{d}V = (1 + \rho)\text{Ţ}\mathrm{d}\left(\frac{\rho V}{\text{Ţ}}\right), \tag{72}$$

which on integration yields

$$\text{Ţ} = c_0 \rho^{\frac{\gamma}{(1+\gamma)}} \,, \tag{73}$$

where c_0 is a constant of integration.

From Eqs. (71) and (73), we obtain

$$s = \left(\frac{1 + \gamma}{c_0}\right) \rho^{\frac{1}{1+\rho}} \,. \tag{74}$$

These equations are not valid for $\gamma = -1$. For the Zel'dovich fluid ($\gamma = 1$), we get

$$\text{Ţ} = c_0 \rho^{\frac{1}{2}} \,, \tag{75}$$

$$s = \frac{2}{c_0}\rho^{\frac{1}{2}}, \tag{76}$$

$$\Rightarrow s \sim \rho^{\frac{1}{2}} \sim \mathcal{T}\,.$$

Thus the entropy density is proportional to the temperature. We have

$$\mathcal{T} = T_0 \exp\left(\frac{LX}{2-b}\right)\cosh^{\frac{2-b}{2b}}\left(\sqrt{|\alpha|}\,T\right)\times$$

$$\left[\frac{(2b+1)|\alpha|}{b}\tanh^2\left(\sqrt{|\alpha|}\,T\right) + \frac{b(4-b)L^2}{4(b-2)^2} + \frac{MLb}{(b-2)} - M^2 - \frac{3|\alpha|}{2}\right]^{\frac{1}{2}}, \tag{77}$$

$$s = s_0 \exp\left(\frac{LX}{2-b}\right)\cosh^{\frac{2-b}{2b}}\left(\sqrt{|\alpha|}\,T\right)\times$$

$$\left[\frac{(2b+1)|\alpha|}{b}\tanh^2\left(\sqrt{|\alpha|}\,T\right) + \frac{b(4-b)L^2}{4(b-2)^2} + \frac{MLb}{(b-2)} - M^2 - \frac{3|\alpha|}{2}\right]^{\frac{1}{2}}, \tag{78}$$

$$S = S_0 \exp\left(\frac{(b+1)LX}{2-b}\right)\cosh^{\frac{3b-2}{2b}}\left(\sqrt{|\alpha|}\,T\right)\times$$

$$\left[\frac{(2b+1)|\alpha|}{b}\tanh^2\left(\sqrt{|\alpha|}\,T\right) + \frac{b(4-b)L^2}{4(b-2)^2} + \frac{MLb}{(b-2)} - M^2 - \frac{3|\alpha|}{2}\right]^{\frac{1}{2}}, \tag{79}$$

where $T_0 = \dfrac{c_0}{E}$, $s_0 = \dfrac{2}{c_0 E}$ and $S_0 = \dfrac{2E}{c_0}$ are constant.
For a radiating fluid ($\gamma = \frac{1}{3}$), we get

$$\mathcal{T} \sim \rho^{\frac{1}{4}}, \tag{80}$$

$$s \sim \rho^{\frac{3}{4}} \sim \mathcal{T}^3. \tag{81}$$

Thus the entropy density is proportional to the cube of the temperature.

Now, the temperature, entropy density and entropy of radiating universe are given by

$$\mathcal{T} = T_{00} \exp\left(\frac{LX}{4-2b}\right)\cosh^{\frac{2-b}{4b}}\left(\sqrt{|\alpha|}\,T\right)\times$$

$$\left[\frac{(2b+1)|\alpha|}{b}\tanh^2\left(\sqrt{|\alpha|}\,T\right)+\frac{b(4-b)L^2}{4(b-2)^2}+\frac{MLb}{(b-2)}-M^2-\frac{3|\alpha|}{2}\right]^{\frac{1}{4}}, \tag{82}$$

$$s = s_{00}\exp\left(\frac{3LX}{4-2b}\right)\cosh^{\frac{6-b}{4b}}\left(\sqrt{|\alpha|}\,T\right)\times$$

$$\left[\frac{(2b+1)|\alpha|}{b}\tanh^2\left(\sqrt{|\alpha|}\,T\right)+\frac{b(4-b)L^2}{4(b-2)^2}+\frac{MLb}{(b-2)}-M^2-\frac{3|\alpha|}{2}\right]^{\frac{3}{4}}, \tag{83}$$

$$S = S_{00}\exp\left(\frac{(2b+1)LX}{2(b-2)}\right)\cosh^{\frac{7b+2}{4b}}\left(\sqrt{|\alpha|}\,T\right)\times$$

$$\left[\frac{(2b+1)|\alpha|}{b}\tanh^2\left(\sqrt{|\alpha|}\,T\right)+\frac{b(4-b)L^2}{4(b-2)^2}+\frac{MLb}{(b-2)}-M^2-\frac{3|\alpha|}{2}\right]^{\frac{3}{4}}, \tag{84}$$

where $T_{00} = \dfrac{c_0}{\sqrt{E}}$, $s_{00} = \dfrac{2}{c_0 E\sqrt{E}}$ and $S_{00} = \dfrac{2\sqrt{E}}{c_0}$ are constant.

5. *Case (iii): $ab > 0$*

In this case we obtain

$$f = n\exp\left(\frac{Lx}{(b-2)}\right), \tag{85}$$

$$\mu = \beta\cos^{\frac{1}{2}}(\sqrt{\alpha}\,t + t_0)\,, \tag{86}$$

$$\nu = r\cos^{\frac{(b-2)}{2b}}(\sqrt{\alpha}\,t + t_0)\,, \tag{87}$$

$$g = N\exp\left(\frac{Lbx}{2(b-2)} + Mx\right), \tag{88}$$

$$h = \frac{1}{N}\exp\left(\frac{Lbx}{2(b-2)} - Mx\right). \tag{89}$$

Therefore, we have

$$A = E\exp\left(\frac{Lx}{(b-2)}\right)\cos^{\frac{(b-2)}{2b}}(\sqrt{\alpha}\,t + t_0)\,, \tag{90}$$

$$B = G \exp\left(\frac{Lbx}{2(b-2)} + Mx\right) \cos^{\frac{1}{2}}(\sqrt{\alpha}\, t + t_0)\,, \tag{91}$$

$$C = H \exp\left(\frac{Lbx}{2(b-2)} - Mx\right) \cos^{\frac{1}{2}}(\sqrt{\alpha}\, t + t_0)\,. \tag{92}$$

Here E, G and H are defined in Section 4.

After using a suitable transformation of coordinates, the metric (1) reduces to the form

$$\mathrm{d}s^2 = E^2 \exp\left(\frac{2LX}{(b-2)}\right) \cos^{\frac{(b-2)}{b}}(\sqrt{\alpha}\, T)(\mathrm{d}X^2 - \mathrm{d}T^2) +$$

$$\exp\left(\frac{LbX}{(b-2)} + 2MX\right) \cos(\sqrt{\alpha}\, T)\mathrm{d}Y^2 +$$

$$\exp\left(\frac{LbX}{(b-2)} - 2MX\right) \cos(\sqrt{\alpha}\, T)\mathrm{d}Z^2. \tag{93}$$

For the specification of displacement vector $\beta(t)$ within the framework of Lyra geometry and for realistic models of physical importance, we consider the following two cases by taking β as constant and also β as function of time.

5.1. β is a constant, $\beta = \beta_0$

Using Eqs. (90), (91) and (92) in Eqs. (11) and (14), the expressions for pressure p and density ρ for the model (93) are given by

$$8\pi p = \frac{1}{E^2} \exp\left(\frac{2LX}{2-b}\right) \cos^{\frac{2-b}{b}}(\sqrt{\alpha}\, T) \times$$

$$\left[\alpha \left\{\frac{(3b-4)}{4b} \tan^2\left(\sqrt{\alpha}\, T\right) + 1\right\} + \frac{b(b+4)L^2}{4(b-2)^2} - M^2 + \frac{MLb}{b-2}\right] - \frac{3}{4}\beta_0^2, \tag{94}$$

$$8\pi \rho = \frac{1}{E^2} \exp\left(\frac{2LX}{2-b}\right) \cos^{\frac{2-b}{b}}(\sqrt{\alpha}\, T) \times$$

$$\left[\frac{\alpha(3b-4)}{4b} \tan^2\left(\sqrt{\alpha}\, T\right) + \frac{b(4-3b)L^2}{4(b-2)^2} - M^2 + \frac{MLb}{b-2}\right] + \frac{3}{4}\beta_0^2. \tag{95}$$

From Eq. (17), the non-vanishing component F_{12} of the electromagnetic field tensor is obtained as

$$F_{12} = \sqrt{\frac{\bar{\mu}}{8\pi} \frac{2ML}{(2-b)}}\; G\; \exp\left\{\left(\frac{Lb}{b-2} + 2M\right) \frac{X}{2}\right\} \cos(\sqrt{\alpha}\, T)\,. \tag{96}$$

From the above equation, it is observed that the electromagnetic field tensor increases with time.

The reality conditions (Ellis [47])

$$(i)\ \rho + p > 0, \quad (ii)\ \rho + 3p > 0$$

lead to

$$\frac{\alpha(3b-4)}{2b}\tan^2(\sqrt{\alpha}\,T) > \frac{b(b-4)L^2}{2(b-2)^2} + \frac{2MLb}{2-b} + 2M^2 - \alpha\,, \tag{97}$$

and

$$\exp\left(\frac{2LX}{2-b}\right)\left[\frac{\alpha(3b-4)}{b}\tan^2(\sqrt{\alpha}\,T) + 3\alpha + \frac{2bL^2}{(b-2)^2} - 4M^2 + \frac{4MLb}{(b-2)}\right]$$

$$> \frac{3}{2}\beta_0^2 E^2 \cos^{b-2} b(\sqrt{\alpha}\,T)\,. \tag{98}$$

The dominant energy conditions (Hawking and Ellis [48])

$$(i)\ \rho - p \geq 0, \quad (ii)\ \rho + p \geq 0$$

lead to

$$\frac{3}{2}\beta_0^2 E^2 \cos^{\frac{b-2}{b}}(\sqrt{\alpha}\,T) \geq \exp\left(\frac{2LX}{2-b}\right)\left[\frac{b^2L^2}{(b-2)^2} + \alpha\right] \tag{99}$$

and

$$\frac{\alpha(3b-4)}{2b}\tan^2(\sqrt{\alpha}\,T) \geq \frac{b(b-4)L^2}{2(b-2)^2} + \frac{2MLb}{2-b} + 2M^2 - \alpha\,. \tag{100}$$

The conditions (98) and (99) impose the restriction on β_0.

5.2. *β is a function of t*

In this case, to find the explicit value of the displacement field $\beta(t)$, we assume that the fluid obeys an equation of state given by Eqs. (48). Here we consider three cases of physical interest.

5.2.1. **Empty universe**

Let us consider $\gamma = 0$ in (48). In this case $p = \rho = 0$. Thus, from Eqs. (11) and (14) we obtain

$$\beta^2(t) = \frac{2}{3E^2}\exp\left(\frac{2LX}{2-b}\right)\cos^{\frac{2-b}{b}}(\sqrt{\alpha}\,T)\left[M^2 + \frac{L^2b^2}{(b-2)^2} + \alpha\right]. \tag{101}$$

Halford [6] has pointed out that the constant vector displacement field ϕ_i in Lyra's geometry plays the role of cosmological constant Λ in the normal general relativistic treatment. From Eq. (101), it is observed that the displacement vector $\beta(t)$ is a decreasing function of time.

5.2.2. **Zel'dovich Universe**

Let us consider $\gamma = 1$. Hence, Eq. (48) gives $\rho = p$.

Using Eqs. (90), (91) and (92) in Eqs. (11) and (14), we obtain

$$\beta^2(t) = \frac{2}{3E^2} \exp\left(\frac{2LX}{2-b}\right) \cos^{\frac{2-b}{b}}(\sqrt{\alpha}\,T) \left[M^2 + \frac{L^2 b^2}{(b-2)^2} + \alpha\right]. \tag{102}$$

From Eq. (102), it is observed that the displacement vector β is a decreasing function of time. The expression for pressure p and energy density ρ is given by

$$8\pi p = 8\pi\rho = \frac{1}{E^2} \exp\left(\frac{2LX}{2-b}\right) \cos^{\frac{2-b}{b}}(\sqrt{\alpha}\,T) \left[\frac{(3b-4)\alpha}{4b} \tan^2\left(\sqrt{\alpha}\,T\right) + \frac{\alpha}{2} + \frac{b(4-b)L^2}{4(b-2)^2} - \frac{3}{2}M^2 + \frac{MLb}{b-2}\right]. \tag{103}$$

The reality conditions (Ellis [47])

$$(i)\ \rho + p > 0, \quad (ii)\ \rho + 3p > 0$$

lead to

$$\frac{(3b-4)\alpha}{4b} \tan^2\left(\sqrt{\alpha}\,T\right) > \frac{b(b-4)L^2}{4(b-2)^2} + \frac{3}{2}M^2 + \frac{MLb}{2-b} - \frac{\alpha}{2}\,. \tag{104}$$

5.2.3. **Radiating universe**

Let us consider $\gamma = \frac{1}{3}$. Hence Eq. (48) reduces to $\rho = 3p$.

In this case, using Eqs. (90) - (92) in Eqs. (11) - (14), the expressions for $\beta(t)$, p and ρ are obtained as

$$\beta^2(t) = \frac{2}{3E^2} \exp\left(\frac{2LX}{2-b}\right) \cos^{\frac{2-b}{b}}(\sqrt{\alpha}\,T) \left[\frac{\alpha(3b-2)}{2b} + \frac{\alpha(5b-4)}{4b} \tan^2\left(\sqrt{\alpha}\,T\right) + \frac{3b^2L^2}{4(b-2)^2} + M^2\right], \tag{105}$$

$$8\pi p = \frac{1}{E^2} \exp\left(\frac{2LX}{2-b}\right) \cos^{\frac{2-b}{b}}(\sqrt{\alpha}T) \left[\frac{MLb}{b-2} + \right.$$

$$\left. \frac{bL^2(4-3b)}{4(b-2)^2} - \frac{\alpha}{2b} - 2M^2 - \frac{1}{2}\alpha \tan^2\left(\sqrt{\alpha}T\right)\right], \tag{106}$$

$$8\pi\rho = \frac{3}{E^2} \exp\left(\frac{2LX}{2-b}\right) \cos^{\frac{2-b}{b}}(\sqrt{\alpha}T) \left[\frac{MLb}{b-2} + \right.$$

$$\left. \frac{bL^2(4-3b)}{4(b-2)^2} - \frac{\alpha}{2b} - 2M^2 - \frac{1}{2}\alpha \tan^2\left(\sqrt{\alpha}T\right)\right]. \tag{107}$$

From Eq. (105), it is observed that the displacement vector β is a decreasing function of time. The reality conditions (Ellis [47])

$$(i)\ \rho + p > 0, \quad (ii)\ \rho + 3p > 0\,,$$

and the dominant energy conditions (Hawking and [48])

$$(i)\ \rho - p \geq 0, \quad (ii)\ \rho + p \geq 0$$

lead to

$$\frac{\alpha}{2} \tan^2\left(\sqrt{\alpha}T\right) > \frac{MLb}{b-2} + \frac{bL^2(4-3b)}{4(b-2)^2} - \frac{\alpha}{2b} - 2M^2\,,$$

and

$$\frac{\alpha}{2} \tan^2\left(\sqrt{\alpha}T\right) \geq \frac{MLb}{b-2} + \frac{bL^2(4-3b)}{4(b-2)^2} - \frac{\alpha}{2b} - 2M^2, \tag{108}$$

respectively.

The expressions for the expansion θ, shear scalar σ^2, deceleration parameter q and proper volume V for the model (103) are given by

$$\theta = \frac{(2-3b)\sqrt{\alpha}}{2bE} \exp\left(\frac{LX}{2-b}\right) \cos^{\frac{2-b}{2b}}\left(\sqrt{\alpha}T\right) \tan\left(\sqrt{\alpha}T\right), \tag{109}$$

where $c_2 = c_1^{\frac{(2-b)}{2b}}$,

$$\sigma^2 = \frac{\alpha}{3b^2E^2} \exp\left(\frac{2LX}{2-b}\right) \cos^{\frac{2-b}{b}}\left(\sqrt{\alpha}T\right) \tan^2\left(\sqrt{\alpha}T\right), \tag{110}$$

$$q = -\frac{8b(b-1)E^2\left(1 - \frac{b-2}{b}\tan^2(\sqrt{|\alpha|}\,T)\right)}{9(3b-2)^2\exp\left(\frac{2LX}{2-b}\right)\cos^2\left(\sqrt{|\alpha|}\,T\right)\tan^4\left(\sqrt{|\alpha|}\,T\right)}, \tag{111}$$

$$V = \sqrt{-g} = E^2\exp\left(\frac{(b+2)LX}{b-2}\right)\cos^{\frac{2(b-1)}{b}}\left(\sqrt{\alpha}\,T\right). \tag{112}$$

From Eqs. (109) and (110), we obtain

$$\frac{\sigma^2}{\theta^2} = \frac{4}{3(3b-2)^2} = \text{constant}. \tag{113}$$

The rotation ω is identically zero.

The Hubble parameter H is given by

$$H = \frac{3(2-3b)\sqrt{\alpha}}{2bE}\exp\left(\frac{LX}{2-b}\right)\cos^{\frac{2-b}{2b}}\left(\sqrt{\alpha}\,T\right)\tan\left(\sqrt{\alpha}\,T\right). \tag{114}$$

For $\dfrac{2}{b} > 3$, the model (93) starts expanding at $T = 0$ and attains its maximum value at $T = \dfrac{\pi}{4\sqrt{\alpha}}$. After that θ decreases to attain its minimum negative value at $T = \dfrac{3\pi}{4\sqrt{\alpha}}$. Thus, the model oscillates with the period $\dfrac{\pi}{2\sqrt{\alpha}}$. Since $\dfrac{\sigma}{\theta} =$ constant, the model does not approach isotropy. The sign of q indicates whether the model inflates or not. A positive sign of q corresponds to the standard decelerating model, while the negative sign $-1 \leq q < 0$ indicates inflation. Recent observations show that the deceleration parameter of the universe is in the range $-1 \leq q < 0$ and the present universe is undergoing an accelerated expansion [49, 50]. This behaviour is clearly shown in Fig. 2. Also, the current observation of SNe Ia and CMBR favour

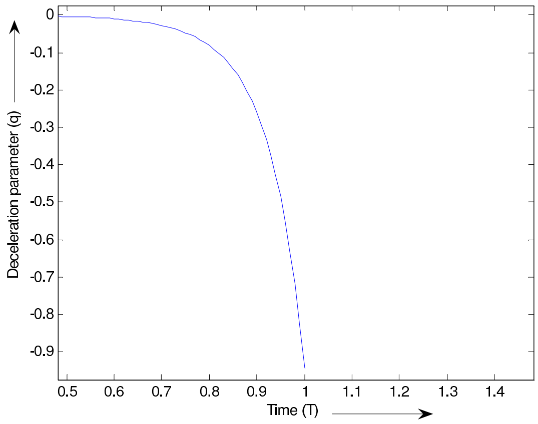

Fig. 2. Plot of the deceleration parameter q vs. time T.

an accerating model ($q < 0$). From Eq. (111) it can be seen that the deceleration parameter $q < 0$ when

$$T < \frac{1}{\sqrt{\alpha}} \tan^{-1}\left(\sqrt{\frac{b}{(b-2)}}\right).$$

It follows that our models of the universe are consistent with recent observations.

5.3. *Thermodynamical behaviour and entropy of the universe*

From Eqs. (75), (76) and (71), the expression for temperature, entropy density and entropy for Zel'dovich fluid ($\gamma = 1$) is given by

$$\c{T} = T_0 \exp\left(\frac{LX}{2-b}\right) \cos^{\frac{2-b}{2b}}\left(\sqrt{|\alpha|}\,T\right)\left[\frac{(3b-4)\alpha}{4b}\tan^2\left(\sqrt{\alpha}\,T\right) + \frac{\alpha}{2}\right.$$
$$\left. + \frac{b(4-b)L^2}{4(b-2)^2} - \frac{3}{2}M^2 + \frac{MLb}{b-2}\right]^{\frac{1}{2}}, \tag{115}$$

$$s = s_0 \exp\left(\frac{LX}{2-b}\right) \cos^{\frac{2-b}{2b}}\left(\sqrt{|\alpha|}\,T\right)\left[\frac{(3b-4)\alpha}{4b}\tan^2\left(\sqrt{\alpha}\,T\right) + \frac{\alpha}{2}\right.$$
$$\left. + \frac{b(4-b)L^2}{4(b-2)^2} - \frac{3}{2}M^2 + \frac{MLb}{b-2}\right]^{\frac{1}{2}}, \tag{116}$$

$$S = S_0 \exp\left(\frac{(b+1)LX}{2-b}\right) \cos^{\frac{3b-2}{2b}}\left(\sqrt{|\alpha|}\,T\right)\left[\frac{(3b-4)\alpha}{4b}\tan^2\left(\sqrt{\alpha}\,T\right) + \frac{\alpha}{2}\right.$$
$$\left. + \frac{b(4-b)L^2}{4(b-2)^2} - \frac{3}{2}M^2 + \frac{MLb}{b-2}\right]^{\frac{1}{2}}, \tag{117}$$

where T_0, s_0 and S_0 are defined in Section 4.3.

From Eqs. (80), (81) and (71), the expression for the temperature, entropy density and entropy for a radiating fluid ($\gamma = \frac{1}{3}$) is given by

$$\c{T} = T_{00} \exp\left(\frac{LX}{4-2b}\right) \cos^{\frac{2-b}{4b}}\left(\sqrt{|\alpha|}\,T\right)\left[\frac{(3b-4)\alpha}{4b}\tan^2\left(\sqrt{\alpha}\,T\right) + \frac{\alpha}{2}\right.$$

$$+\frac{b(4-b)L^2}{4(b-2)^2}-\frac{3}{2}M^2+\frac{MLb}{b-2}\Bigg]^{\frac{1}{4}}, \tag{118}$$

$$s = s_{00}\exp\left(\frac{3LX}{4-2b}\right)\cos^{\frac{6-b}{4b}}\left(\sqrt{|\alpha|}\,T\right)\left[\frac{(3b-4)\alpha}{4b}\tan^2\left(\sqrt{\alpha}\,T\right)+\frac{\alpha}{2}\right.$$

$$\left.+\frac{b(4-b)L^2}{4(b-2)^2}-\frac{3}{2}M^2+\frac{MLb}{b-2}\right]^{\frac{3}{4}}, \tag{119}$$

$$S = S_{00}\exp\left(\frac{(2b+1)LX}{2(b-2)}\right)\cos^{\frac{7b+2}{4b}}\left(\sqrt{|\alpha|}\,T\right)\left[\frac{(3b-4)\alpha}{4b}\tan^2\left(\sqrt{\alpha}\,T\right)+\frac{\alpha}{2}\right.$$

$$\left.+\frac{b(4-b)L^2}{4(b-2)^2}-\frac{3}{2}M^2+\frac{MLb}{b-2}\right]^{\frac{3}{4}}, \tag{120}$$

where T_{00}, s_{00} and S_{00} are defined in Section 4.3.

6. *Case (ii): $ab = 0$*

In this case, we obtain

$$f = n\exp\left(\frac{Lx}{(b-2)}\right), \tag{121}$$

$$\mu = (c_1 t + t_0)^{\frac{1}{2}}, \tag{122}$$

$$\nu = \left(\frac{1}{m}\right)^{\frac{(b-2)}{b}}(c_1 t + t_0)^{\frac{(b-2)}{2b}}, \tag{123}$$

$$g = N\exp\left(\frac{Lbx}{2(b-2)}+Mx\right), \tag{124}$$

$$h = \frac{1}{N}\exp\left(\frac{Lbx}{2(b-2)}-Mx\right). \tag{125}$$

Therefore, we have

$$A = E_0\exp\left(\frac{Lx}{(b-2)}\right)(c_1 t + t_0)^{\frac{(b-2)}{2b}}, \tag{126}$$

$$B = N \exp\left(\frac{Lbx}{2(b-2)} + Mx\right)(c_1 t + t_0)^{\frac{1}{2}}, \tag{127}$$

$$C = H_0 \exp\left(\frac{Lbx}{2(b-2)} - Mx\right)(c_1 t + t_0)^{\frac{1}{2}}, \tag{128}$$

where $E_0 = n\,m^{\frac{(2-b)}{b}}$, $H_0 = \dfrac{1}{N}$.

After using suitable transformation of coordinates, the metric (1) reduces to the form

$$\mathrm{d}s^2 = E_0^2 \exp\left(\frac{2LX}{(b-2)}\right)(c_1 T)^{\frac{(b-2)}{b}}(\mathrm{d}X^2 - \mathrm{d}T^2) +$$

$$\exp\left(\frac{LbX}{(b-2)} + 2MX\right)(c_1 T)\mathrm{d}Y^2 + \exp\left(\frac{LbX}{(b-2)} - 2MX\right)(c_1 T)\mathrm{d}Z^2. \tag{129}$$

It is observed that the model (129) is the same as obtained by Pradhan and Shyam Sunder [34]. Thus the physical and geometrical properties of the model are similar to the model obtained by Pradhan and Shyam Sundar [34].

7. *Discussion and concluding remarks*

In the present study, I investigate the plane-symmetric inhomogeneous cosmological models with perfect fluid distribution and electromagnetic field based on Lyra's geometry. The source of the magnetic field is due to an electric current produced along the z-axis. The free gravitational field is assumed to be of Petrov-type II non-degenerate. It is observed that the gauge function $\beta(t)$ is large at the beginning and reduces fast with the evolution of universe for all cases. Also, it is found that $\beta(t)$ decreases as time increases, i.e., $\beta(t)$ is a decreasing function of time and it plays the same role in Lyra's geometry as the cosmological constant $\Lambda(t)$ in general relativity. It means that the displacement vector $\beta(t)$ coincides with the nature of the cosmological constant Λ. The nontrivial role of vacuum in the early universe generates Λ-term that lead to inflationary phase. Therefore, the study of cosmological models in Lyra's geometry may be relavant to inflationary models. There seems a good possibility of Lyra's geometry to provide a theorectical foundation of relativistic gravitation and cosmology. However, the astrophysical bodies are still an open question. In fact, it needs a fair trail by experiment. It is seen that solution obtained by Pradhan, Yadav and Singh [34] are a particular case of my solution. Generally, these models represent expanding, shearing and Petrov type-II non-degenerate universe in which flow vector is geodesic. Also, $\dfrac{\sigma}{\theta}\neq 0$, thus the models do not approach isotropy. The value of deceleration parameter q is found to be negative ($-1 < q < 0$), implying that our universe is accelerating, which is supported by SNe Ia and CMBR observations. This behaviour is clearly depicted in Figs. 1 and 2. From Fig. 1, we note that at the early stage of universe, the deceleration parameter q oscillates between $q < 0$ and $q > -1$ and afterwards it is uniform (negative) for ever. This has physical meaning.

The idea of premordinal magnetism is appealing because it can potentially explain all large-scale fields seen in the universe today, especially those found in remote proto-galaxies. As a result, the literature contains many studies that examine the role and implications of magnetic field in cosmology. Maarteens [56] in his study explained that magnetic fields are observed not only in stars but also in galaxies. In principle, these fields could play a significant role in the structure formation, but also affect the anisotropies in cosmic microwave background radiation [CMB]. Since the electric and magnetic fields are interrelated, their independent nature disappears when we consider them depending on time. Hence, it would be proper to look on these fields as a single field - the electromagnetic field. It is worth mentioning here that the magnetic field affects all physical and kinematical quantities but it does not affect the rate of expansion. Also, we see that in the absence of magnetic field, inhomogeneity of the universe dies out. This signifies the role of magnetic field. The present study also extends the work of Yadav and Bagora [57] within the framework of Lyra's geometry and clarifies the thermodynamics of plane-symmetric universe by introducing the integrability condition and temperature. A new general equation of state describing the Zel'dovich fluid and radiating fluid models as functions of temperature and volume is found. The basic equations of thermodynamics for plane-symmetric universe has been deduced which may be useful for better understanding of the evolution of universe.

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LYRAOVA KOZMOLOGIJA NEHOMOGENOG SVEMIRA S ELEKTROMAGNETSKIM POLJEM

Izvode se ravninsko-simetrični nehomogeni kozmološki modeli za raspodjelu perfektne tekućine s elektromagnetskim poljem za Lyraovu geometriju. Radi postizanja određenog rješenja, pretpostavlja se $A = f(x)\nu(t)$, $B = g(x)\mu(t)$ i $C = h(x)\mu(t)$, gdje su A, B i C metrički koeficijenti. Postignuta rješenja su poopćenja ranijih rješenja Pradhana, Yadava i Singha (2007) i u suglasju su s novim opažanjima supernova Ia. Podrobno se proučavaju fizička i kinematička svojstva tih modela.