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FIVE-DIMENSIONAL STRING COSMOLOGICAL MODELS WITH MASSIVE SCALAR FIELD

GANESHWAR MOHANTY[a] and GAURANGA CHARANA SAMANTA[b]

[a]*703, Rameswarpatna, Bhubaneswar-751002, India, and*
P. G. Department of Mathematics, Sambalpur University, Jyoti Vihar-768019, India
E-mail address: ganeshwarm@yahoo.com

[b]*Department of Mathematics, Gandhi Institute for Technological Advancement, Madanpur-752054, India E-mail address: gauranga1981@yahoo.co.in*

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We have constructed five-dimensional plane-symmetric cosmological models in the presence of massive scalar field in Lyra manifold when the source of the gravitational field is a massive string with $\rho = (1 + \omega)\lambda$ (Takabayasi string). The role of massive scalar field in the evolution of the universe has been studied. The physical and kinematical behavior of the models is discussed.

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1. Introduction

The study of higher-dimensional cosmological models is important because they reveal the nature of the universe at early stages of the evolution. These higher-dimensional scenarios are based on Kaluza-Klein theories. In the context of the Kaluza-Klein and super-string theories, higher-dimensional cosmological problems have recently acquired much significance. Kaluza-Klein theories have been used as a way of unifying all gauge interactions with gravity. The experimental detection of the time variation of the fundamental constants could provide strong evidence for the existence of extra dimensions [1 − 3].

It has been a subject of interest of cosmologists to study the nature of scalar field with or without the mass parameter, interacting with a cloud of strings, in order to draw an analogy of the physics of the cosmos with experimental results. Moreover, it has been established in quantum physics that a massive scalar field is associated with zero-spin chargeless particles, like the π and K-mesons, and the study of such fields in general theory of relativity has been extensively worked out to obtain a picture of the space-time and gravitational field associated with neutral elementary particles of zero spin. Mohanty and Pradhan [4] constructed a FRW cosmological model in the presence of massive scalar field in general theory of relativity and obtained a closed elliptic model. Various authors, viz. Rao et al. [5], Uhlir and Dittrich [6], Patel [7], Rao and Singh [8], Singh and Deo [9], Singh [10], Reddy [11], Venkateswarlu and Reddy [12], Shanti and Rao [13], Roy and Maiti [14], Bhattacharjee and Baruah [15], Mohanty and Sahoo [16], Mohanty et al. [17], Mohanty et al. [18, 19], Rahaman et al. [20], Mohanty et al. [21], Mohanty and Sahu [22], constructed different cosmological models with zero-mass scalar field in four-dimensional space-time. Mohanty and Sahoo [23, 24] constructed different cosmological models with zero-mass scalar field in five-dimensional space-time. The massive scalar field in relativistic mechanics yields some significant conclusions regarding both, the singularities involved and Mach's principle. However, only a massive scalar meson field satisfying the Klein-Gordon equation is physically meaningful. Candelas [25] has shown that by introducing a massive scalar field into general relativity, one can avoid the emergence of singularities. Roy and Rao [26] have shown that massive scalar field can not be a source of gravitation in axially-symmetric gravitational field. Reddy and Innaiah [27] showed the non-existence of static spherically-symmetric conformally-flat zero-mass scalar field with perfect fluid distribution. Mohanty and Pattanaik [28] showed non-existence of scalar meson field in Berttotti-Robinson type space-time. Aygun and Tarhan [29] studied massive scalar field with viscous and heat flow and showed the decay of massive scalar field with and without source density in Gödel universe. Bali and Anjali [30] studied a string-cosmological model in general theory of relativity. Subsequently, Bali and Chandnani [31 − 33] and Rahaman et al. [34] constructed various cosmological models in Lyra geometry. So far, the problem of interactions of string with massive scalar fields in Kaluza-Klein space time in Lyra manifold is not found in the literature. Therefore, in this paper, we construct a five-dimensional mesonic-string cosmological model in Lyra manifold. The energy-momentum tensor is assumed to be the simple extension of the usual four-dimensional case. In Section 2, we set up the field equations. In Section 3, explicit solutions are obtained and some physical and kinematical properties are discussed. In Section 4, some concluding remarks are given.

2. *Metric and field equations*

We consider the five-dimensional space-time in the form

$$ds^2 = -\mathrm{d}t^2 + R^2\left(\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2\right) + A^2\mathrm{d}m^2. \tag{1}$$

The fifth co-ordinate is taken to be space-like and the metric coefficients are assumed to be functions of cosmic time only, i.e. $A = A(t)$ and $R = R(t)$.

The Einstein's field equations, based on Lyra manifold as proposed by Sen [35] and Sen and Dunn [36] in normal gauge, may be written as

$$R_{ij} - \frac{1}{2}g_{ij}R + \frac{3}{2}\phi_i\phi_j - \frac{3}{4}g_{ij}\phi_a\phi^a = -\chi T_{ij}\,, \tag{2}$$

where ϕ_j is the displacement vector and other symbols have their usual meanings as in the Riemannian geometry. The displacement vector ϕ_j is taken in the form

$$\phi_j = \Big(\beta(t),\, 0,\, 0,\, 0,\, 0\Big). \tag{3}$$

The energy-momentum tensor for a cloud of string dust with massive scalar fields along the direction of the string is given by

$$T_{ij} = T^s_{ij} + T^\nu_{ij}\,. \tag{4}$$

The energy-momentum tensors corresponding to a massive string and massive scalar field are given by

$$T^s_{ij} = \rho u_i u_j - \lambda w_i w_j\,, \tag{5}$$

and

$$T^\nu_{ij} = V_{,i}V_{,j} - \frac{1}{2}g_{ij}\big(V_{,k}V^{,k} - M^2V^2\big), \tag{6}$$

respectively, where ρ is the proper energy density for a cloud of strings with particles attached to them, λ is the string tension density, w^i represents the direction of the strings satisfying

$$u_i u^i = -w_i w^i = -1\,, \tag{7}$$

and

$$u^i w_i = 0\,, \tag{8}$$

where u^i is the five-velocity vector of the particles which has components $(1,0,0,0,0)$. Without loss of generality, we choose

$$w^i = (0,\, 0,\, 0,\, 0,\, A^{-1}). \tag{9}$$

The scalar meson field satisfies the Klein-Gordon equation

$$g^{ij}V_{;ij} + M^2V = 0\,. \tag{10}$$

The field equation (2) for the metric (1) yields the following system of equations

$$-3\left(\frac{R'}{R}\right)^2 - 3\frac{R'A'}{RA} + \frac{3}{4}\beta^2 = -\chi\left(\rho + \frac{1}{2}(V'^2 - M^2V^2)\right), \tag{11}$$

$$2\frac{R''}{R} + \left(\frac{R'}{R}\right)^2 + 2\frac{R'A'}{RA} + \frac{A''}{A} + \frac{3}{4}\beta^2 = -\frac{\chi}{2}\left(V'^2 + M^2V^2\right), \tag{12}$$

$$3\frac{R''}{R} + 3\left(\frac{R'}{R}\right)^2 + \frac{3}{4}\beta^2 = -\chi(-\lambda + \frac{1}{2}\left(V'^2 + M^2V^2\right)). \tag{13}$$

The Klein-Gordon equation (10) for the metric (1) becomes

$$-V'' - \left(3\frac{R'}{R} + \frac{A'}{A}\right)V' + M^2V = 0\,. \tag{14}$$

3. *Solution of the field equations*

Here there are six unknowns, viz. R, A, β, V, ρ and λ, involved in four field equations. In order to overcome the under-determinacy, we consider

$$A = R^n, \tag{15}$$

and following Rao et al. [37] and Mohanty and Pradhan [4] we further consider

$$V'^2 - M^2V^2 = 0\,. \tag{16}$$

On integration, Eq. (16) yields either

$$V = k\mathrm{e}^{Mt}, \tag{17}$$

or

$$V = k\mathrm{e}^{-Mt}, \tag{18}$$

where $k\,(\neq 0)$ is a constant of integration. Using Eqs. (15), (17) and (18) in Eq. (14), we get

$$R'(n+3) = 0\,. \tag{19}$$

Equation (19) yields three cases:

Case 1: $n + 3 = 0$,
Case 2: $R' = 0$,
Case 3: $n + 3 = 0$ and $R' = 0$,

which are studied in the following sections.

$$\textbf{Case 1}: n+3=0 \tag{20}$$

Using Eqs. (15), (17) and (18) in field equations (11) – (13), we get

$$-\frac{R''}{R}+7\left(\frac{R'}{R}\right)^2+\frac{3}{4}\beta^2=-\chi M^2k^2\mathrm{e}^{\pm 2Mt}, \tag{21}$$

$$3\frac{R''}{R}+3\left(\frac{R'}{R}\right)^2+\frac{3}{4}\beta^2=-\chi M^2k^2\mathrm{e}^{\pm 2Mt}+\chi\lambda\,, \tag{22}$$

$$6\left(\frac{R'}{R}\right)^2+\frac{3}{4}\beta^2=-\chi\rho\,. \tag{23}$$

In this case, there are three field equations and four unknowns. To solve the field equations, one more relation connecting these variables is needed. Therefore we assume here

$$\rho=(1+\omega)\lambda \qquad \text{(Takabayasi string)}. \tag{24}$$

Using Eq. (24) in the field equations (21) – (23) we get either

$$R=k_2\exp\left(\chi\,\frac{k^2}{4(5+4\omega)}\,\mathrm{e}^{2Mt}+k_1t\right), \tag{25}$$

$$A=\left\{k_2\exp\left(\chi\,\frac{k^2}{4(5+4\omega)}\,\mathrm{e}^{2Mt}+k_1t\right)\right\}^{-3}, \tag{26}$$

or

$$R=k_2\exp\left(\chi\,\frac{k^2}{4(5+4\omega)}\,\mathrm{e}^{-2Mt}+k_1t\right), \tag{27}$$

$$A=\left\{k_2\exp\left(\chi\,\frac{k^2}{4(5+4\omega)}\,\mathrm{e}^{-2Mt}+k_1t\right)\right\}^{-3}, \tag{28}$$

where k_2 is a non-zero integration constant.

In these cases, the structure of the model is described by the metric

$$\mathrm{d}s^2=-\mathrm{d}t^2+\left\{k_2\exp\left(\chi\,\frac{k^2}{4(5+4\omega)}\,\mathrm{e}^{2Mt}+k_1t\right)\right\}^2\left(\mathrm{d}x^2+\mathrm{d}y^2+\mathrm{d}z^2\right)$$
$$+\left\{k_2\exp\left(\chi\,\frac{k^2}{4(5+4\omega)}\,\mathrm{e}^{2Mt}+k_1t\right)\right\}^{-6}\mathrm{d}m^2, \tag{29}$$

and

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + \left\{ k_2 \exp\left(\chi \frac{k^2}{4(5+4\omega)} \mathrm{e}^{-2Mt} + k_1 t \right) \right\}^2 \left(\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2 \right)$$

$$+ \left\{ k_2 \exp\left(\chi \frac{k^2}{4(5+4\omega)} \mathrm{e}^{-2Mt} + k_1 t \right) \right\}^{-6} \mathrm{d}m^2, \tag{30}$$

respectively. The rest energy density (ρ), string tension density (λ), the particle density (ρ_p), the gauge function (β), the scalar expansion (θ) and the shear (σ) for the models (29) and (30) are given by

$$\rho = \frac{4(1+\omega)}{5+4\omega} M^2 k^2 \mathrm{e}^{2Mt}, \tag{31}$$

$$\lambda = \frac{4}{5+4\omega} M^2 k^2 \mathrm{e}^{2Mt}, \tag{32}$$

$$\rho_p = \frac{4\omega}{5+4\omega} M^2 k^2 \mathrm{e}^{2Mt}, \tag{33}$$

$$\frac{3}{4}\beta^2 = -6 \left(\frac{\chi k^2 M}{10+8\omega} \mathrm{e}^{2Mt} + k_1 \right)^2 - \frac{4(1+\omega)}{5+4\omega} \chi M^2 k^2 \mathrm{e}^{2Mt}, \tag{34}$$

$$\theta = 0\,, \tag{35}$$

$$\sigma^2 = \frac{1}{2} \left[\frac{4}{9} + 3 \left(\frac{\chi M k^2}{2(5+4\omega)} \mathrm{e}^{2Mt} + k_1 + \frac{1}{3} \right)^2 + \left(\frac{3\chi M k^2}{2(5+4\omega)} \mathrm{e}^{2Mt} + 3k_1 + \frac{1}{3} \right)^2 \right], \tag{36}$$

and

$$\rho = \frac{4(1+\omega)}{5+4\omega} M^2 k^2 \mathrm{e}^{-2Mt}, \tag{37}$$

$$\lambda = \frac{4}{5+4\omega} M^2 k^2 \mathrm{e}^{-2Mt}, \tag{38}$$

$$\rho_p = \frac{4\omega}{5+4\omega} M^2 k^2 \mathrm{e}^{-2Mt}, \tag{39}$$

$$\frac{3}{4}\beta^2 = -6 \left(-\frac{\chi k^2 M}{10+8\omega} \mathrm{e}^{-2Mt} + k_1 \right)^2 - \frac{4(1+\omega)}{5+4\omega} \chi M^2 k^2 \mathrm{e}^{-2Mt}, \tag{40}$$

$$\theta = 0\,, \tag{41}$$

$$\sigma^2=\frac{1}{2}\left[\frac{4}{9}+3\left(-\frac{\chi Mk^2}{2(5+4\omega)}\mathrm{e}^{-2Mt}+k_1+\frac{1}{3}\right)^2+\left(-\frac{3\chi Mk^2}{2(5+4\omega)}\mathrm{e}^{-2Mt}+3k_1+\frac{1}{3}\right)^2\right]. \quad (42)$$

$$\textbf{Case 2}: R'=0 \quad (43)$$

In this case, the metric becomes flat. The physical and kinematical quantities have the following respective expressions

$$\rho = M^2k^2\mathrm{e}^{\pm 2Mt}, \quad (44)$$

$$\lambda = 0\,, \quad (45)$$

$$\frac{3}{4}\beta^2 = -\chi M^2k^2\mathrm{e}^{\pm 2Mt}, \quad (46)$$

$$\theta = 0\,, \quad (47)$$

$$\sigma^2 = \frac{2}{3}\,. \quad (48)$$

In this case the string-tension density vanishes and $\rho = \rho_p$. This model represents a dust universe.

$$\textbf{Case 3}: R'=0 \ \text{ and } \ n+3=0 \quad (43)$$

This case leads to case 2.

4. *Conclusion*

In this paper, we have constructed five-dimensional cosmological models generated by a cloud of strings coupled with massive scalar field in Kaluza-Klein space time in the framework of Lyra manifold. The models (29) and (30) constructed in this paper are physically realistic at initial epoch. Equations (40) and (46) indicate negative value of β^2 which is acceptable. Halford [38] showed that β^2 can be considered as positive or negative for various realistic physical situations. In the mixture of mesonic string, the big-bang singularity does not occur at initial epoch. This situation is similar to that of the case studied by Mohanty and Pradhan [4] in four-dimensional space-time. In the case of expanding model represented by Eq. (29), there may be a big crunch at infinite future, since $\rho \to \infty$, $\lambda \to \infty$, $\rho_p \to \infty$, and $\beta \to \infty$ as $t \to \infty$. Thus, the model leads to unphysical situations at infinite future. In the case of a physically realistic model represented by (30), we have $\rho \to 0$, $\lambda \to 0$, $\rho_p \to 0$, and $\beta \to 0$ as $t \to \infty$.

Since σ is positive, it indicates that the shape of the universe is preserved during the evolution in the case of both models. However, both models do not admit rotation, i.e. $\omega_{ij} = 0$. It can be clearly observed for the model (29) that the scalar field increases exponentially, whereas for the model (30) the scalar field decreases exponentially. It is well known that the massive scalar field, considered in the present investigation with mass parameter M, is related to the mass m of zero-spin particles by $M = m/\hbar$. Here we observe that the models in the presence of massive scalar field are free from singularities. But in the absence of the massive scalar field, the model admits singularities discussed earlier by Rahaman et al. [34]. Hence, we may say that the presence of massive scalar field removes singularity.

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PET-DIMENZIJSKI KOZMOLOŠKI MODELI SA STRUNAMA I MASIVNIM SKALARNIM POLJEM

Postavili smo pet-dimenzijske ravninsko-simetrične modele uz prisustvo masivnog skalarnog polja u Lyraovoj geometriji, kada su masivne strune s $\rho = (1+\omega)\lambda$ (Takabayasijeve strune) izvor gravitacijskog polja. Proučavamo ulogu masivnog skalarnog polja u razvoju svemira. Raspravljamo fizičke i kinematske odlike modela.