Printed ISSN 1330–0016
Online ISSN 1333–9133
CD ISSN 1333–8390
CODEN FIZBE7

LETTER TO THE EDITOR

BIANCHI TYPE-III STRING COSMOLOGICAL MODELS WITH BULK VISCOSITY AND TIME-DEPENDENT Λ TERM

R. K. TIWARI[a] and U. K. DWIVEDI[b]

[a]*Department of Mathematics, Govt. Model Science College, Rewa (MP) India*

[b]*Department of Mathematics, Govt. Engineering College, Rewa (MP) India*

Received 2 April 2008; Accepted 15 July 2009
Online 7 October 2009

We study Einstein's field equations in Bianchi type-III string cosmological models with bulk viscosity and variable cosmological term Λ. Exact solutions of the field equations are obtained by assuming the conditions: the bulk viscosity is proportional to the expansion scalar, $\xi \propto \theta$, expansion scalar is proportional to shear scalar, $\theta \propto \sigma$, and Λ is proportional to the Hubble parameter. The corresponding physical interpretations of the cosmological solutions are also discussed.

PACS numbers: 04.20.Jb, 98.80.Cq UDC 524.83,

Keywords: Bianchi type-III string cosmological model, variable Λ, bulk viscosity, expansion scalar, shear scalar, Hubble parameter

1. *Introduction*

One outstanding problem in cosmology is the cosmological constant problem [1, 2]. Since, its introduction, its significance has been studied from time to time by various workers [3 – 5]. In modern cosmological theories, the cosmological constant remains a focal point of intrest. A wide range of observations now suggests compellingly that the universe possesses a non-zero cosmological constant [6].

On other hand, the string models have an important role in cosmology, as strings are believed to have played an important role during the early stage of the universe [7] and can generate density fluctuations which lead to galaxy formations [8]. The study of string-theory cosmological models was initiated by Letelier [9] and Stachel

[10]. Their model was used as a source for the Bianchi type-I and Kantowski-Sachs cosmologies by Letelier, Krori et al. [11]. Wang [12, 15] discussed the solutions of the Bianchi type-I – IX cosmological models for string cloud. Tikekar and Patel [16] and Chakraborty and Chakraborty [17] presented exact solutions for the Bianchi type-III cosmological models for string cloud. Recently, Singh et al. investigated Bianchi type-III cosmological models with gravitational constant G and the cosmological constant Λ [18].

In this letter, we study Bianchi type-III string cosmological models in the presence of bulk viscosity and time-dependent cosmological term Λ. To obtain an explict solution, we assume that the coefficient of the viscosity is proportional to the expansion scalar, $\xi \propto \theta$, and the expansion scalar is proportional to the shear scalar, $\theta \propto \sigma$, and for the cosmological term Λ, we assume that it is proportional to the Hubble parameter $\Lambda \propto H$.

We consider the Bianchi type-III metric in the form

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + A^2\mathrm{d}x^2 + B^2\mathrm{e}^{2x}\mathrm{d}y^2 + C^2\mathrm{d}z^2, \tag{1}$$

where A, B and C are functions of time t.

The energy-momentum tensor for a cloud of strings with bulk viscosity is [15]

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j - \xi\theta(u_i u_j + g_{ij})\,, \tag{2}$$

where $\rho = \rho_p + \lambda$ is the rest energy density of the cloud of strings with particles attached to them, ρ_p is the rest energy density of particles, λ is the tension density of the cloud of strings, $\theta = u^i_{;i}$ is the scalar of expansion and ξ is the coefficient of bulk viscosity. Letelier [9] assumed that the energy density for the coupled system ρ and ρ_p is positive, while the tension density λ may be positive or negative. The u^i is the cloud four-velocity vector and ξ^i represents the direction of anisotropy, i.e. the direction of strings. They satisfy the relation [14].

$$u^i u_i = -x^i x_i = -1, \quad u^i x_i = 0\,. \tag{3}$$

The expressions for scalar of expansion θ and shear scalar σ are

$$\theta = u^{;i}_i = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} = 3H\,, \tag{4}$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{3}\left(\frac{\dot{A}^2}{A^2} + \frac{\dot{B}^2}{B^2} + \frac{\dot{C}^2}{C^2} - \frac{\dot{A}\dot{B}}{AB} - \frac{\dot{B}\dot{C}}{BC} - \frac{\dot{C}\dot{A}}{CA}\right), \tag{5}$$

where H is the Hubble parameter.

Einstein's field equations with $8\pi G = 1$ and variable cosmological term $\Lambda(t)$ in suitable units are

$$R_{ij} - \frac{1}{2}Rg_{ij} = T_{ij} - \Lambda(t)g_{ij}\,. \tag{6}$$

For the metric (1), Einstein's field equations can be written as

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = \xi\theta - \Lambda\,, \tag{7}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} = \xi\theta - \Lambda\,, \tag{8}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} = \lambda + \xi\theta - \Lambda\,, \tag{9}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} - \frac{1}{A^2} = \rho + \Lambda\,, \tag{10}$$

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = 0\,, \tag{11}$$

where dots on A, B and C denote the ordinary differentiation with respect to t. From Eq. (11), we have

$$A = k_1 B\,, \tag{12}$$

where k_1 is the constant of integration. In order to obtain a more general solution, we assume that the coefficient of the bulk viscosity is proportional to the expansion scalar, i.e.

$$\xi = k\theta\,, \tag{13}$$

where k is a positive constant.

Now, there are five independent equations (8) – (11) and (13) in the seven unknowns A, B, C, λ, ρ, ξ and Λ. Thus, two more relations are needed to solve the system completely. We assume that one condition is that the scalar of expansion is proportional to the shear scalar $\theta \propto \sigma$, which leads to

$$B = C^n\,, \tag{14}$$

where n is a constant, and the second condition is [19, 20]

$$\Lambda = aH\,, \tag{15}$$

where H is the Hubble parameter and a is a positive constant.

Substituting Eq. (14) into Eq. (4) and using Eq. (13), we have

$$\theta = (2n+1)\frac{\dot{C}}{C}\,, \tag{16}$$

$$\xi = k(2n+1)\frac{\dot{C}}{C}\,. \tag{17}$$

With the help of Eqs. (4), (15), (16) and (17), Eq. (7) reduces to

$$\frac{\ddot{C}}{C} + \left\{\frac{n - k(2n+1)^2}{n+1}\right\}\left(\frac{\dot{C}}{C}\right)^2 = \frac{-3a(2n+1)}{n+1}\;\frac{\dot{C}}{C}\,. \tag{18}$$

Equation (18) can be rewritten as

$$\left\{\dot{\overline{\frac{\dot{C}}{C}}}\right\}\Big/\left\{\frac{\dot{C}}{C}\right\} + \left[\frac{(2n+1) - k(2n+1)^2}{(n+1)}\right]\frac{\dot{C}}{C} = \frac{-3a(2n+1)}{n+1}\,. \tag{19}$$

On integrating Eq. (19), we get

$$C = \left\{\frac{(2n+1)-k(2n+1)^2}{(n+1)}\left[\frac{-(n+1)}{3a(2n+1)}k_3\exp\left(\frac{-3a(2n+1)t}{n+1}\right) + k_4\right]\right\}^{\frac{(n+1)}{(2n+1)-k(2n+1)^2}}, \tag{20}$$

where k_3 and k_4 are constants of integration.

By using Eqs. (12), (14) and (20), the line element (1) reduces to

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + \left(k_1^2\mathrm{d}x^2 + \mathrm{e}^{2x}\mathrm{d}y^2\right) \tag{21}$$

$$\times\left\{\frac{(2n+1)-k(2n+1)^2}{(n+1)}\left[\frac{-(n+1)}{3a(2n+1)}k_3\exp\left(\frac{-3a(2n+1)t}{n+1}\right) + k_4\right]\right\}^{\frac{2n(n+1)}{(2n+1)-k(2n+1)^2}}$$

$$+\left\{\frac{(2n+1)-k(2n+1)^2}{(n+1)}\left[\frac{-(n+1)}{3a(2n+1)}k_3\exp\left(\frac{-3a(2n+1)t}{n+1}\right) + k_4\right]\right\}^{\frac{2(n+1)}{(2n+1)-k(2n+1)^2}}\mathrm{d}z^2.$$

Under suitable transformation of coordinates, the line element (21) reduces to

$$\mathrm{d}s^2 = -\mathrm{d}T^2 + \left[\exp\left(\frac{-3a(2n+1)}{(n+1)}T\right) + k_5\right]^{\frac{2n(n+1)}{(2n+1)-k(2n+1)^2}}\left(\mathrm{d}X^2 + \mathrm{e}^{2x}\mathrm{d}Y^2\right)$$

$$+\left[\exp\left(\frac{-3a(2n+1)}{(n+1)}T\right) + k_5\right]^{\frac{2(n+1)}{(2n+1)-k(2n+1)^2}}\mathrm{d}z^2, \tag{22}$$

where k_5 is a constant.

For the model (22), the expressions for the energy density ρ, the string tension density λ, the particle density ρ_p, the coefficient of bulk viscosity ξ, the expansion scalar θ, the shear scalar σ and the cosmological term Λ are, respectively, given by

$$\rho = \frac{9(n+2)na^2}{\big[k(2n+1)-1\big]^2}\left[1+k_5\exp\left(\frac{3a(2n+1)T}{n+1}\right)\right]^{-2} \tag{23}$$

$$-\frac{9(2n+1)a^2}{\big[k(2n+1)-1\big]}\left[1+k_5\exp\left(\frac{3a(2n+1)T}{n+1}\right)\right]^{-1}-\left[\exp\left(\frac{-3a(2n+1)}{(n+1)}T\right)+k_5\right]^{\frac{2n(n+1)}{(2n+1)[k(2n+1)-1]}},$$

$$\lambda = \left[1+k_5\exp\left(\frac{3a(2n+1)T}{n+1}\right)\right]^{-2} \tag{24}$$

$$\times\left\{\frac{27a^2n^2}{\big[k(2n+1)-1\big]^2}-\frac{18k_3na^2(2n+1)}{(n+1)\big[k(2n+1)-1\big]}\exp\left(\frac{3a(2n+1)}{(n+1)}T\right)-\frac{9a^2k(2n+1)^2}{\big[k(2n+1)-1\big]^2}\right\}$$

$$-\left[\exp\left(\frac{-3a(2n+1)}{(n+1)}T\right)+k_5\right]^{\frac{-2n(n+1)}{(2n+1)-k(2n+1)^2}}+\frac{9a^2(2n+1)}{\big[k(2n+1)-1\big]}\left[1+k_5\exp\left(\frac{3a(2n+1)T}{n+1}\right)\right]^{-1},$$

$$\rho_p = \left[1+k_5\exp\left(\frac{3a(2n+1)T}{n+1}\right)\right]^{-2} \tag{25}$$

$$\times\left[\frac{18a^2n(1-n)+9a^2k(2n+1)^2}{\big[k(2n+1)-1\big]^2}+\frac{18a^2nk_5(2n+1)}{(n+1)\big[k(2n+1)-1\big]}\exp\left(\frac{3a(2n+1)}{(n+1)}T\right)\right]$$

$$\times\frac{-18a^2(2n+1)}{\big[k(2n+1)-1\big]}\left[1+k_5\exp\left(\frac{3a(2n+1)T}{n+1}\right)\right]^{-1},$$

$$\xi = \frac{3ak(2n+1)}{\big[k(2n+1)-1\big]}\left[1+k_5\exp\left(\frac{3a(2n+1)T}{n+1}\right)\right]^{-1}, \tag{26}$$

$$\theta = \frac{3a(2n+1)}{\big[k(2n+1)-1\big]}\left[1+k_5\exp\left(\frac{3a(2n+1)T}{n+1}\right)\right]^{-1}, \tag{27}$$

$$\sigma = \frac{\sqrt{3}a(n-1)}{\big[k(2n+1)-1\big]}\left[1+k_5\exp\left(\frac{3a(2n+1)T}{n+1}\right)\right]^{-1}, \tag{28}$$

$$\Lambda = \frac{9a^2(2n+1)}{\left[k(2n+1)-1\right]} \left[1 + k_5 \exp\left(\frac{3a(2n+1)T}{n+1}\right)\right]^{-1}, \tag{29}$$

The volume V of the model is given by

$$V^3 = ABC\,. \tag{30}$$

For $\rho > 0$, we require $(2n+1) < 1/k$. The model has a singularity at $T = (n+1)/[3\alpha(2n+1)]\log(-1/k_5) = T_0$(say), provided $k_5 < 0$. The spatial volume is zero at T_0, the expansion scalar is infinite at $T = T_0$, which shows that the big-bang starts evolving with zero volume at $T = T_0$ with an infinite rate of expansion. The scalar factors also vanish at $T = T_0$ and hence the model has a "point type singularity" at the initial epoch. The string tension density, the particle density, the coefficient of bulk viscosity, the coefficient of shear scalar and the cosmological term diverge at the initial singularity. As T increases, the scale factors, spatial volume and expansion scalar decrease. Thus, the rate of expansion slows down with the increase in time. Also, ρ, λ, ρ_p, ξ, Λ and σ^2 decrease as T increases, As $T \to \infty$, the scalar factors and volume become constant whereas ρ, λ, ρ_p, ξ, Λ and σ^2 tend to zero. Therefore, the model would essentially give an empty universe for a large time T. $\lim_{t\to\infty} \sigma/\theta = constant$, so that the model does not approach isotropy for large value of T. Hence, the model represents the shearing and non-rotating expanding universe with the big-bang start (at $T = T_0$). In addition, we observe that when $\Lambda = 0$, the model reduces to the string model of constant coefficient of bulk viscosity, which was previously given by X. X. Wang [15].

In summary, we studied the Bianchi type-III string cosmological models with bulk viscosity and cosmological term $\Lambda(t)$. We adopt the conditions $\xi \propto \theta$, $\theta \propto \sigma$ and $\Lambda \propto H$ [15, 19]. Thus, the cosmological model for a string cosmology with bulk viscosity and cosmological term is obtained, and the physical and geometrical aspects of the model are also discussed. The model describes a shearing non-rotating continuously expanding universe with a big-bang start. In the absence of cosmological term Λ, the model reduces to the Wang model [15].

References

[1] Y. J. Ng, Int. J. Mod. Phys. D **1** (1992) 145.

[2] S. Weinberg, Rev. Mod. Phys. **61** (1989) 1.

[3] Ya. B. Zeldovich, Sov. Phys. Usp. **11** (1968) 381.

[4] A. D. Linde, ZETP. Lett. **19** (1974) 183.

[5] L. M. Krauss and M. S. Turner, Gen. Rel. Grav. **27** (1995) 1137.

[6] A. G. Reiss et al., Astron. J. **607** (2004) 665.

[7] A. Vilenkin, Phys. Rep. **121** (1985) 263.

[8] Ya. B. Zeldovich, Mon. Nat. R. Astron. Soc. **192** (1980) 663.

[9] P. S. Letelier, Phys. Rev. D **28** (1983) 2414.

[10] J. Stachel, Phys. Rev. D **21** (1980) 217.

[11] K. D. Krori, A. K. Goswami and A. D. Purkayastha, J. Math. Phys. **36** (1995) 1347.

[12] X. X. Wang, Chin. Phys. Lett. **20** (2003) 615.

[13] X. X. Wang, Chin. Phys. Lett. **21** (2004) 1205.

[14] X. X. Wang, Chin. Phys. Lett. **22** (2005) 29.

[15] X. X. Wang, Chin. Phys. Lett. **23** (2006) 1702.

[16] L. K. Tikekar Rana Patel, Gen. Relativ. Gravit. **24** (1992) 394.

[17] N. C. Chakraborty and S. Chakraborty, Int. J. Mod. Phys. D **10** (2001) 723.

[18] J. P. Singh, R. K. Tiwari and Pratibha Shukla, Chin. Phys. Lett. **24** (2007) 12.

[19] R. Schützhold, Phys. Rev. Lett. **89** (2002) 081302.

[20] R. Schützhold, Int. J. of Mod. Phy. A **17** (2002) 4359.

BIANCHIJEV KOZMOLOŠKI MODEL TIPA-III SA STRUNAMA, VOLUMNOM VISKOZNOŠĆU I PROMJENLJIVIM ČLANOM Λ

Proučavamo Einsteinove jednadžbe polja u Bianchijevim kozmološkim modelima tipa III sa strunama, volumnom viskoznošću i promjenljivim kozmološkim članom Λ. Postigli smo egzaktna rješenja jednadžbi polja pretpostavivši uvjete: volumna viskoznost je razmjerna skalaru širenja, $\xi \propto \theta$, skalar širenja je razmjeran skalaru posmika, $\theta \propto \sigma$, i Λ je razmjerno Hubbleovom parametru. Raspravljaju se fizikalna tumačenja kozmoloških rješenja.