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THERMODYNAMICS OF PLANE-SYMMETRIC INHOMOGENEOUS UNIVERSE IN GENERAL RELATIVITY

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Thermodynamics of plane-symmetric inhomogeneous cosmological models of perfect fluid distribution with electromagnetic field is studied. The source of magnetic field is due to an electric current produced along z-axis. F_{12} is the non-vanishing component of the electromagnetic field tensor. The free gravitational field is assumed to be Petrov type-II non-degenerate. We study the thermodynamical properties of plane-symmetric inhomogeneous universe. Some physical aspects of the models are discussed and the entropy distribution is given explicity.

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1. Introduction

The standard FRW cosmological model prescribes a homogeneous and isotropic distribution of matter in the description of the present state of the universe. At the present state of evolution, the universe is spherically symmetric and the matter distribution of universe is on the whole isotropic and homogeneous. But in the early stages of evolution, it could have not had such smooth characteristics. Right after the big bang singularity, neither the assumption of spherical symmetry nor that of isotropy can be strictly valid. We consider plane-symmetry and provide an avenue to study inhomogeneities. Inhomogeneous cosmological models play an important role in our understanding of some essential features of the universe,

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such as the formation of galaxies during the early stage of evolution and process of homogenization. The early attempts of construction of such model have been done by Tolman [1] and Bondi [2] who considered spherically symmetric models. Inhomogeneous plane-symmetric models were considered by Taub [3, 4] and later by Tomimura [5], Szekeres [6], Collins and Szafron [7] and Szafron and Collins [8]. Recently, Senovilla [9] obtained a new class of exact solutions of Einstein's equation without big bang singularity, representing a cylindrically symmetric inhomogeneous cosmological model filled with perfect fluid which is smooth and regular everywhere, satisfying energy and causality conditions. Later, Ruiz and Senovilla [10] examined a fairly large class of singularity-free models through a comprehensive study of general cylindrically-symmetric metric with separable function of space and time as metric coefficients. Dadhichi et al. [11] established a link between the FRW models and singularity-free family by deducing the latter through a natural and simple inhomogenization and anisotropization of the former. Recently, Patel et al. [12] presented a general class of inhomogeneous cosmological models filled with non-thermalised perfect fluid by assuming that the background space-time admits two space like commuting Killing vectors and has separable metric coefficients. Singh, Mehta and Gupta [13] obtained inhomogeneous cosmological models of perfect fluid distribution with electromagnetic field. Recently Pradhan, Yadav and Singh [14] investigated inhomogeneous cosmological models in general relativity. The occurrence of magnetic field on the galactic scale is well-established fact today, and its importance for a variety of astrophysical phenomena is generally acknowledged as pointed out by Zeldovich et al. [15]. Also, Harrison [16] suggested that magnetic field could have a cosmological origin. As a natural consequence, we should include magnetic fields in the energy momentum tensor of the early universe. The choice of anisotropic cosmological models in Einstein's system of field equations leads to the cosmological models more general than Robertson-Walker models [17]. The presence of primordial magnetic field in the early stages of evolution of the universe has been discussed by several authors [18 − 27]. Strong magnetic field can be created due to the adiabatic compressions in clusters of galaxies. Large-scale magnetic field gives rise to anisotropies in the universe. The anisotropic pressure created by the magnetic fields dominates the evolution of the shear anisotropy and it decays slower than in the case when the pressure was isotropic [28, 29]. Such field can be generated at the end of an inflationary epoch [30 − 38]. Anisotropic magnetic field models give significant contribution in the evolution of galaxies and stellar objects. Bali and Ali [35] obtained a magnetized cylindrically symmetric universe with electrically neutral perfect fluid as a source of matter. Bali and Tyagi [36] investigated a plane-symmetric inhomogeneous cosmological model of perfect fluid distribution with electromagnetic fluid. In the present paper, we revisit their solution and obtain a new solution of plane-symmetric inhomogeneous cosmological models of perfect fluid distribution with electromagnetic field. We discuss the thermodynamical properties of the model, and the entropy distribution is also given explicity. The dissipative mechanism not only modifies the nature of singularity that usually occurrs for perfect fluid, but also can successfully account for the large entropy per baryon in the present universe.

2. *The metric and field equations*

We consider the metric in the form

$$ds^2 = A^2(dx^2 - dt^2) + B^2 dy^2 + C^2 dz^2, \tag{1}$$

where the metric potentials A, B and C are functions of x and t. The energy momentum tensor is taken as

$$T_i^j = (\rho + p) v_i v^i + p g_i^j + E_i^j, \tag{2}$$

where E_i^j is the electromagnetic field given by Lichnerowicz [38] as

$$E_i^j = \bar{\mu}\Big[h_i h^j (v_i v^j + \frac{1}{2} g_i^j) - h_i h^j\Big], \tag{3}$$

where ρ and p are the energy density and isotropic pressure, respectively, and v_i is the flow vector satisfying the relation

$$g_{ij} v^i v^j = -1 \,. \tag{4}$$

$\bar{\mu}$ is the magnetic permeability and h_i the magnetic flux vector defined by

$$h_i = \frac{1}{\bar{\mu}} \, {}^\star\mathcal{F}_{ij} v^j \,, \tag{5}$$

where ${}^\star\mathcal{F}_{ij}$ is the dual magnetic field tensor defined by Synge [39],

$${}^\star\mathcal{F}_{ij} = \frac{\sqrt{-g}}{2} \, \epsilon_{ijkl} F^{kl}. \tag{6}$$

F_{ij} is the electromagnetic field tensor and is ϵ_{ijkl} the Levi-Civita tensor density. The coordinates are considered commoving so that $v^1 = 0 = v^2 = v^3$ and $v^4 = 1/A$. We consider that the current is flowing along the z-axis so that $h_3 \neq 0$, $h_1 = 0 = h_2 = h_4$. The only non-vanishing component of F_{ij} is F_{12}. The Maxwell's equations read

$$F_{ij;k} + F_{jk;i} + F_{ki;j} = 0 \tag{7}$$

and

$$\left[\frac{1}{\bar{\mu}} F^{ij}\right]_{;j} = J_i \,. \tag{8}$$

We require that F_{12} be function of x alone. We assume that the magnetic permeability is a function of both x and t. Here the semicolon represents a covariant differentiation.

The Einstein's field equations (in gravitational units $c = 1$, $G = 1$)

$$R_i^j - \frac{1}{2} R g_i^j + \Lambda g_i^j = -8\pi T_i^j \,. \tag{9}$$

For the line element (1) we find the following relations

$$8\pi A^2 \left(p + \frac{F_{12}^2}{2\bar{\mu} A^2 B^2} \right) \tag{10}$$

$$= - \frac{B_{44}}{B} - \frac{C_{44}}{C} + \frac{A_4}{A} \left(\frac{B_4}{B} + \frac{C_4}{C} \right) + \frac{A_1}{A} \left(\frac{B_1}{B} + \frac{C_1}{C} \right) + \frac{B_1 C_1}{BC} - \frac{B_4 C_4}{BC} - \Lambda A^2,$$

$$8\pi A^2 \left(p + \frac{F_{12}^2}{2\bar{\mu} A^2 B^2} \right) = - \left(\frac{A_4}{A} \right)_4 + \left(\frac{A_1}{A} \right)_1 - \frac{C_{44}}{C} + \frac{C_{11}}{C} - \Lambda A^2, \tag{11}$$

$$8\pi A^2 \left(p - \frac{F_{12}^2}{2\bar{\mu} A^2 B^2} \right) = - \left(\frac{A_4}{A} \right)_4 + \left(\frac{A_1}{A} \right)_1 - \frac{B_{44}}{B} + \frac{B_{11}}{B} - \Lambda A^2, \tag{12}$$

$$8\pi A^2 \left(\rho + \frac{F_{12}^2}{2\bar{\mu} A^2 B^2} \right) \tag{13}$$

$$= - \frac{B_{11}}{B} - \frac{C_{11}}{C} + \frac{A_4}{A} \left(\frac{B_4}{B} + \frac{C_4}{C} \right) + \frac{A_1}{A} \left(\frac{B_1}{B} + \frac{C_1}{C} \right) - \frac{B_1 C_1}{BC} + \frac{B_4 C_4}{BC} - \Lambda A^2,$$

$$0 = \frac{B_{14}}{B} + \frac{C_{14}}{C} - \frac{A_1}{A} \left(\frac{B_4}{B} + \frac{C_4}{C} \right) - \frac{A_4}{A} \left(\frac{B_1}{B} + \frac{C_1}{C} \right), \tag{14}$$

where the sub-indices 1 and 4 in A, B, C and elsewhere indicate ordinary differentiation with respect to x and t, respectively.

3. Solution of the field equation

Eqs. (10) – (12) lead to

$$\left(\frac{A_4}{A} \right)_4 - \frac{B_{44}}{B} + \frac{A_4}{A} \left(\frac{B_4}{B} + \frac{C_4}{C} \right) - \frac{B_4 C_4}{BC}$$

$$= \left(\frac{A_1}{A} \right)_1 + \frac{C_{11}}{C} - \frac{A_1}{A} \left(\frac{B_1}{B} + \frac{C_1}{C} \right) - \frac{B_1 C_1}{BC} = a \,, \tag{15}$$

where a is constant, and

$$\frac{8\pi F_{12}^2}{\bar{\mu} B^2} = \frac{B_{44}}{B} - \frac{B_{11}}{B} + \frac{C_{11}}{C} - \frac{C_{44}}{C} \,. \tag{16}$$

Equations (10) – (14) represent a system of five equations in six unknowns, A, B, C, ρ, p and F_{12}. For a complete determination of these unknowns, one extra condition is needed. As in the case of general-relativistic cosmologies, the introduction of inhomogeneities into the cosmological equations produces a considerable increase in mathematical difficulty: non-linear partial differential equations must now be solved. In practice, this means that we must proceed either by means of approximations which render the non-linearities tractable, or we must introduce particular symmetries into the metric of space-time in order to reduce the number of degrees of freedom which the in homogeneities can exploit. In the present case, we assume that the metric is Petrov type-II non-degenerate. This requires that

$$\frac{B_{11}+B_{44}+2B_{14}}{B} - \frac{C_{11}+C_{44}+2C_{14}}{C} = \frac{2(A_1+A_4)(B_1+B_4)}{AB} - \frac{2(A_1+A_4)(C_1+C_4)}{AC} . \tag{17}$$

Let us consider that

$$A = f(x)\vartheta(t), \qquad B = g(x)\mu(t), \qquad C = h(x)\mu(t) \,. \tag{18}$$

Using Eqs. (18) in (14) and (17), we get

$$\frac{\dfrac{g_1}{g} + \dfrac{h_1}{h}}{\dfrac{f_1}{f}} = \frac{\dfrac{2\mu_4}{\mu}}{\dfrac{\mu_4}{\mu} - \dfrac{\vartheta_4}{\vartheta}} = b \,, \tag{19}$$

where b is a constant, and

$$\frac{\dfrac{g_{11}}{g} + \dfrac{h_{11}}{h}}{\dfrac{g_1}{g} - \dfrac{h_1}{h}} - \frac{2f_1}{f} = 2\left(\frac{\mu_4}{\mu} - \frac{\vartheta_4}{\vartheta}\right) = L \,, \tag{20}$$

where L is a constant. Equation (19) leads to

$$f = n(gh)^{1/b}, \qquad b \neq 0 \tag{21}$$

and

$$\mu = m\vartheta^{b/(b-2)}, \tag{22}$$

where m and n are constants of integration. From Eqs. (15), (18) and (19), we have

$$\frac{1}{b}\,\frac{g_{11}}{g} + \frac{1+b}{b}\,\frac{h_{11}}{h} - \frac{2}{b}\left(\frac{g_1^2}{g^2} + \frac{h_1^2}{h^2}\right) - \frac{2+b}{b}\,\frac{g_1 h_1}{gh} = a \tag{23}$$

and

$$\frac{2}{b}\left(\frac{\mu_{44}}{\mu}+\frac{\mu_4^2}{\mu^2}\right)=-a\,. \tag{24}$$

Let us assume

$$g=\mathrm{e}^{U+W}, \qquad h=\mathrm{e}^{U-W}, \tag{25}$$

Equations (20) and (25) lead to

$$V_1=M\exp\left(Lx+\frac{2(2-b)}{b}U\right), \tag{26}$$

where M is an integration constant. From Eqs. (23), (25) and (26), we have

$$\frac{2+b}{b}\,U_{11}-\frac{4}{b}\,U_1^2-2bM\exp\left(Lx+\frac{2(2-b)}{b}\,U\right)-ML\exp\left(Lx+\frac{2(2-b)}{b}\,U\right)$$

$$+2M^3L\exp\left(2Lx+\frac{4(2-b)}{b}\,U\right)=a\,. \tag{27}$$

Equation (27) leads to

$$U=\frac{Lbx}{2(b-2)}\,, \qquad b\neq 2\,. \tag{28}$$

Equations (26) and (28) lead to

$$W=Mx+\log N\,, \tag{29}$$

where N is the constant of integration. Equation (24) leads to

$$\mu=\beta\sin^{1/2}\left(\sqrt{\alpha}\,t+t_0\right), \tag{30}$$

where $\alpha=ab$, β is constant and t_0 is a constant of integration.

Using Eqs. (21), (22), (25), (28), (29) and (30), we obtain

$$f=n\exp\left(\frac{Lx}{b-2}\right), \tag{31}$$

$$\vartheta=\gamma_0\sin^{(b-2)/(2b)}\left(\sqrt{\alpha}\,t+t_0\right), \tag{32}$$

$$g=N\exp\left(\frac{Lbx}{2(b-2)}+Mx\right), \tag{33}$$

$$h = \frac{1}{N} \exp\left(\frac{Lbx}{2(b-2)} - Mx\right), \tag{34}$$

where $\gamma_0 = (\beta/m)^{(b-2)/b}$. Therefore, we have

$$A = E \exp\left(\frac{Lx}{b-2}\right) \sin^{(b-2)/(2b)}\left(\sqrt{\alpha}\, t + t_0\right), \tag{35}$$

$$B = G \exp\left(\frac{Lbx}{2(b-2)} + Mx\right) \sin^{1/2}\left(\sqrt{\alpha}\, t + t_0\right), \tag{36}$$

$$C = H \exp\left(\frac{Lbx}{2(b-2)} - Mx\right) \sin^{1/2}\left(\sqrt{\alpha}\, t + t_0\right), \tag{37}$$

where $E = n\gamma_0$, $G = N\beta$ and $H = \beta/N$.

After using a suitable transformation of coordinates, the metric (1) reduces to the form

$$\mathrm{d}s^2 = E^2 \exp\left(\frac{2Lx}{b-2}\right) \sin^{(b-2)/b}\left(\sqrt{\alpha}\,\tau\right)\left(\mathrm{d}X^2 - \mathrm{d}\tau^2\right) \tag{38}$$

$$+ \exp\left(\frac{LbX}{b-2} + 2MX\right) \sin\left(\sqrt{\alpha}\,\tau\right)\mathrm{d}Y^2 + \exp\left(\frac{LbX}{b-2} - 2MX\right) \sin\left(\sqrt{\alpha}\,\tau\right)\mathrm{d}Z^2,$$

Here $\sqrt{\alpha}\, t + t_0 = \sqrt{\alpha}\,\tau$, $x = X$, $By = Y$ and $Cz = Z$.

The expressions for pressure p and density ρ for model (38) are given by

$$8\pi p = \frac{1}{g^2} \exp\left(\frac{2LX}{2-b}\right) \sin^{(2-b)/b}\left(\sqrt{\alpha}\,\tau\right) \tag{39}$$

$$\times \left[\alpha\left\{\frac{3b-4}{4b}\cot^2\sqrt{\alpha}\,\tau + 1\right\} + \frac{b(b+4)L^2}{4(b-2)^2} - M^2 + \frac{MLb}{b-2}\right] - \Lambda,$$

$$8\pi\rho = \frac{1}{g^2} \exp\left(\frac{2LX}{2-b}\right) \sin^{(2-b)/b}\left(\sqrt{\alpha}\,\tau\right) \tag{40}$$

$$\left[\frac{(3b-4)\alpha}{4b}\cot^2\sqrt{\alpha}\,\tau + \frac{b(4-3b)L^2}{4(b-2)^2} - M^2 + \frac{MLb}{b-2}\right] + \Lambda.$$

The non-vanishing component F_{12} of electromagnetic field tensor is given by

$$F_{12} = \sqrt{\frac{\bar{\mu}2MLb}{8\pi(2-b)}} G \exp\left\{\left(\frac{Lb}{b-2} + 2M\right)\frac{X}{2}\right\} \sin\left(\sqrt{\alpha}\,\tau\right). \tag{41}$$

The scalar expansion θ calculated for the flow vector v^i is given by

$$\theta = \frac{(3b-2)\sqrt{\alpha}}{2bE} \exp\left(\frac{LX}{2-b}\right) \sin^{(b-2)/(2b)}\left(\sqrt{\alpha}\,\tau\right) \cot\left(\sqrt{\alpha}\,\tau\right). \tag{42}$$

The shear scalar σ^2, acceleration vector v^i and proper volume V are given by

$$\sigma^2 = \frac{\alpha}{3b^2E^2} \exp\left(\frac{2LX}{2-b}\right) \sin^{(b-2)/b}\left(\sqrt{\alpha}\,\tau\right) \cot^2\left(\sqrt{\alpha}\,\tau\right), \tag{43}$$

$$\dot{v}^i = \left(\frac{L}{b-2}, 0, 0, 0\right), \tag{44}$$

$$V = \sqrt{-g} = E^2 \exp\left(\frac{(b+2)LX}{b-2}\right) \sin^{2(b-1)/b}\left(\sqrt{\alpha}\,\tau\right). \tag{45}$$

4. *Thermodynamical properties*

From the thermodynamics [37, 38], we apply the combination of the first and second law of thermodynamics to the system with volume V. As is well known,

$$T\mathrm{d}S = \mathrm{d}(\rho V) + p\mathrm{d}V. \tag{46}$$

Equation (46) may be written as

$$T\mathrm{d}S = \mathrm{d}\big[(\rho+p)V\big] - V\mathrm{d}p. \tag{47}$$

The integrability condition is necessary to define the perfect fluid as a thermodynamical system [39, 40, and 41]. It is given by

$$\frac{\partial^2 S}{\partial T \partial V} = \frac{\partial^2 S}{\partial V \partial T}, \tag{48}$$

which leads to the relation between pressure p and temperature T,

$$\mathrm{d}p = \frac{\rho+p}{T}\,\mathrm{d}T. \tag{49}$$

Plugging Eq. (49) in Eq. (47), we have the differential equation

$$\mathrm{d}S = \frac{1}{T}\mathrm{d}\big[(\rho+p)V\big] - (\rho+p)V\frac{\mathrm{d}T}{T^2}. \tag{50}$$

We rewrite Eq. (50) as

$$\mathrm{d}S = \mathrm{d}\left[\frac{(\rho+p)V}{T} + c\right], \tag{51}$$

where c is constant. Hence the entropy is defined as

$$S = \frac{\rho + p}{T} V \,. \tag{52}$$

From Eqs. (46) and (52), we get the following expression for the entropy production rate in the plane-symmetric inhomogeneous universe,

$$\frac{\mathrm{d}S}{S} = \frac{\cot\left(\sqrt{\alpha}\,\tau\right)\left[\frac{\sqrt{\alpha}}{b}T_1\right] - (3b-4)\mathrm{cosec}^2\left(\sqrt{\alpha}\,\tau\right)]}{\alpha\left[\frac{3b-4}{2b}\cot^2\left(\sqrt{\alpha}\,\tau\right)+1\right] + \frac{b(4-b)L^2}{2(b-2)^2} + \frac{2MLb}{b-2} - 2M^2}\,, \tag{53}$$

where the expression T_1 in the nominator is given by

$$\frac{\alpha(3b-4)(3b-2)}{4b}\cot^2\!\left(\sqrt{\alpha}\tau\right) + (b-1)\alpha + \frac{b(10b-b^2-3)L^2}{(b-2)^2} + \frac{MLb(3b-2)}{b-2} - M^2(3b-2).$$

It gives the rate of change of entropy with time. Clearly $\mathrm{d}S/S > 0$ implies that

$$\frac{\sqrt{\alpha}}{b}\left[\frac{\alpha(3b-4)(3b-2)}{4b}\cot^2\!\left(\sqrt{\alpha}\tau\right) + (b-1)\alpha + \frac{b(10b-b^2-3)L^2}{(b-2)^2}\right.$$

$$\left. + \frac{MLb(3b-2)}{b-2} - M^2(3b-2)\right] > (3b-4)\mathrm{cosec}\left(\sqrt{\alpha}\,\tau\right).$$

Hence $\mathrm{d}S > 0$, which implies that total entropy always increases with the change of proper time, irrespectively of the expanding model. Also $\mathrm{d}S/S \to \infty$, as

$$\cot^2\left(\sqrt{\alpha}\,\tau\right) = \frac{\alpha^{-1}\left(\frac{b(4-b)L^2}{2(b-2)^2} + \frac{2MLb}{b-2} - 2M^2\right) - 1}{\frac{3b-4}{2b}}$$

and universe becomes homogeneous.

Let the entropy density be s, so that

$$s = \frac{S}{T} = \frac{\rho + p}{T} = \frac{(1+\gamma)\rho}{T}\,, \tag{54}$$

where $p = \gamma\rho$ and $0 < \gamma \leq 1$, if we define the entropy density in terms of temperature. The first law of thermodynamics may be written as

$$\mathrm{d}(\rho V) + \gamma\rho\,\mathrm{d}V = (1+\gamma)T\,\mathrm{d}\left(\frac{\rho V}{T}\right), \tag{55}$$

which on integration yields

$$T \sim \rho^{\gamma/(1+\gamma)}. \tag{56}$$

From Eqs. (54) and (56), we obtain

$$s \sim \rho^{1/(1+\gamma)}. \tag{57}$$

The entropy of the commoving volume then varies according to

$$S \sim sV\,. \tag{58}$$

These equations are not valid for $\gamma = -1$, i.e., for the vacuum fluid. For the Zel'dovich fluid ($\gamma = 1$), we get

$$T \sim \rho^{1/2}, \tag{59}$$

$$s \sim \rho^{1/2} \sim T\,. \tag{60}$$

Thus the entropy density is proportional to the temperature. We have

$$T = T_0 \left\{ \frac{1}{E^2} \exp\left(\frac{2LX}{2-b}\right) \sin^{(2-b)/b}\left(\sqrt{\alpha}\,\tau\right) \right. \tag{61}$$

$$\left. \times \left[\frac{(3b-4)\alpha}{4b} \cot^2\left(\sqrt{\alpha}\,\tau\right) + \frac{b(4-3b)L^2}{4(b-2)^2} - M^2 + \frac{MLb}{b-2} \right] + \Lambda \right\}^{1/2},$$

$$s = s_0 \left\{ \frac{1}{E^2} \exp\left(\frac{2LX}{2-b}\right) \sin^{(2-b)/b}\left(\sqrt{\alpha}\,\tau\right) \right. \tag{62}$$

$$\left. \times \left[\frac{(3b-4)\alpha}{4b} \cot^2\left(\sqrt{\alpha}\,\tau\right) + \frac{b(4-3b)L^2}{4(b-2)^2} - M^2 + \frac{MLb}{b-2} \right] + \Lambda \right\}^{1/2},$$

$$S = S_0 \exp\left(\frac{(b+1)LX}{2-b}\right) \sin^{(3b-2)/2b}\left(\sqrt{\alpha}\,\tau\right) \times \tag{63}$$

$$\left[\frac{(3b-4)\alpha}{4b} \cot^2\left(\sqrt{\alpha}\,\tau\right) + \frac{b(4-3b)L^2}{4(b-2)^2} - M^2 + \frac{MLb}{b-2} + \Lambda E^2 \exp\left(\frac{2LX}{b-2}\right) \sin^{(b-2)/b}\left(\sqrt{\alpha}\,\tau\right) \right]^{1/2}.$$

Here T_0, s_0 and S_0 are constant. For a radiating fluid ($\gamma = 1/3$),

$$T \sim \rho^{1/4}, \tag{64}$$

$$s \sim \rho^{3/4} \sim T^3. \tag{65}$$

Thus the entropy density is proportional to the cube of temperature. Now,

$$T = T_{00}\left\{\frac{1}{E^2}\exp\left(\frac{2LX}{2-b}\right)\sin^{(2-b)/b}\left(\sqrt{\alpha}\,\tau\right)\right. \tag{66}$$

$$\left.\times\left[\frac{(3b-4)\alpha}{4b}\cot^2\left(\sqrt{\alpha}\,\tau\right)+\frac{b(4-3b)L^2}{4(b-2)^2}-M^2+\frac{MLb}{b-2}\right]+\Lambda\right\}^{1/4},$$

$$s = s_{00}\left\{\frac{1}{E^2}\exp\left(\frac{2LX}{2-b}\right)\sin^{(2-b)/b}\left(\sqrt{\alpha}\,\tau\right)\right. \tag{67}$$

$$\left.\times\left[\frac{(3b-4)\alpha}{4b}\cot^2\left(\sqrt{\alpha}\,\tau\right)+\frac{b(4-3b)L^2}{4(b-2)^2}-M^2+\frac{MLb}{b-2}\right]+\Lambda\right\}^{3/4},$$

$$S = S_{00}\exp\left(\frac{(b+1)LX}{2-b}\right)\sin^{(3b-2)/2b}\left(\sqrt{\alpha}\,\tau\right)\times \tag{68}$$

$$\left[\frac{(3b-4)\alpha}{4b}\cot^2\left(\sqrt{\alpha}\,\tau\right)+\frac{b(4-3b)L^2}{4(b-2)^2}-M^2+\frac{MLb}{b-2}+\Lambda E^2\exp\left(\frac{2LX}{b-2}\right)\sin^{(b-2)/b}\left(\sqrt{\alpha}\,\tau\right)\right]^{3/4}.$$

where T_{00}, s_{00} and S_{00} are constant.

The rates of expansion along the x-, y- and z-axes are given by

$$H_1 = \frac{A_4}{A} = \frac{(b-2)\sqrt{\alpha}}{2b}\cot\left(\sqrt{\alpha}\,\tau\right), \tag{69}$$

$$H_2 = \frac{B_4}{B} = \frac{\sqrt{\alpha}}{2}\cot\left(\sqrt{\alpha}\,\tau\right), \qquad H_3 = \frac{C_4}{C} = \frac{\sqrt{\alpha}}{2}\cot\left(\sqrt{\alpha}\,\tau\right). \tag{70}$$

We see that at the beginning stage, the rates of expansion along the x-, y- and z-axes are infinitely large. With increase in time, the expansion rate decreases. It is also observed that the expansion rate along the y- and z-axes remains the same for all possible values of b but along the x-axis expansion rate depends upon b, i.e., for $b < 0$, the rate of expansion along the x-axis will be greater than along the y- and z-axes, while for $b > 0$, the rate of expansion along the x-axis will be smaller than along the y- and z-axes.

5. *Solution in the absence of magnetic field*

When $M = 0$ and $L = 0$, we see that the magnetic field in the model (38) vanishes and the geometry of space time takes the form

$$\mathrm{d}s^2 = E^2\sin^{(b-2)/b}\left(\sqrt{\alpha}\,\tau\right)(\mathrm{d}X^2-\mathrm{d}\tau^2)+\sin\left(\sqrt{\alpha}\,\tau\right)\mathrm{d}Y^2+\sin\left(\sqrt{\alpha}\,\tau\right)\mathrm{d}Z^2. \tag{71}$$

The expressions for physical and kinematical parameters are given by

$$8\pi p = \frac{1}{E^2} \sin^{(2-b)/b}\left(\sqrt{\alpha}\,\tau\right)\alpha \left\{\frac{3b-4}{4b}\cot^2\left(\sqrt{\alpha}\,\tau\right)+1\right\} - \Lambda\,, \tag{72}$$

$$8\pi\rho = \frac{1}{E^2} \sin^{(2-b)/b}\left(\sqrt{\alpha}\,\tau\right)\left\{\frac{(3b-4)\alpha}{4b}\cot^2\left(\sqrt{\alpha}\,\tau\right)\right\} + \Lambda\,, \tag{73}$$

$$\theta = \frac{(3b-2)\sqrt{\alpha}}{2bE}\sin^{(b-2)/(2b)}\left(\sqrt{\alpha}\,\tau\right)\cot\left(\sqrt{\alpha}\,\tau\right), \tag{74}$$

$$\sigma^2 = \frac{\alpha}{3b^2E^2}\exp\left(\frac{2LX}{2-b}\right)\sin^{(b-2)/b}\left(\sqrt{\alpha}\,\tau\right)\cot^2\left(\sqrt{\alpha}\,\tau\right), \tag{75}$$

$$\dot{v}^i = (0,\,0,\,0,\,0)\,, \tag{76}$$

$$V = \sqrt{-g} = E^2\exp\left(\frac{(b+2)LX}{b-2}\right)\sin^{2(b-1)/b}\left(\sqrt{\alpha}\,\tau\right). \tag{77}$$

For the Zel'dovich fluid ($\gamma = 1$), we get

$$T = T_0\left[\frac{1}{E^2}\sin^{(2-b)/b}\left(\sqrt{\alpha}\,\tau\right)\left\{\frac{(3b-4)\alpha}{4b}\cot^2\left(\sqrt{\alpha}\,\tau\right)\right\}+\Lambda\right]^{1/2}, \tag{78}$$

$$s = s_0\left[\frac{1}{E^2}\sin^{(2-b)/b}\left(\sqrt{\alpha}\,\tau\right)\left\{\frac{(3b-4)\alpha}{4b}\cot^2\left(\sqrt{\alpha}\,\tau\right)\right\}+\Lambda\right]^{1/2}, \tag{79}$$

$$S = S_0\sin^{(3b-2)/2b}\left(\sqrt{\alpha}\,\tau\right)\left[\frac{(3b-4)\alpha}{4b}\cot^2\left(\sqrt{\alpha}\,\tau\right)+\Lambda E^2\sin^{(b-2)/b}\left(\sqrt{\alpha}\,\tau\right)\right]^{1/2}. \tag{80}$$

Here T_0, s_0 and S_0 are constant.

For a radiating fluid ($\gamma = 1/3$),

$$T = T_{00}\left[\frac{1}{E^2}\sin^{(2-b)/b}\left(\sqrt{\alpha}\,\tau\right)\left\{\frac{(3b-4)\alpha}{4b}\cot^2\left(\sqrt{\alpha}\,\tau\right)\right\}+\Lambda\right]^{1/4}, \tag{81}$$

$$s = s_{00}\left[\frac{1}{E^2}\sin^{(2-b)/b}\left(\sqrt{\alpha}\,\tau\right)\left\{\frac{(3b-4)\alpha}{4b}\cot^2\left(\sqrt{\alpha}\,\tau\right)\right\}+\Lambda\right]^{3/4}, \tag{82}$$

$$S = S_{00}\sin^{(3b-2)/2b}\left(\sqrt{\alpha}\,\tau\right)\left[\frac{(3b-4)\alpha}{4b}\cot^2\left(\sqrt{\alpha}\,\tau\right)+\Lambda E^2\sin^{(b-2)/b}\left(\sqrt{\alpha}\,\tau\right)\right]^{3/4}, \tag{83}$$

where T_{00}, s_{00} and S_{00} are constant.

6. *Concluding remarks*

We have obtained a new plane-symmetric inhomogeneous cosmological model of electromagnetic perfect fluid as the source of matter. The model (38) starts expanding with big-bang singularity at $\tau = 0$. Generally, the model represents expanding, shearing, non-rotating and Petrov type-II non-degenerate universe in which flow vector is geodetic. It is also observed that model (38) is oscillatory. In this model $\sigma/\theta = 2/\left[\sqrt{3}(3b-2)\right] = constant$, and is no approach to isotropy. The idea of primordial magnetism is appealing because it can potentially explain all large-scale fields seen in the universe today, especially those found in remote proto-galaxies. As a result, the literature contains many studies that examine the role and implications of magnetic field in cosmology. It is worth mentioning here that magnetic field affects all physical and kinematical quantities, but it does not affect the rate of expansion. Also, we see that in the absence of magnetic field inhomogeneity, the universe dies out. It signifies the role of magnetic field.

We clarify thermodynamics of plane-symmetric universe by introducing the integrability condition and temperature. All thermal quantities are derived as functions of either temperature or volume. In this case we see that the third law of thermodynamics is satisfied. Furthermore, we find a new general equation of state, describing the Zel'dovich model and radiating fluid model as function of temperature and volume. The total entropy never vanishes and goes on increasing as an evolution process. The basic equation of thermodynamics for plane-symmetric universe has been deduced and thermodynamics of the model is discussed.

The deceleration parameter of model (38) is given by

$$q = -1 + \frac{b}{2(b-1)\cos^2\left(\sqrt{\alpha}\tau\right)} \,. \tag{84}$$

The sign of q indicates whether the model inflates or not. A positive sign of q corresponds to standard decelerating model, whereas the negative sign $-1 \leq q < 0$, indicates the inflation.

Recent observations show that the deceleration parameter of the universe is in the range $-1 \leq q < 0$ and the present universe is undergoing an accelerated expansion [42, 43]. Also, the current observations of SNe Ia and CMBR favor an accelerating model ($q < 0$). From Eq. (84) it can be seen that the deceleration parameter $q < 0$ when

$$\tau > \frac{1}{\sqrt{\alpha}} \cos^{-1} \sqrt{\frac{b}{2(b-1)}} \,.$$

It follows that our model of the universe is consistent with the recent observations for $b = 0$ and $q = -1$ which is the case of the de Sitter universe. Finally, the solution presented in this paper is new and may be useful for better understanding of the evolution of universe in plane-symmetric space-time with electromagnetic field in general relativity.

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TERMODINAMIKA RAVNINSKO-SIMETRIČNOG NEHOMOGENOG SVEMIRA U OPĆOJ TEORIJI RELATIVNOSTI

Proučavamo termodinamiku ravninsko-simetričnih kozmoloških modela s perfektnom raspodjelom tekućine i elektromagnetskim poljem. Izvor magnetskog polja je električna struja u smjeru z-osi. Jedina komponenta elektromagnetskog tenzora polja je F_{12}. Pretpostavlja se nedegenerirano slobodno gravitacijsko polje Petrova tipa II. Proučavamo termodinamička svojstva ravninsko-simetričnog nehomogenog svemira. Raspravljaju se neke fizikalne osobine modela, a raspodjela entropije daje se eksplicitno.